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A

TREATISE

OF

OPTICS:

CONTAINING

ELEMENTS OF THE SCIENCE;

IN TWO BOOKS.

By JOSEPH HARRIS, Esq.

Late His MAJESTY's ASSAY-MASTER of the MINT.

LONDON:

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BIBLIOTHECA REGIA MONACENSIS

ELEMENTS OF THE SCIENCE,

CONTAINING

OPTICS:

IN TWO BOOKS.

ADVERTISEMENT.

THE Author of the following Work, in the year 1742 firſt propoſed printing *a Treatiſe upon Microſcopes* only. The plan laid down was entirely new ; and to render it more compleat, a prefatory diſcourſe was intended, to introduce ſuch general principles as were neceſſary for underſtanding the ſubject ſcientifically. This diſcourſe increaſed ſo faſt, under the Author's pen, that it was ſoon thought adviſeable to enlarge the original deſign, into a general *Treatiſe of Optics*. Two books, containing the Elementary parts of this ſcience, being finiſhed with all convenient expedition, were immediately printed off---both Plates and Letter-preſs---with uncommon care and correctneſs. And upon this foundation, it was propoſed to explain the Theory and Mechaniſm of Optical Inſtruments, in a third Book, under the general head of Teleſcopes and Microſcopes. A Taſk of this kind, required a multitude of accurate experiments ; and perhaps nobody could be better qualified for making or reaſoning upon them, than Mr. Harris. But, while this buſineſs was forwarding with unuſual celerity, our Author was called from the purſuit, by an extraordinary demand of duty in His Majeſty's Mint : And from that time, a variety of avocations, for the publick ſervice, with a ſeries of bad health, the conſequence of laborious application and ſtudy, prevented any regular renewal of the ſubject for ſeveral years. Mr. Harris however, did not loſe ſight of his favourite object, but employed moſt opportunities for the advancement of it ; till at length, finding himſelf once more at leiſure, he reſumed his application to this work with an earneſtneſs that probably haſtened his diſſolution : For, in the actual proſecution of ſeveral intereſting experiments he was taken ſick, and, unfortunately for the Publick, expired 26th September 1764.

After Mr Harris's deceaſe, a numerous collection of Manuſcripts were ſubmitted to the conſideration of ſome Gentlemen, well acquainted with the Science of Optics, in hopes of compleating ſo valuable

valuable a work. But it appeared, upon examination, that though there ſeemed to be ample materials for the purpoſe, and many of them prepared for the preſs; yet the greater part, having never been digeſted, was incapable of any arrangement, without aſſiſtance of the Author---no longer to be had---or taking up the ſubject *de novo*, and purſuing it through an almoſt infinite variety of experiments--A Labour, neither to be expected or deſired of a Stranger!

Under theſe circumſtances, the whole Treatiſe has long lain dormant, againſt the importunities of many friends, who have frequently recommended to publiſh this Elementary Part alone. They have repeatedly alledged---That the Elements of Science are always the ſame---That theſe Elements, being quite diſtinct, are compleat in themſelves---That very little has been written on the ſubject---And That the following ſheets, containing much new matter, would be a valuable acquiſition to the ſcience. Such have been the reaſons advanced for this publication, by Gentlemen whoſe names might do honour to any Book: And they have at laſt prevailed over every ſcruple of the Proprietor. But as nothing here is intended to bias the favour of the Public, the true hiſtory of this work is fairly related, and the performance ſubmitted, upon its own merit, to the candid judgment of the impartial Reader.

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A TREATISE OF OPTICS.

BOOK I.

The Elementary Part of Optics.

THE object or design of OPTICS is to explain the *Theory of Vision*, as it is founded upon known and established properties of light. These properties may be considered in the abstract, that is, without an immediate application to Vision; and this properly makes the elementary part of the science: for upon these are founded the surprising effects of all sorts of optical instruments; the principal of which are *Microscopes* and *Telescopes*. Both these enlarge the apparent magnitudes of objects: the Microscope being adapted for magnifying small objects that are near at hand; whilst the Telescope is applied to remote objects.

Optics is distinguished into two principal heads: *Catoptrics*, which treats of the reflection of light by smooth or polish'd surfaces of given figures; and *Dioptrics*, whose object is the progress of light through transparent bodies, terminated also by given surfaces. But of all the kinds of surfaces, the most useful in Optics are the *Plain* and *Spherical*, and to consider others is seldom necessary: nor is it requisite to make a formal distinction as to Catoptrics and Dioptrics, it being often more advantageous to blend them together.

SECTION I.

Of the chief Properties of Light, with an Explanation, as they occur, of some general Terms.

Art. 1. IT is manifest that rays of light are incessantly propagated in all possible directions, from every physical point of a luminous body; and when they enlighten other bodies upon which they fall, they

Light in continual motion.

they are alſo reflected and diſperſed from every part of theſe bodies. For opaque bodies ſo enlightened are * viſible to the eye, as well as thoſe that are luminous.

Ray of Light. 2. By *a Ray of light* is meant, the leaſt light, or part of light, which may be ſtopt alone without the reſt of the light, or propagated alone, or do or ſuffer any thing alone, which the reſt of the light doth not or ſuffers not †.

Light conſiſts of ſmall material Particles. 3. It is very probable that rays of light are not continued filaments or threads; but conſiſt of ſmall and diſtinct particles of matter ‖, which following in a cloſe ſucceſſion do as it were make one continued thread.

Light not inſtantaneous, but its Velocity exceeding great. 4. That light ſhould be propagated in time, and not inſtantaneouſly, is very reaſonable to ſuppoſe: And that its velocity is much quicker than that of ſound, we know, becauſe the flaſh of a gun fired at a conſiderable diſtance will be ſeen ſome time before the report is heard **. The *Aſtronomers* have diſcovered the velocity of light ‡ to be ſuch, as to reach from the ſun to us in about 8 minutes of time.

Medium. 5. *Definition.* Any *tranſparent Body*, as *Air*, *Glaſs*, *Water*, &c. alſo *empty Space*, is called a Medium: And mediums are ſaid to be *denſer* or *rarer*, as they are ſpecifically heavier or lighter. *Note*, By *a Medium is always underſtood a denſe Medium*, unleſs it be otherwiſe expreſſed.

Object, Naked Viſion, Radiant. 6. *Defin.* All bodies ſeen by rays propagated directly from them to the eye, without being at all bent out of their courſe in their paſſage, are called *Objects*. And the viſion of objects in this manner is called *Naked Viſion*. A ſmall object, or a phyſical point of an object, conſidered as propagating light towards a certain part, is ſometimes called a *Radiant* or *Radiating Point*.

7. The

* It cannot be doubted, but that light by its impulſe upon the proper organ is the mediate cauſe of viſion: and therefore wherever bodies are viſible, light muſt be propagated from them to the eye. A continued uninterrupted motion is eſſential to the very being of light, according to what ideas we can form of it; for the moment it ſtops, it ceaſes to be light to us.

† Sir *Iſaac Newton*'s Defin. Optics, page 2. Light is ſo ſubtile and fine, that it is probable the ſmalleſt part of it which we can ſtop, may yet conſiſt of ſeveral rays.

‖ How exquiſitely minute muſt theſe particles be, that myriads of them can move promiſcuouſly all manner of ways, without impinging one another! And that this is the caſe, we need not doubt, ſince different bodies and different parts of the ſame body are diſtinctly viſible at the ſame time. And how curious muſt the eye be framed, to be ſenſible of theſe ſmall impulſes, and to diſtinguiſh at the ſame time thoſe from different objects!

** It is for the ſame cauſe that a clap of thunder is not heard till ſome time after the flaſh of lightening has been ſeen.

‡ This diſcovery was firſt made by *Monſieur Romer*, who having obſerved that the eclipſes of the *Satellites of Jupiter* appeared ſooner or later than according to the theory, as the earth was nearer to or farther from *Jupiter*, concluded from thence that the progreſs of light is not inſtantaneous: And by obſerving the different times of the appearances of theſe eclipſes, according to the different diſtances of the earth from *Jupiter*, the time the ſun's light takes in coming from him to us, has been diſcovered. This hypotheſis, that light is not inſtantaneouſly propagated thro' all ſpace, is now univerſally agreed to; and is farther confirmed by a notable diſcovery of an apparent motion of the fixed ſtars, by the *Reverend* and ingenious *Dr. James Bradley*, *Reg. Profeſſor of Aſtronomy*, who has elegantly accounted for that phænomenon, by the theory of the light's motion.

7. The progreſs or path of light, whilſt it continues uninterrupted in a medium of the ſame uniform denſity, is in * ſtrait lines. For all bodies ceaſe to be viſible, when any opaque body is interpoſed between them and the ſpectator.

Thus from a radiating point Q of an object PQR, are propagated inceſſantly in ſtrait lines, and in all poſſible directions the rays Q*q*, Q*q*, Q*q*, &c. And in like manner rays of light are inceſſantly propagated from every other phyſical point of the object, as well as from Q; otherwiſe the whole object would not be viſible, and that all at once, and to all poſitions of the eye. Fig. 1.

8. A ray of light, as AC, meeting obliquely with a ſurface RS, if it be turned back into the medium from whence it came, it will take ſome new direction, as CB: And if it enters the medium RG, inſtead of proceeding directly forward in the ſtrait line ACG, it will be bent at C into ſome † new direction CD, in which it will proceed ſtrait‖ whilſt it continues in that medium. The diſpoſition of the rays of light to be turned back into the medium from whence they came, is called *Reflexibility*: as is that to be bent at their entrance into another medium, called *Refrangibility*. The changes which the rays undergo in theſe two caſes, are called *Reflection* and *Refraction*. The original or primary ray AC, is called the *incident Ray*; C *the Point of Incidence*; CB, *the reflected Ray*; and CD, *the refracted Ray*. Fig. 2.

Reflection and Refraction &c.

In like manner, if BC or DC be an incident ray, which at C is bent into CA; the *Ray* CA is accordingly called the *reflected or refracted Ray*.

9. Let PCO be a ſtrait line perpendicular to the ſurface RS at the point of incidence C; the angle ACP which the incident ray makes with this perpendicular, is called the *Angle of Incidence*; BCP the *Angle of Reflection*; and DCF the *Angle of Refraction*. From the point of incidence C, with any radius deſcribe a circle cutting the incident and refracted rays in A and D; and from the points A, D upon the line PO let fall the perpendiculars AE, DF. Now the line AE is the Sine of the angle of incidence; and DF is the ſine of the angle of refraction. The line AE for ſhortneſs of expreſſion is called *the Sine of Incidence*, and DF *the Sine of Refraction*.

Sines of Incidence and Refraction.

* The propagation of light is in this reſpect different from that of ſound, which ſeems to ſuffer no obſtruction by the crookedneſs of its paſſage; whereas no object can be ſeen through a crooked pipe. Sound is undoubtedly propagated to the ear by an undulating motion of the air, (like that of water after a ſtone has been thrown into it) cauſed by the tremor or vibrating motion of the ſounding body; and not by any particles of matter emitted from that body.

† The advantages we derive from the refrangibility of light are ineſtimable: without this property in light, we ſhould have figured glaſſes in vain, and Teleſcopes and Microſcopes would not have exiſted.

‖ It ſeems that light is not reſiſted, at leaſt ſenſibly, by any medium through which it paſſeth; for if it was, its motion would be curvilineal, and not ſtrait.

The following *Axioms* about Light, are by repeated experiments found to be general without any exception; and are by ſome called the *Laws of Reflection and Refraction.*

10. Axiom I. *The Angles of Reflection and Refraction lie in one and the ſame Plane with the Angle of Incidence.* That is, AC, CB, CD and the perpendicular PO are all in the ſame plane.

11. Axiom II. *The Angle of Reflection* (BCP, or *b*CP) *is* * *equal to the Angle of Incidence.* (ACP or *a*CP.)

12. *Corol.* Hence it follows, that when the incident ray is perpendicular to the reflecting ſurface, it ſhall be reflected directly back along the ſame perpendicular. For in that caſe the Angles ACP, BCP both vaniſh, and AC, BC coincide with PC.

13. Axiom III. *If the Reflected or Refracted Ray* (BC or DC) *be returned directly back to the Point of Incidence* (C); *it ſhall be reflected or refracted into the ſame Line* (CA) *before deſcribed by the incident Ray.*

14. Axiom IV. *Refraction out of the rarer Medium into the denſer, is made towards the Perpendicular; that is, ſo that the Angle of Refraction be leſs than the Angle of Incidence. And the converſe.* That is, the angle of incidence ACP from a rare medium, as air, into a denſer RSO, is greater than the angle of refraction DCO. And if DC be an incident ray, the angle of incidence DCO ſhall be leſs than the angle of refraction ACP.

15. Axiom V. *The Sine of Incidence is either accurately, or very nearly, in a given Ratio to the Sine of Refraction* †. That is, any other incident ray *a*C will be ſo refracted into C*d*, that whatever ratio there be between

* This ſeems to agree with the law of reflection of hard bodies of a ſenſible bulk.

† The refraction of light at the ſurfaces of tranſparent bodies was taken notice of by the Ancients. *Ariſtotle* has a problem concerning the apparent curvity of an oar in water: And *Archimedes* is ſaid to have written a book about the appearance of a ring or circle under water. *Alhazen* the *Arabian* and *Vitellio* thought, that the angles of incidence and refraction were in a given ratio; but this proportion being found erroneous in large angles, the Moderns began to examine the matter more ſtrictly. *Kepler*, among the reſt, made ſeveral experiments about it; and tho' he miſs'd his aim, his attempts and conjectures became uſeful to others. After the invention of Teleſcopes, the ſubject of this enquiry being thought more valuable than before, was farther purſued; and *Willebrardus Snellius* (a *Dutchman*) found out the truth: but ſtill he did not throughly comprehend his own invention. This ſhort hiſtory is extracted from *M. Huygens*'s *Dioptricks*, pag. 1, 2. To which may be added, that *Des Cartes*, ſome time after it had been diſcovered by *Snellius*, was the firſt who publiſhed, that *the Sines of Incidence and Refraction* (and not their angles) *are in a given ratio.*

The cauſe of refraction ſeems to be, an attraction of light by the ſurfaces of mediums of greater denſities: Upon this ſuppoſition, the reaſon

tween the ſines AE, DF; there will be the ſame ratio between the ſines *ae*, *df*: i. e. AE : DF :: *ae* : *df*.

16. *Corol.* 1. Hence it follows, that if the angles of incidence be increaſed or diminiſhed, the angles of refraction will be alſo increaſed or diminiſhed; but not in the ſame proportion, becauſe angles and their ſines have not the ſame ratio; ſmaller angles and their ſines being nearer to a ratio of equality, than larger angles and their ſines. Alſo if two angles of incidence be equal to each other, the angles of refraction ſhall be likewiſe equal to each other.

17. *Corol.* 2. If the incident ray is perpendicular to the refracting ſurface, as PC, it will then undergo no refraction, but proceed directly in the ſtrait line PCO. For the angle of incidence being now vaniſhed, the angle of refraction muſt alſo vaniſh.

18. *Corol.* 3. If the ratio between the ſines of incidence and refraction at the ſurface of a given medium, be known, and an angle of incidence ACE be given; the angle of refraction FCD may be thus found: About the point of incidence C, with any radius, deſcribe a circle cutting the incident ray AC ſomewhere in A; and from A upon the perpendicular PCO to the refracting ſurface, let fall the perpendicular AEB; then make AE to EH, as the ſine of incidence to the ſine of refraction; and through H draw HD parallel to PO, and where this line cuts the circle as in D, is a point through which the refracted ray CD will paſs. Or, the angle FCD may be found by a table of ſines, thus: *Sine of Incidence* : *Sine of Refraction* :: *Sine* ∠ ACE : *Sine* ∠ FCD.

19. *Scholium.* A ray of light as AC falling upon a ſpherical ſurface ECF, either convex as *Fig.* 3. or concave as *Fig.* 4. will be reflected or refracted after the ſame manner, as it would be by a plane RS touching the ſpherical ſurface at the point of incidence C. For in that caſe a line PO, which is perpendicular to the ſpherical ſurface ECF, will be alſo perpendicular to the plane RS; and therefore in both caſes, the angle of incidence ACP being the ſame, the angles of reflection and refraction will be the ſame alſo. Fig. 3, 4.

20. It is found by experience, that if the refraction be made out of air into water, the ſine of incidence (AE) is to the ſine of refraction (FD) as 4 to 3: If out of air into glaſs, the ſines are as 3 to 2 nearly; or more accurately as 17 to 11, or 14 to 9. And reverſely, if the refraction

ſon is manifeſt why light at its entrance into a denſe medium is refracted nearer to the perpendicular, and farther from the perpendicular at its entrance into a rarer medium. And from the ſame principle Sir *Iſaac Newton* hath ſhewed, why the refractions are in the ratio of the ſines. *Newton*'s *Opticks*, 3d Edition, pag. 68 to 70.

refraction be made from water into air, the ſines will be as 3 to 4: if from glaſs into air, as 2 to 3, or 11 to 17 *.

Fig. 2. 21. *Corol.* Hence if the refraction be from water or glaſs into air, and the angle of incidence GCF be greater than about 48½ degrees in water, or greater than about 40 deg. in glaſs; the ray GC will not be refracted into air, but will be reflected into the line C*g*, making the angle of reflection *g*CF = GCF the angle of incidence: Becauſe the ſines of 48½ deg. and 40 deg. are to the radius (C*k*) as 3 to 4, and as 11 to 17 nearly; and therefore when the ſine G*h* has a greater proportion to the radius than as above, the ray will not be refracted.

22. *Scholium.* The ſurfaces of bodies, as well thoſe that are tranſparent, as opaque, being for the moſt part uneven; it neceſſarily follows that abundance of light is diſſipated by bodies even the moſt tranſparent: Some being reflected and ſome refracted towards different parts by their uneven ſurfaces, whilſt ſome is reflected and ſome refracted pretty uniformly. This accounts, why a perſon can ſee through water, and his image reflected by it at the ſame time: Why, in the duſk, the furniture in a room may be ſeen by the reflection of a window glaſs, whilſt objects that are without are ſeen through it.

[22.] Sir *Iſaac Newton* † hath ſhewed that the reflection and refraction of light are probably both owing to the ſame power or cauſe differently exerciſed in different circumſtances. And that this power exerts itſelf at ſome diſtance from the reflecting ſurface; ſo that light is not reflected by impinging on the ſolid parts of bodies, but by ſome principle, which is uniformly diffuſed to a certain diſtance from the body. And this accounts why poliſh'd ſurfaces reflect light ſo uniformly as they do, which otherwiſe would be inexplicable. For the beſt poliſhed ſurface is full of ſcratches or fiſſures made by the powder wherewith it was poliſhed, and therefore utterly incapable of reflecting light uniformly, if reflection was made by the impulſes of light upon the ſolid parts of bodies. Nevertheleſs we find that rough or uneven ſurfaces reflect light all manner of ways; and that the ſmoother they are, the more uniform will be their reflections. The reaſon of which ſeems to be, that the reflective power reaches every where to the ſame diſtance from the body nearly; and that when the depths of the fiſſures are leſs than ſome aſſignable quantity, the reflective power may be ſo extended as to form as it were a film or ſurface round the body that ſhall be perfectly ſmooth. And hence perhaps there may be a degree of ſmoothneſs of bodies attainable by human art, beyond which no optical advantages are to be expected.

* The rays of light are not all equally refrangible, but this inequality need not in this place be conſidered. Alſo the numbers 2, 3, for expreſſing the ratio of refraction between glaſs and air, are ſufficiently accurate for our preſent purpoſe. How to find, by experiments, the refractive power of all ſorts of mediums both fluid and ſolid, is ſhewed in Sir *Iſaac Newton*'s Optical Lectures, §. 2.

† Vid. *Newton*'s Opticks, p. 237, to 244.

expected. What has been ſaid as to the reflection of bodies, may be alſo applied to their refractions.

23. *Defin.* Any parcel of rays, as QAC, QBC, or QAB conſidered apart from the reſt, is called a *Pencil of Rays.* When the baſe of the pencil is a circle, as the pupil of the eye, an optic glaſs, *&c.* or conſidered as being a circle, it is alſo called a *Cone of Rays.* That ray as QA, *fig.* 5, 9, which falls perpendicular upon any ſurface, is called the *Axis* of the pencil or cone; alſo the *Principal Ray:* [a]And this ray undergoes no change either by reflection or refraction. Fig. 1,5,9. Pencil or Cone of Rays.

The rays of any pencil before they have undergone any change by reflection or refraction, are called *Incident Rays:* afterwards they are called *Reflected* or *Refracted Rays*, according to the change they underwent. Both the incident and reflected or refracted rays, are conſidered as making one pencil. Incident and reflected or refracted Rays. [a] 12&17. I. Fig. 5, 9.

24. *Defin.* Rays proceeding from a certain point as Q, and as they go on, ſpread or recede farther aſunder, are called *Diverging Rays.* And rays, which, on the contrary, convene towards a certain point at which they would at laſt unite, if not intercepted in their paſſage, are called *Converging Rays.* *Rays* are ſaid to be *parallel* when their inclination, if they have any, is inſenſible: or, when the diſtance of the point from, or towards, which they diverge, or converge, is ſo great, that the breadth of the pencil bears no ſenſible proportion thereto. Thus, the ſolar rays are reckoned parallel, becauſe their inclination at the diſtance we are from the ſun, is altogether inſenſible. In like manner, the rays which enter the eye from an object at a mile or even leſs diſtance, may be reckoned parallel, becauſe the breadth of the eye's pupil bears no ſenſible proportion to that diſtance. Diverging, converging, and parallel Rays.

25. *Defin.* The point from which rays diverge, or towards which they converge (tho' they never arrive at it) is called their *Focus.* Every pencil of rays after reflection or refraction has two focus's, the one as Q being the *Focus of incident Rays*; and the other as *q*, the *Focus of the reflected or refracted Rays.* Theſe conſidered as being the focus's of the ſame pencil, are called *Conjugate Focus's.* Parallel rays are conſidered as having focus's at an infinite diſtance. The Line Q*q* joining the conjugate focus's, is called the *Axis of the Pencil.* The point Q from whence the rays diverge, or in which they would unite after reflection or refraction, if *q* be the focus of incident rays converging, is called a *real Focus.* But *q* in the three figures referred to, is called a *virtual* or *imaginary Focus*; becauſe the rays are not really there united *. Fig. 5,9,10. Focus, real, imaginary: conjugate Focus's. Geometrical Focus. Aberration, proper Focus.

All

* Theſe definitions are placed here, becauſe they are general; but a beginner need not dwell upon them, their meaning will be more obvious after a peruſal of part of the next ſection.

All the rays in a pencil will not be always reflected or refracted into one point, as will be ſhewed hereafter; but only thoſe rays that are equidiſtant from the axis on different ſides, all thoſe that have different inclinations having different focus's. And, ſtrictly ſpeaking, by the focus of a pencil is meant the focus of thoſe rays that are neareſt to the axis: this is alſo called the *Geometrical Focus*. And the diſtance between this focus and the *proper or reſpective Focus* of any ray, is called the *Aberration* of that ray. Thus, *fig.* 13, 14, if Q be the focus of incident rays, and *q* be the focus of thoſe rays that are next the axis after refraction, *x* the focus of any particular ray CD; then is *q* called the *Geometrical Focus*, *x* the *reſpective* or *proper Focus* of the ray CD, and *qx* its *Aberration*.

Image. 26. *Defin.* A phyſical point Q (*fig.* 5, 9, 10.) being conſidered as an object, the correſponding point *q* from or towards which the rays diverge or converge after reflection or refraction, is called the *Image* of the ſaid point Q. In like manner the images of all the ſeveral points of an object PQR, (*fig.* 6, 7, 8, 11, 12.) being taken collectively or as one image, as *pqr*, is called the *Image* of the ſaid *Object* PQR.

27. Light the farther it is propagated from any radiating point, becomes continually rarer: And if it met no obſtruction in its courſe, the decreaſe of its denſity would be as the ſquare of the diſtance from the radiating point. This is manifeſt from the principles of geometry.

Fig. 5. Let BQC be a pencil of light propagated from the point Q, and let Q*a*, Q*b* be equal reſpectively to half QB, QC. Now a circle whoſe diameter is BC will be four times as big as a circle deſcribed upon *ab*; but the quantity of light contained in theſe two circles is equal; and therefore the light at the diſtance Q*a* is four times denſer than at the diſtance QB. In like manner, if Q*e*, Q*f* be one fourth of QB, QC, the light at *ef* will be 16 times denſer than at BC. For a circle deſcribed upon *ef* contains all the light that can come into the circle deſcribed upon BC, and theſe two circles are to one another as 1 to 16: That is, *Light the farther it is propagated decreaſes in its denſity in a duplicate Ratio of the diſtance.*

28. *Corol.* Hence luminous bodies, or thoſe that ſhine with their own light, are vaſtly brighter than opaque bodies illuminated by them. For opaque bodies diſperſe the light falling upon them, alſo all manner of ways; whence, ſuppoſing that all the light was reflected, the quantity of it received by the eye from the opaque body, compared to that which it receives from the luminous body, is only as the viſible illuminated ſurface of the opaque body, to the ſurface of an hemiſphere whoſe radius is the diſtance of the opaque body from the eye; ſuppoſing the breadth of the pupil to be the ſame in both caſes, and that the ſum of the diſtances of the opaque body from the eye and from the luminous body,

differs

differs inſenſibly from the direct diſtance of the luminous body from the eye.

29. Hence again, the light of the full moon at a medium is about 100,000 times fainter or rarer than the ſun's light when at the ſame height above the horizon, all other circumſtances being the ſame alſo; as will appear by comparing the moon's apparent diſk to the ſurface of a hemiſphere. Whence it is eaſy to conceive, that ſince we can bear the ſun's heat, why we are not ſenſible of any from the moon; and much leſs can we be ſenſible of any heat from any of the other planets.

No Medium perfectly tranſparent.

30. We do not know any medium that is perfectly tranſparent. Even the air, the moſt rare and tranſparent medium we know of, is full of opaque particles that impede light; this is manifeſt from the phænomenon of a beam of light let in through a ſmall hole into a darkened chamber, which will be viſible like a *luminous Cone* from every part of the room: and this ſhews that the whole light does not go forward in its rectilineal courſe, but that at every point of the medium through which it paſſeth, ſome part of it is reflected every way; for the viſibility of the luminous cone is cauſed by this reflection. The greater faintneſs of the ſun and moon when near the horizon, than when elevated higher up, ſhews alſo that their light is more obſtructed by the longer tract of air and vapours in the one caſe, than in the other. The loſs of light in paſſing through glaſs is ſtill greater.

Hence Light decreaſes in its denſity in a greater proportion than as the ſquares of the diſtances.

31. Hence the quantity of light contained in any pencil is continually diminiſhed, the greater the diſtance from the radiant: and this diminution is more or leſs, according to the more or leſs opacity of the medium through which it paſſes. This makes it impoſſible to aſſign exactly the different proportions of the denſity of light, at different diſtances from the radiant; but in all caſes, the decreaſe of the denſity of light is greater than as the ſquares of the ſaid diſtances. And this is the cauſe why objects appear leſs bright the farther they are from the ſpectator. For if none of the rays were ſtopt in their paſſage, the degree of brightneſs of the picture of an object upon the retina, would be the ſame at all diſtances between the eye and object, ſuppoſing the aperture of the eye's pupil to remain the ſame. For in that caſe the magnitude of the picture upon the retina, and the denſity of the light would increaſe or decreaſe together in the ſame proportion, *viz.* reciprocally as the ſquares of the diſtances of the eye from the object; and therefore the denſity of the light upon the retina would be invariable at all diſtances *.

* For example; when the eye approaches as near again to the object, the picture upon the retina becomes quadruple; and the quantity of light received from the object thro' the ſame aperture

Luminous bodies ſhining in the dark, as a *lamp* or *torch*, &c. emit to very great diſtances much more light than in that caſe is neceſſary for viſion. And tho' their light ſuffers a continual diminution by the heterogeneity of the medium, the farther it goes; yet there being ſtill more left than is requiſite for viſion, their ſplendour is not ſenſibly increaſed or diminiſhed, by leſſening or increaſing their diſtances.

Light not all reflected.

32. Light is not only diverted out of its courſe by the reflection of bodies it meets with, but alſo a great part of it is frequently ſuffocated, or as it were deſtroyed by them. For it is manifeſt, that black or dark bodies reflect much leſs light than others of darker colours; and it is probable, that no bodies whatever reflect all the light they receive *. Hence the ſurface of the moon having in it ſeveral large dark ſpots, the quantity of light we receive from her muſt be leſs than according to the computation in *Art.* 29.

33. Hence the power of the eye to diſcern objects without inconvenience, by different quantities of light, is vaſtly extenſive. For not only objects of different colours placed in the ſame light, are ſeen with equal eaſe; but even objects ſo ſmall as the letters of a book may be diſtinguiſhed and read by a clear moon-light. And admitting the ſurface of the eye's pupil to be ten times greater in a weak light than in a ſtrong light, yet the proportion of the weakeſt light to the ſtrongeſt by which the eye can conveniently ſee objects, perhaps does not exceed that of 1 to 10,000 †. How exquiſite are our faculties!

Speculum. Plane and ſpherical Speculums.

34. *Defin.* Any ſmooth or poliſhed ſurface, reflecting light, is called a *Speculum*. Reflecting ſurfaces that are *flat*, as a *Looking-glaſs*, are called ‖ *Plane Speculums*; and thoſe that are *Segments of a Sphere*, are accordingly called *Spherical Speculums*; and as they are convex or concave as ECF, *fig.* 3, or *fig.* 4. they are called *convex* or *concave Speculums*.

35. *N. B.* There will be frequent occaſion to conſider the ſines of incidence and refraction; and therefore both for the ſake of brevity and perſpicuity,

aperture of the pupil at half the diſtance, is alſo quadruple: and this being equally ſpread over four times the ſurface of the retina, the light is juſt as denſe as before when the object was at twice the diſtance.

* Light, whatever afterwards become of its component particles, ceaſes to be light as ſoon as its motion is ſtopt. Whether there be a decreaſe in the velocity of light, is impoſſible for us to know from any experiment. But it cannot well be doubted, that ſince light is continually propagated from the ſun and other ſelf-ſhining bodies, ſo it muſt alſo ſuffer a continual waſte and decay, to make room for theſe continual ſupplies.

† By a well-contrived experiment M. *Bouguer* found full moon-light at a medium to be about 300,000 times weaker than the ſun's light. Vide the learned Dr. *Smith*'s Remark 96. upon his compleat ſyſtem of Optics. But it is to be obſerved, that ſky-light, which properly is day-light, is always much weaker than the direct ſun's light, and the degrees of it do not only differ conſiderably, at different times of the year, but alſo frequently in the ſame day; and thoſe different degrees are not eaſily meaſured, ſo as to be reduced to numbers. However, it is manifeſt that the difference of the degrees of light ſuitable for viſion, is very conſiderable.

‖ Theſe are alſo called *Mirrours*, but Opticians generally make uſe only of the word *Speculum*.

perſpicuity, we ſhall often abbreviate theſe expreſſions, by calling the *Sine of Incidence*, I; and the *Sine of Refraction*, R. And when the ratio between theſe ſines is the only thing under conſideration, without regarding which is the ſine of incidence, or which the ſine of refraction, we ſhall call the *greater* of theſe ſines, *m*; and the *leſſer* of them, *n*. Hence, and from *Art.* 14. if the incidence is from a rare medium into a denſe, it will be I : R :: *m* : *n*; and when the incidence is from a denſe medium into a rare, it will be I : R :: *n* : *m*. If the refraction be betwixt air and glaſs, *m* : *n* :: [a]3 : 2; if betwixt air and water, *m* : *n* :: [a]4 : 3. [a] Art. 20.

36. *Lemma.* Small arches and their ſines are very near in a ratio of equality *, and therefore have the ſame ratio to each other, as their ſines nearly; that is, if the Arches DO, *d*O, are but ſmall, it will be DO : *d*O :: DF : *df* nearly. And the ſmaller are the arches, the leſs will be the error in ſuppoſing them equal to their ſines. Thus, if the arch DO does not exceed 20 *min.* and the radius CD or CO be divided into 10,000,000 of equal parts; the exceſs of the arch DO above its ſine DF will be leſs than one of thoſe parts. Fig. 2.

Alſo the verſed ſines OF, O*f* of very ſmall arches OD, O*d* are as the ſquares of the ſines FD, *fd* of thoſe arches nearly. For OF : FD :: FD : FP; and O*f* : *fd* :: *fd* : *f*P; but when the arches OD, O*d* are very ſmall, FP and *f*P will be nearly in a ratio of equality, and therefore OF : O*f* :: FD^2 : *fd*^{2} nearly.

SECTION II.

Of the Reflection and Refraction of a Pencil of Rays at a plane Surface.

§. 1. *Of Reflection at a plane Surface.*

PROPOSITION I.

37. *A Pencil of rays diverging from a point* Q, *will after reflection by a plane ſpeculum* BAC, *diverge in ſuch a manner, that if the reflected rays were produced backwards, they would all meet in a point* q, *taken in the perpendicular* QA *to that plane produced, as far below* A *as* Q *is above it; that is,* QAq *is a ſtrait line, and* qA = AQ. Fig. 5.

* Mathematicians will at firſt ſight underſtand this lemma; and this is not a proper place to prove propoſitions purely geometrical. And tho' the lemma is never ſtrictly true till the arch vaniſhes, yet as by diminiſhing the arch the error may be reduced to a leſs quantity than any aſſignable one, no objection can lie againſt it.

Let QB be any incident ray, and let QA, BF be each perpendicular to the plane or ſtrait line BAC, and make the angle DBF equal to
[a] 11. I. the angle QBF; then will BD be the [a]reflected ray. Produce DB backwards until it meets QA produced in q; I ſay q is the focus of all the rays diverging from Q upon BAC, and that $Aq = AQ$. For BF, Qq being [b]parallel, the angle $QBF = {}^{c}\angle BQq = {}^{b}FBD = {}^{c}BqQ$;
[b] Conſtruct. [c] 29. I *Eucl.* wherefore the right-angled triangles BAQ, BAq having beſides the right angles at A, the ∠s BQA, BqA equal, and the ſide BA common;
[d] 26. I *Eucl.* the ſides AQ, Aq are alſo [d]equal. The ſame argument holds good with reſpect to any other incident ray, as QC; and therefore all the rays diverging from Q, will after reflection by the plane BC, belong to the focus q. *Q. E. D.*

38. *Corol.* 1. Hence, and from *Art.* 13. if q be the focus of incident rays, Q will be the focus of the reflected ones; that is, if rays DB, EC, *&c.* converge towards the focus q, they will all meet in Q after reflection.

39. *Corol.* 2. If the ſpeculum BC be inclined a little, as into the poſition kCK, ſo that CH ſhall be a perpendicular thereto at the point C; the incident ray QC ſhall be reflected into CI, ſo that the ∠ ECI contained between the two reflected rays CE, CI at both poſitions of the ſpeculum ſhall be double the angle GCH, the inclination given the ſpeculum. And hence, *Any error in the figure of a ſpeculum, as well as any inclination given it, produces a double error in its effect.*

PROP. II.

40. *Images formed by the reflection of a plane ſurface, are equal and ſimilar to the objects; and are at the ſame diſtance from, and have a like ſituation with reſpect to the backſide of the plane; that the objects are from, and have with reſpect to the foreſide.*

Fig. 6, 7, 8. For the images p, q, r of the radiating points P, Q, R [a]being in the perpendiculars PBp, Q Aq, RCr to the reflecting plane BC, and at
[a] 37. I. the ſame diſtance from one ſide of the plane, [a]that the radiants are from the other; that is, ſo that Bp, Aq, Cr are equal reſpectively to BP, AQ, CR: And this being true of the images of every point of the object PQR, it is manifeſt that the whole image p, q, r is equal and ſimilar to the object PQR; and has a like ſituation with reſpect to one ſide of the plane, that the object has with reſpect to the other. *Q. E. D.*

41. *Corol.* 1. If the poſition of the object PQR, and of the eye e, be given, ſuppoſing the pupil of the eye to be only a point *, the parts of

* In the figures the breadth of the pupil is taken in, and it is manifeſt that it receives light from a larger part of the ſpeculum than it would do if it was no bigger than a point. And in the next

of the ſpeculum *a*,*b* which reflect to the eye the rays proceeding from any points P,R of the object, may be found by drawing from the correſponding points *p*,*r* of the image the ſtrait lines *pae*, *rbe*. For the rays diverging from the points P,R upon the parts *a*,*b*, do after reflection belong to the [a]focus's *p*,*r*.

[a] 37. I.

Or the point *a* of the ſpeculum which reflects to the eye at *e*, the rays diverging from P, may be found without having the place of the image *p*: Thus, upon the ſpeculum let fall the perpendiculars PB, *eu*. Now if *a* be a point from whence the reflected ray comes to the eye, the angles P*a*B, *eau* will be equal. Wherefore PB : B*a* :: *eu* : *au*; and therefore PB + *eu* : B*a* + *au* (B*u*) :: *eu* : *au*. After the ſame manner may be found the point *b*, which reflects to the eye a ray which came from R, *&c.*

42. *Corol.* 2. The eye at *e* will ſee the whole object PQR by rays reflected from the part of the ſpeculum contained between *a*,*b*, and from no other part; all the reſt of the ſpeculum in this caſe being uſeleſs. But if any part of *ab* be covered, ſome part of the image will be accordingly loſt. If either the eye or the object change their poſitions, the rays that come to the eye will be reflected from a different part of the ſpeculum, which part may be found as in the preceding *Article*.

43. *Corol.* 3. The diſtance of any point of the image from the eye, is equal to the incident and reflected rays taken both together; that is, *pe*, or *re*, is equal to P*a*+*ae*, or R*b*+*be*. And tho' with reſpect to the different ſides of the ſpeculum, the object and image have a like ſituation; yet to a ſpectator who firſt looks at one and then at the other, the caſe will be different.

44. *Corol.* 4. If *ab* be the breadth of the ſpeculum, and *e* the place of the eye; the breadth of the images that can be ſeen by the eye at *e*, will be contained between the lines *ea*, *eb* produced indefinitely. And therefore having the diſtances and ſituations of objects before the ſpeculum given, the extent of thoſe that are viſible by the reflection of a ſpeculum having a given breadth, may be found from the extent of the viſible image.

45. *Corol.* 5. The nearer is the eye, and the leſs oblique to the ſpeculum, the greater will be the field of viſible images.

46. *Corol.* 6. If an object PR be placed parallel to a plane ſpeculum BC, and two lines as P*p*, P*r* be drawn from any point P thereof to the extremities of its image *pr*; the part BA of the ſpeculum included

Fig. 6.

next article, the eye will ſee the object by rays reflected from a ſmaller part of the ſpeculum, than it could do if its pupil was only a point. The differences in both caſes is manifeſt from the figures.

cluded between theſe two lines will be equal to half the length of the object or of its image *pr*. For the triangles PBA, P*pr* being ſimilar, and PB being equal to half P*p*, it will be BA $=\frac{1}{2}$ *pr* $=\frac{1}{2}$ PR. Whence,

47. *Corol.* 7. A perſon viewing himſelf in a plane looking-glaſs placed upright, will ſee his image compleat in a part of the glaſs, whoſe length and breadth is equal to half the length and breadth of the correſponding parts of his own body. And this will be always the caſe at whatever diſtance he ſtands from the glaſs.

48. *Corol.* 8. *A ſpectator will ſee his own image as far beyond the ſpeculum, as he is before it; and as he moves to or from the ſpeculum, his image will at the ſame time, move towards or from him on the other ſide, but apparently with a double velocity, becauſe the two motions are equal, and contrarily. In like manner, if while the ſpectator is at reſt, an object be in motion, its image behind the ſpeculum will be ſeen to move at the ſame rate. And if the ſpectator moves, the images of objects that are at reſt will appear to approach or recede from him, after the ſame manner as when he moves towards real objects.

49. *Corol.* 9. Whatever has been ſaid of the image of any object, or primary radiant, holds alſo true of the image of another image, &c. whence by two † plane ſpeculums duly placed, a multiplication of images may be made; and the diſtance of any image will be equal to the length of a ray propagated from the primary radiant through all the intermediate reflections to the eye.

§. 2. *Of Refraction at a plane Surface.*

PROP. III.

Fig. 9, 10. 50. *If* Q *be the focus of incident rays upon a refracting plane* BAC, *and a point* q *be taken in the perpendicular* QA *(produced if need be) to that plane, ſo that* Aq *ſhall be to* AQ, *as the ſine of incidence to the ſine of refraction; or if it be made* R : I :: AQ : Aq, *the point* q *will be the focus of thoſe rays that are neareſt to the perpendicular* QA *after refraction: that is, if the points of incidence* B, C *of the rays* QB, QC *are not remote from* A; *the refracted rays* Be, Cf *produced backwards would meet in* q *nearly.*

Let QC be any incident ray, and C*f* the refracted ray, which produce backwards until it meets the perpendicular AQ ſomewhere, as in *q*. Through the point of incidence C draw EH parallel to QA; then will [a]29.I.*Eucl.* QCE $=$ [a] AQC be the angle of incidence, and HC*f* $=$ [a] A*q*C be the angle

* It is here preſuppoſed that images formed by plane ſpeculums, appear in their true places; and this ſuppoſition is confirmed by univerſal experience. *Vide* Book II. Sect. 3.

† *Vide* Ibid.

angle of refraction. And in the triangle QqC, it will be *As the ſine of* the $\angle$ CQq (AQC), is *to the ſine of the* $\angle$ CqQ (AqC*) *ſo* Cq *to* CQ: that is, I : R :: [b]Cq : CQ. But when the angle of incidence AQC is very ſmall, it will be Cq : CQ :: †Aq : AQ nearly; that is, I : R :: Aq : AQ nearly. And the nearer any incident ray as Qb, or Qa is to the perpendicular QA, the nearer will the ſides qb, Qb, or qa, Qa of the $\triangle$s Qqb or Qqa approach to a †ratio of equality with qA, QA until at length they coincide. And therefore the point q found as above, is without ſenſible error, the focues of the refracted rays. *Q. E. D.*

[b] Conſtruct.

51. *Corol.* 1. If q be the focus of converging rays (eB, fC, *&c.*) Q will be their focus after refraction.

52. *Corol.* 2. Becauſe Aq : AQ :: [a]I : R; it will be, Qq (Aq ~ AQ) : Aq :: $m-n$ (I ~ R) : m or n (I) according as the incidence is into a rare or denſe medium. And if the point of incidence of a ray QC is ſo near to A, that the angles and their ſines may be reckoned proportional, it will be

[a] 50. I.

$\left\{\begin{matrix} m \\ n \end{matrix}\right\}$ (I) : $m-n$ :: $\angle$ *Incidence* AQC : *refracted* $\angle$ Qcq = bCf.

53. *Corol.* 3. The image q of a ſmall object Q placed under water is one fourth nearer to the ſurface than the object. And hence the bottom of a pond of water is one third part deeper than it appears to a ſpectator above. For Aq being [a] $\frac{3}{4}$ of AQ; qQ is $\frac{1}{3}$ of Aq. This may be an uſeful caution to perſons who are not ſwimmers, to prevent their plunging out of their depths unwarily ‖.

Fig. 10. Water is $\frac{1}{3}$ deeper than it appears.

[a] 20 & 52 I.

Hence we have the reaſon of the common phænomenon of a ſhilling or other object placed in an empty veſſel, appearing to be elevated higher and higher as the veſſel is filling with water. Suppoſe the veſſel empty, Bk its ſide, and Q the object at the bottom: If the eye be at e, the object will be hid by the ſide Bk; but by filling the veſſel with water, it will become viſible, and be ſeen at q, the ray QB being refracted into Be. And if the eye be ſo placed as to ſee the object at Q when the veſſel was empty, while it is filling the object will appear to riſe gradually in the line Qq.

Why an object at the bottom of an empty veſſel appears elevated as the veſſel is filling with water.

54. *Corol.* 4. If the refracting medium is glaſs, and the angles of incidence and refraction as AQC, AqC be ſo ſmall as to be reckoned proportional to their ſines; the refracted angle QCq = fch, that is, the angle contained between the incident ray produced and the refracted ray, will be [a] $\frac{1}{3}$ of the greateſt of thoſe angles, and [a] $\frac{1}{2}$ of the leaſt of them. And hence if the angle of incidence be increaſed or diminiſhed, the refracted angle will be increaſed or diminiſhed by $\frac{1}{3}$ of that augmented

Fig. 9, 10.

[a] 20 & 52 I.

or

* By the known rules of Trigonometry.

† This is manifeſt from an inſpection of the figures.

‖ If a ſtrait ſtick be immerſed perpendicularly into water, until the part immerſed appears of equal length with the part above; theſe two parts being meaſured, will be found to be to one another, about as 4 to 3.

or diminiſhed part, if the incidence is from air to glaſs; and by $\frac{1}{3}$ if the incidence is from glaſs to air.

55. *Corol.* 5. Hence, and from *Art.* 39. the aberration ariſing from a fault in the figure of glaſs, if a ray be refracted, and the refraction be from air to glaſs, is but $\frac{1}{6}$ of the aberration of a reflected ray: And the aberration of refraction from glaſs to air is $\frac{1}{4}$ of the aberration of reflection; the angle of incidence in both the caſes of reflection and refraction being the ſame. And if the medium is water, the aberration of refraction will be $\frac{1}{8}$ and $\frac{1}{6}$ of the aberration of reflection ariſing from a fault in the ſurface.

Fig. 13, 14. 56. Scholium. *When an incident ray as* QC *is oblique to the plane refracting ſurface* AC; *the proper focus* x *of the refracted ray* CD, *will be farther from* Q *than the geometrical focus* q; that is, Qx *is always greater than* Qq. *And the more oblique is the incidence*, that is, *the greater is the angle* AQC, *the greater will be the aberration* qx *of the point* x *from* q, *the focus of thoſe rays that are next to the axis or perpendicular* QA. In each figure ſuppoſe a ſtrait line drawn from *q* to C. Now if *q* be ſuppoſed to be the focus of the refracted ray CD, it ought
[a] 50. I. to be, AQ : A*q* :: [a]CQ : C*q* :: [a]I : R. But it is manifeſt that theſe ratio's are not equal. For if QA, and *q*A be made radius's, CQ, and C*q* will be the ſecants of the unequal angles AQC, A*q*C. And becauſe it muſt be AQ : A*q* :: [a]CQ : C*x*, it is manifeſt that in *fig.* 13. C*x* is greater than C*q*; and in *fig.* 14. C*x* muſt be leſs than C*q*; and in both caſes Q*x* is greater than Q*q*. And it is manifeſt alſo, that the greater is the angle AQC, the greater will be *qx*. Therefore, *&c. Q. E. D.*

PROP. IV.

Fig. 13, 14. 57. *The focus* Q *of any incident ray as* QC *upon a refracting plane* AC, *being given*; *to find the point* x *where the refracted ray* CD *produced backwards would cut the axis or perpendicular* QA *(produced if need be) to the refracting plane.*

In the perpendicular AQ, take the point *q* ſuch that AQ : A*q* :: R : I; and through the point *q* parallel to the refracting plane AC, draw the indefinite line *qt*. Let *t* be the point where CQ (produced if need be) cuts the line *qt*: From the center C with the radius C*t* deſcribe the arch *tx*, and the point *x*, where this arch meets the perpendicular AQ, is the focus of the refracted ray CD. For the right-angled triangles
[a] 4. VI *Euc.* QAC, Q*qt* being ſimilar, it will be CQ : Q*t* :: [a]AQ : Q*q*; and there-
[b] Conſtruct. fore C*t* (CQ $\pm$ Q*t*) $=$ [b]C*x* : CQ :: A*q* (AQ $\pm$ Q*q*) : AQ :: [b]I : R, and conſequently the $\angle$ A*x*C $=$ HCD is the angle of refraction *. Therefore, *&c. Q. E. I.*

Or,

* The author of this elegant conſtruction was Dr. *Barrow*.

Or the proper focus x of the refracted ray CD, may be otherwiſe found, thus: ſay,

I : R :: *Sine* ∠ AQC : *Sine* ∠ AxC; or R : I :: *Co-ſecant* AQC : *Coſec.* AxC. then, *Co-tangent* AQC : *Co-tangent* AxC :: AQ : Ax.

58. *Lemma.* The verſed ſines of very ſmall arches of unequal circles, having the ſame right ſine common to them all, are reciprocally proportional to the diameters of theſe circles, very nearly. Fig. 13, 14.

From the centers Q, x deſcribe the arches Ca, Cb; then will Qa = QC, and $xb = x$C. I ſay the verſed ſines Aa, Ab of the unequal arches Ca, Cb having the ſame right ſine CA, are reciprocally as the radii Qa, xb nearly.

For Aa : AC :: [a]AC : 2AQ + Aa; and, [a] 13. VI. *Eu.*

Ab : AC :: [a]AC : 2Ax + Ab; therefore,

Aa : Ab :: 2Ax + Ab : 2AQ + Aa.

But when Aa and Ab are but ſmall in proportion to the diameters 2Ax + 2Ab, 2AQ + 2Aa, it will be without ſenſible error; Aa : Ab :: 2Ax + 2Ab : 2AQ + 2Aa. And the two laſt terms being divided by 2, it will be Aa : Ab :: xb : Qa nearly. *Q. E. D.*

59. *Corol.* Aa : Ab :: I : R nearly; and therefore Aa = $\frac{I}{R}$ Ab; and Ab = $\frac{R}{I}$ Aa. In *fig.* 13. Aa = $\frac{m}{n}$ Ab; and in *fig.* 14 Aa = $\frac{n}{m}$ Ab.

60. *Schol.* The aberration qx is always in the ſame ratio to Aa the verſed ſine of the angle of incidence nearly. And $qx = \frac{m}{n}$ Aa − $\frac{n}{m}$ Aa nearly. For,

CASE 1. *When the incidence is into a denſe medium; it will be,* Fig. 13.

$n : m$ :: [a] AQ : Aq :: AQ + Aa (CQ) : Aq + [†] $\frac{m}{n}$ Aa = [a] Cx = [a] bx = Ax + Ab. Wherefore Ax = Aq + $\frac{m}{n}$ Aa − Ab; and conſequently $qx = \frac{m}{n}$ Aa − Ab = [b] $\frac{m}{n}$ Aa − $\frac{n}{m}$ Aa. *Q. E. D.*

[a] Conſtruct. & 35. I.

[b] 59. I.

CASE 2. *When the incidence is into a rare medium; it will be,* Fig. 14.

$m : n$ :: [a] AQ : Aq :: AQ + Aa (CQ) : Aq + $\frac{n}{m}$ Aa = [a] Cx = [a] bx = Ax + Ab; wherefore Aq = Ax + Ab − $\frac{n}{m}$ Aa; and therefore qx = Ab − $\frac{n}{m}$ Aa = [b] $\frac{m}{n}$ Aa − $\frac{n}{m}$ Aa. *Q. E. D.*

61. *Corol.* Hence, and from *Art.* 20. when the refraction is between glaſs and air, the aberration qx is $\frac{5}{6}$ of Aa, the verſed ſine of the angle of incidence; for $\frac{m}{n} - \frac{n}{m} = \frac{3}{2} - \frac{4}{3} = \frac{9}{6} - \frac{4}{6} = \frac{5}{6}$. In like manner when the refraction is between air and water, $qx = \frac{7}{11}$ Aa.

† For $n : m$ :: Aa : $\frac{m}{n}$ Aa.

PROP. V.

62. *Images formed by refractions at plane ſurfaces, are erect, or alike ſituated with the objects, and on the ſame ſide of the refracting planes.*

Fig. 11, 12. From each point P, R, &c. of an object PQR radiating upon the refracting plane BC; upon the ſaid plane let fall the perpendiculars PB, RC, &c. Now if in the ſaid perpendiculars, the points *p*, *r*, &c. be ſo taken that R : I :: BP : B*p* :: CR : C*r*, &c. the points *p*, *r*, &c. ſo [a] 50. I. found will be the [a]images of the points P, R, &c. and therefore all theſe points connected together, will give the *image p q r* of the whole *object* PQR. Whence the whole of the propoſition is ſelf-evident.

63. *Corol.* 1. If the object be rectilinear, its image will be ſo too; and the correſponding parts of both will be ſimilar. All other objects and their images, will be ſimilar, if they lie parallel to the refracting plane; otherwiſe not. For if they be oblique, the image of a ſquare may be an oblong; and the image of an oblong parallellogram, may be a ſquare; as is manifeſt from the preceding, and the inſpection of the figures.

64. *Corol.* 2. If *e* be the pupil of the eye, the points P, R of the object will be ſeen by rays *ae*, *be* diverging from the focus's *p*, *r* nearly. When the eye is ſo ſituated that the axis of the pencil entering the pupil is perpendicular to the refracting plane; the focus of that pen- [a] 50. I. cil is [a] accurately the correſponding point of the image. But when the rays of the ſaid pencil are oblique to the refracting plane, thoſe rays if [b] 56. I. produced backwards would unite in a point [b] farther from the object than the correſponding point of the image; and the diſtance of that point from the image will be more or leſs, according to the greater or leſſer obliquity of the rays to the refracting ſurface. And hence it is that objects in a different medium from that wherein is the ſpectator, generally appear a little diſtorted.

65. *Corol.* 3. Objects under water, to a ſpectator above, appear one fourth higher than they really are; and alſo ſomewhat diminiſhed, and diſtorted, if they are not parallel to the ſurface. And to a ſpectator under water, objects that are above appear one third higher than they really are: And if they are oblique to the ſurface of the water, they'll appear ſomewhat magnified and diſtorted.

Fig. 10. Why a ſtrait ſtick immerſed in water appears bent.

66. *Corol.* 4. From what has been above delivered, we have alſo the reaſon why a ſtrait ſtick STQ immerſed obliquely into water, appears bent at the ſurface: the end Q appearing in *q*, and every other part by the refraction of the rays at the ſurface, being elevated in proportion, the whole ſtick appears under the form ST*q*; that is, the part under water

water appears alſo *ſtrait, as T*q*, the only bending being at the ſurface T.

§. 3. *Of the Progreſs of Light through a Medium terminated by plane Surfaces.*

PROP. VI.

67. *A ray of light as* QA, *falling obliquely upon a medium terminated by two parallel plane ſurfaces* RS, TV, *will after its ſecond refraction at* B, *emerge in a line* BC *parallel to the incident ray* QA. Fig. 15.

Produce the incident ray QA to D, and the emergent ray BC to E; and through the points A, B draw the perpendiculars PAO, FBG to the ſides RS, TV. Now becauſe the ſides RS, TV are parallel, the ray AB within the medium is equally inclined to them both; that is, the ∠ OAB = [a]FBA; wherefore if theſe be conſidered as angles of incidence, that is, if the ray be conceived to go both ways along AB, the ∠s PAQ, GBC will be [b]equal; and conſequently their oppoſite ∠s OAD, FBE are [c]equal; and taking away from theſe the equal angles OAB, FBA, the alternate angles DAB, EBA will be equal; and therefore QAD is [d]parallel to CBE. *Q. E. D.*

[a] 29. I. *Euc.*
[b] 13 & 16. I.
[c] 15. I. *Euc.*
[d] 27. I. *Euc.*

68. *Schol.* It is manifeſt that the leſs is the thickneſs of the medium RV, or the leſs is the obliquity of the incident ray, the leſs will be the diſtance between the parallel lines AD, BC: And when RV is very thin, or the angle QAP is very ſmall, the ſaid diſtance will be ſo inconſiderable, that the ray QABC may without ſenſible error be reckoned as one ſtrait line.

PROP. VII.

69. *If a ray* QABC *be tranſmitted through a medium* RST *terminated by two plane ſides* RS, TS *inclined to one another in an angle* RST; *the emergent ray* BC, *will incline from the incident ray* QA, *towards the thickeſt part* RT *of the medium: And if the angle* S, *and the angles of incidence and refraction be ſo ſmall, as to be reckoned proportionable to their ſines; the angle of deviation* QEq, *contained between the incident ray* QAE, *and the emergent ray* CBE*q*, *produced till they meet in* E, *will be to the refracting angle* RST, *as the difference of the ſines of incidence and refraction is to the leſſer of them; that is,* ∠ QE*q* : ∠ RST :: *m*—*n* : *n*. Fig. 16.

Let A*a*, B*b* be perpendicular to the ſides RS, TS reſpectively, and produce them till they meet in D: Alſo produce the incident and emergent rays QA, CB till they meet in E. Now if it be made, *m* : *n* :: Sine ∠ *a*AE : Sine ∠ *a*AB; and *n* : *m* :: Sine ∠ *b*BA : *b*B*q*; *q*B produced will be the emergent ray: whence it is manifeſt,

* For if perpendiculars be drawn from every point of the ſtrait object TQ to the plane ſurface TA, the images of theſe points will divide all the ſaid perpendiculars proportionably. Whence from the 2 of the VI. of *Euclid*, it is eaſy to ſhew, that all theſe images lie in a ſtrait line.

A

TREATISE

OF

OPTICS:

CONTAINING

ELEMENTS OF THE SCIENCE;

IN TWO BOOKS.

By JOSEPH HARRIS, Esq.

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By JOSEPH HARRIS

IN TWO BOOKS.

ELEMENTS OF THE SCIENCE,

CONTAINING

OPTICS:

TRACTISE

BIBLIOTHECA REGIA MONACENSIS

ADVERTISEMENT.

THE Author of the following Work, in the year 1742 firſt propoſed printing *a Treatiſe upon Microſcopes* only. The plan laid down was entirely new ; and to render it more compleat, a prefatory diſcourſe was intended, to introduce ſuch general principles as were neceſſary for underſtanding the ſubject ſcientifically. This diſcourſe increaſed ſo faſt, under the Author's pen, that it was ſoon thought adviſeable to enlarge the original deſign, into a general *Treatiſe of Optics*. Two books, containing the Elementary parts of this ſcience, being finiſhed with all convenient expedition, were immediately printed off---both Plates and Letter-preſs---with uncommon care and correctneſs. And upon this foundation, it was propoſed to explain the Theory and Mechaniſm of Optical Inſtruments, in a third Book, under the general head of Teleſcopes and Microſcopes. A Taſk of this kind, required a multitude of accurate experiments ; and perhaps nobody could be better qualified for making or reaſoning upon them, than Mr. Harris. But, while this buſineſs was forwarding with unuſual celerity, our Author was called from the purſuit, by an extraordinary demand of duty in His Majeſty's Mint : And from that time, a variety of avocations, for the publick ſervice, with a ſeries of bad health, the conſequence of laborious application and ſtudy, prevented any regular renewal of the ſubject for ſeveral years. Mr. Harris however, did not loſe ſight of his favourite object, but employed moſt opportunities for the advancement of it ; till at length, finding himſelf once more at leiſure, he reſumed his application to this work with an earneſtneſs that probably haſtened his diſſolution : For, in the actual proſecution of ſeveral intereſting experiments he was taken ſick, and, unfortunately for the Publick, expired 26th September 1764.

After Mr Harris's deceaſe, a numerous collection of Manuſcripts were ſubmitted to the conſideration of ſome Gentlemen, well acquainted with the Science of Optics, in hopes of compleating ſo valuable

valuable a work. But it appeared, upon examination, that though there ſeemed to be ample materials for the purpoſe, and many of them prepared for the preſs; yet the greater part, having never been digeſted, was incapable of any arrangement, without aſſiſtance of the Author---no longer to be had---or taking up the ſubject *de novo*, and purſuing it through an almoſt infinite variety of experiments--A Labour, neither to be expected or deſired of a Stranger !

Under theſe circumſtances, the whole Treatiſe has long lain dormant, againſt the importunities of many friends, who have frequently recommended to publiſh this Elementary Part alone. They have repeatedly alledged---That the Elements of Science are always the ſame---That theſe Elements, being quite diſtinct, are compleat in themſelves---That very little has been written on the ſubject---And That the following ſheets, containing much new matter, would be a valuable acquiſition to the ſcience. Such have been the reaſons advanced for this publication, by Gentlemen whoſe names might do honour to any Book: And they have at laſt prevailed over every ſcruple of the Proprietor. But as nothing here is intended to bias the favour of the Public, the true hiſtory of this work is fairly related, and the performance ſubmitted, upon its own merit, to the candid judgment of the impartial Reader.

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A TREATISE OF OPTICS.

BOOK I.

The Elementary Part of Optics.

THE object or design of OPTICS is to explain the *Theory of Vision*, as it is founded upon known and established properties of light. These properties may be considered in the abstract, that is, without an immediate application to Vision; and this properly makes the elementary part of the science: for upon these are founded the surprising effects of all sorts of optical instruments; the principal of which are *Microscopes* and *Telescopes*. Both these enlarge the apparent magnitudes of objects: the Microscope being adapted for magnifying small objects that are near at hand; whilst the Telescope is applied to remote objects.

Optics is distinguished into two principal heads: *Catoptrics*, which treats of the reflection of light by smooth or polish'd surfaces of given figures; and *Dioptrics*, whose object is the progress of light through transparent bodies, terminated also by given surfaces. But of all the kinds of surfaces, the most useful in Optics are the *Plain* and *Spherical*, and to consider others is seldom necessary: nor is it requisite to make a formal distinction as to Catoptrics and Dioptrics, it being often more advantageous to blend them together.

SECTION I.

Of the chief Properties of Light, with an Explanation, as they occur, of some general Terms.

Art. 1. IT is manifest that rays of light are incessantly propagated in all possible directions, from every physical point of a luminous body; and when they enlighten other bodies upon which they fall, they Light in continual motion.

they are alſo reflected and diſperſed from every part of theſe bodies. For opaque bodies ſo enlightened are * viſible to the eye, as well as thoſe that are luminous.

Ray of Light.

2. By *a Ray of light* is meant, the leaſt light, or part of light, which may be ſtopt alone without the reſt of the light, or propagated alone, or do or ſuffer any thing alone, which the reſt of the light doth not or ſuffers not †.

Light conſiſts of ſmall material Particles.

3. It is very probable that rays of light are not continued filaments or threads; but conſiſt of ſmall and diſtinct particles of matter ‖, which following in a cloſe ſucceſſion do as it were make one continued thread.

Light not inſtantaneous, but its Velocity exceeding great.

4. That light ſhould be propagated in time, and not inſtantaneouſly, is very reaſonable to ſuppoſe: And that its velocity is much quicker than that of ſound, we know, becauſe the flaſh of a gun fired at a conſiderable diſtance will be ſeen ſome time before the report is heard **. The *Aſtronomers* have diſcovered the velocity of light ‡ to be ſuch, as to reach from the ſun to us in about 8 minutes of time.

Medium.

5. *Definition.* Any *tranſparent Body*, as *Air*, *Glaſs*, *Water*, &c. alſo *empty Space*, is called a Medium: And mediums are ſaid to be *denſer* or *rarer*, as they are ſpecifically heavier or lighter. *Note*, By *a Medium is always underſtood a denſe Medium*, unleſs it be otherwiſe expreſſed.

Object, Naked Viſion, Radiant.

6. *Defin.* All bodies ſeen by rays propagated directly from them to the eye, without being at all bent out of their courſe in their paſſage, are called *Objects*. And the viſion of objects in this manner is called *Naked Viſion*. A ſmall object, or a phyſical point of an object, conſidered as propagating light towards a certain part, is ſometimes called a *Radiant* or *Radiating Point*.

7. The

* It cannot be doubted, but that light by its impulſe upon the proper organ is the mediate cauſe of viſion: and therefore wherever bodies are viſible, light muſt be propagated from them to the eye. A continued uninterrupted motion is eſſential to the very being of light, according to what ideas we can form of it; for the moment it ſtops, it ceaſes to be light to us.

† Sir *Iſaac Newton*'s Defin. Optics, page 2. Light is ſo ſubtile and fine, that it is probable the ſmalleſt part of it which we can ſtop, may yet conſiſt of ſeveral rays.

‖ How exquiſitely minute muſt theſe particles be, that myriads of them can move promiſcuouſly all manner of ways, without impinging one another! And that this is the caſe, we need not doubt, ſince different bodies and different parts of the ſame body are diſtinctly viſible at the ſame time. And how curious muſt the eye be framed, to be ſenſible of theſe ſmall impulſes, and to diſtinguiſh at the ſame time thoſe from different objects!

** It is for the ſame cauſe that a clap of thunder is not heard till ſome time after the flaſh of lightening has been ſeen.

‡ This diſcovery was firſt made by *Monſieur Romer*, who having obſerved that the eclipſes of the *Satellites of Jupiter* appeared ſooner or later than according to the theory, as the earth was nearer to or farther from *Jupiter*, concluded from thence that the progreſs of light is not inſtantaneous: And by obſerving the different times of the appearances of theſe eclipſes, according to the different diſtances of the earth from *Jupiter*, the time the ſun's light takes in coming from him to us, has been diſcovered. This hypotheſis, that light is not inſtantaneouſly propagated thro' all ſpace, is now univerſally agreed to; and is farther confirmed by a notable diſcovery of an apparent motion of the fixed ſtars, by the *Reverend* and ingenious *Dr. James Bradley*, *Reg. Profeſſor of Aſtronomy*, who has elegantly accounted for that phænomenon, by the theory of the light's motion.

7. The progreſs or path of light, whilſt it continues uninterrupted in a medium of the ſame uniform denſity, is in * ſtrait lines. For all bodies ceaſe to be viſible, when any opaque body is interpoſed between them and the ſpectator.

Thus from a radiating point Q of an object PQR, are propagated inceſſantly in ſtrait lines, and in all poſſible directions the rays Q*q*, Q*q*, Q*q*, &c. And in like manner rays of light are inceſſantly propagated from every other phyſical point of the object, as well as from Q; otherwiſe the whole object would not be viſible, and that all at once, and to all poſitions of the eye. Fig. 1.

8. A ray of light, as AC, meeting obliquely with a ſurface RS, if it be turned back into the medium from whence it came, it will take ſome new direction, as CB: And if it enters the medium RG, inſtead of proceeding directly forward in the ſtrait line ACG, it will be bent at C into ſome † new direction CD, in which it will proceed ſtrait‖ whilſt it continues in that medium. The diſpoſition of the rays of light to be turned back into the medium from whence they came, is called *Reflexibility*: as is that to be bent at their entrance into another medium, called *Refrangibility*. The changes which the rays undergo in theſe two caſes, are called *Reflection* and *Refraction*. The original or primary ray AC, is called the *incident Ray*; C *the Point of Incidence*; CB, *the reflected Ray*; and CD, *the refracted Ray*. Fig. 2. Reflection and Refraction &c.

In like manner, if BC or DC be an incident ray, which at C is bent into CA; the *Ray* CA is accordingly called the *reflected or refracted Ray*.

9. Let PCO be a ſtrait line perpendicular to the ſurface RS at the point of incidence C; the angle ACP which the incident ray makes with this perpendicular, is called the *Angle of Incidence*; BCP the *Angle of Reflection*; and DCF the *Angle of Refraction*. From the point of incidence C, with any radius deſcribe a circle cutting the incident and refracted rays in A and D; and from the points A, D upon the line PO let fall the perpendiculars AE, DF. Now the line AE is the Sine of the angle of incidence; and DF is the ſine of the angle of refraction. The line AE for ſhortneſs of expreſſion is called *the Sine of Incidence*, and DF *the Sine of Refraction*. Sines of Incidence and Refraction.

* The propagation of light is in this reſpect different from that of ſound, which ſeems to ſuffer no obſtruction by the crookedneſs of its paſſage; whereas no object can be ſeen through a crooked pipe. Sound is undoubtedly propagated to the ear by an undulating motion of the air, (like that of water after a ſtone has been thrown into it) cauſed by the tremor or vibrating motion of the ſounding body; and not by any particles of matter emitted from that body.

† The advantages we derive from the refrangibility of light are ineſtimable: without this property in light, we ſhould have figured glaſſes in vain, and Teleſcopes and Microſcopes would not have exiſted.

‖ It ſeems that light is not reſiſted, at leaſt ſenſibly, by any medium through which it paſſeth; for if it was, its motion would be curvilineal, and not ſtrait.

The following *Axioms* about Light, are by repeated experiments found to be general without any exception; and are by ſome called the *Laws of Reflection and Refraction.*

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15. Axiom V. *The Sine of Incidence is either accurately, or very nearly, in a given Ratio to the Sine of Refraction* †. That is, any other incident ray *a*C will be ſo refracted into C*d*, that whatever ratio there be between

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17. *Corol.* 2. If the incident ray is perpendicular to the refracting ſurface, as PC, it will then undergo no refraction, but proceed directly in the ſtrait line PCO. For the angle of incidence being now vaniſhed, the angle of refraction muſt alſo vaniſh.

18. *Corol.* 3. If the ratio between the ſines of incidence and refraction at the ſurface of a given medium, be known, and an angle of incidence ACE be given; the angle of refraction FCD may be thus found: About the point of incidence C, with any radius, deſcribe a circle cutting the incident ray AC ſomewhere in A; and from A upon the perpendicular PCO to the refracting ſurface, let fall the perpendicular AEB; then make AE to EH, as the ſine of incidence to the ſine of refraction; and through H draw HD parallel to PO, and where this line cuts the circle as in D, is a point through which the refracted ray CD will paſs. Or, the angle FCD may be found by a table of ſines, thus: *Sine of Incidence* : *Sine of Refraction* :: *Sine* ∠ACE : *Sine* ∠FCD.

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20. It is found by experience, that if the refraction be made out of air into water, the ſine of incidence (AE) is to the ſine of refraction (FD) as 4 to 3: If out of air into glaſs, the ſines are as 3 to 2 nearly; or more accurately as 17 to 11, or 14 to 9. And reverſely, if the refraction

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Fig. 2. 21. *Corol.* Hence if the refraction be from water or glaſs into air, and the angle of incidence GCF be greater than about 48½ degrees in water, or greater than about 40 deg. in glaſs; the ray GC will not be refracted into air, but will be reflected into the line C*g*, making the angle of reflection *g*CF = GCF the angle of incidence: Becauſe the ſines of 48½ deg. and 40 deg. are to the radius (C*k*) as 3 to 4, and as 11 to 17 nearly; and therefore when the ſine G*h* has a greater proportion to the radius than as above, the ray will not be refracted.

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[22.] Sir *Iſaac Newton* † hath ſhewed that the reflection and refraction of light are probably both owing to the ſame power or cauſe differently exerciſed in different circumſtances. And that this power exerts itſelf at ſome diſtance from the reflecting ſurface; ſo that light is not reflected by impinging on the ſolid parts of bodies, but by ſome principle, which is uniformly diffuſed to a certain diſtance from the body. And this accounts why poliſh'd ſurfaces reflect light ſo uniformly as they do, which otherwiſe would be inexplicable. For the beſt poliſhed ſurface is full of ſcratches or fiſſures made by the powder wherewith it was poliſhed, and therefore utterly incapable of reflecting light uniformly, if reflection was made by the impulſes of light upon the ſolid parts of bodies. Nevertheleſs we find that rough or uneven ſurfaces reflect light all manner of ways; and that the ſmoother they are, the more uniform will be their reflections. The reaſon of which ſeems to be, that the reflective power reaches every where to the ſame diſtance from the body nearly; and that when the depths of the fiſſures are leſs than ſome aſſignable quantity, the reflective power may be ſo extended as to form as it were a film or ſurface round the body that ſhall be perfectly ſmooth. And hence perhaps there may be a degree of ſmoothneſs of bodies attainable by human art, beyond which no optical advantages are to be expected.

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Fig. 1,5,9. Pencil or Cone of Rays.

[a] 12&17. I.

The rays of any pencil before they have undergone any change by reflection or refraction, are called *Incident Rays:* afterwards they are called *Reflected* or *Refracted Rays*, according to the change they underwent. Both the incident and reflected or refracted rays, are conſidered as making one pencil.

Fig. 5, 9. Incident and reflected or refracted Rays.

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All

* Theſe definitions are placed here, becauſe they are general; but a beginner need not dwell upon them, their meaning will be more obvious after a peruſal of part of the next ſection.

All the rays in a pencil will not be always reflected or refracted into one point, as will be ſhewed hereafter; but only thoſe rays that are equidiſtant from the axis on different ſides, all thoſe that have different inclinations having different focus's. And, ſtrictly ſpeaking, by the focus of a pencil is meant the focus of thoſe rays that are neareſt to the axis: this is alſo called the *Geometrical Focus*. And the diſtance between this focus and the *proper or reſpective Focus* of any ray, is called the *Aberration* of that ray. Thus, *fig.* 13, 14, if Q be the focus of incident rays, and *q* be the focus of thoſe rays that are next the axis after refraction, *x* the focus of any particular ray CD; then is *q* called the *Geometrical Focus*, *x* the *reſpective* or *proper Focus* of the ray CD, and *qx* its *Aberration*.

Image. 26. *Defin.* A phyſical point Q (*fig.* 5, 9, 10.) being conſidered as an object, the correſponding point *q* from or towards which the rays diverge or converge after reflection or refraction, is called the *Image* of the ſaid point Q. In like manner the images of all the ſeveral points of an object PQR, (*fig.* 6, 7, 8, 11, 12.) being taken collectively or as one image, as *pqr*, is called the *Image* of the ſaid *Object* PQR.

27. Light the farther it is propagated from any radiating point, becomes continually rarer: And if it met no obſtruction in its courſe, the decreaſe of its denſity would be as the ſquare of the diſtance from the radiating point. This is manifeſt from the principles of geometry.

Fig. 5. Let BQC be a pencil of light propagated from the point Q, and let Q*a*, Q*b* be equal reſpectively to half QB, QC. Now a circle whoſe diameter is BC will be four times as big as a circle deſcribed upon *ab*; but the quantity of light contained in theſe two circles is equal; and therefore the light at the diſtance Q*a* is four times denſer than at the diſtance QB. In like manner, if Q*e*, Q*f* be one fourth of QB, QC, the light at *ef* will be 16 times denſer than at BC. For a circle deſcribed upon *ef* contains all the light that can come into the circle deſcribed upon BC, and theſe two circles are to one another as 1 to 16: That is, *Light the farther it is propagated decreaſes in its denſity in a duplicate Ratio of the diſtance.*

28. *Corol.* Hence luminous bodies, or thoſe that ſhine with their own light, are vaſtly brighter than opaque bodies illuminated by them. For opaque bodies diſperſe the light falling upon them, alſo all manner of ways; whence, ſuppoſing that all the light was reflected, the quantity of it received by the eye from the opaque body, compared to that which it receives from the luminous body, is only as the viſible illuminated ſurface of the opaque body, to the ſurface of an hemiſphere whoſe radius is the diſtance of the opaque body from the eye; ſuppoſing the breadth of the pupil to be the ſame in both caſes, and that the ſum of the diſtances of the opaque body from the eye and from the luminous body,

differs

differs insensibly from the direct distance of the luminous body from the eye.

29. Hence again, the light of the full moon at a medium is about 100,000 times fainter or rarer than the sun's light when at the same height above the horizon, all other circumstances being the same also; as will appear by comparing the moon's apparent disk to the surface of a hemisphere. Whence it is easy to conceive, that since we can bear the sun's heat, why we are not sensible of any from the moon; and much less can we be sensible of any heat from any of the other planets.

No Medium perfectly transparent.

30. We do not know any medium that is perfectly transparent. Even the air, the most rare and transparent medium we know of, is full of opaque particles that impede light; this is manifest from the phænomenon of a beam of light let in through a small hole into a darkened chamber, which will be visible like a *luminous Cone* from every part of the room: and this shews that the whole light does not go forward in its rectilineal course, but that at every point of the medium through which it passeth, some part of it is reflected every way; for the visibility of the luminous cone is caused by this reflection. The greater faintness of the sun and moon when near the horizon, than when elevated higher up, shews also that their light is more obstructed by the longer tract of air and vapours in the one case, than in the other. The loss of light in passing through glass is still greater.

Hence Light decreases in its density in a greater proportion than as the squares of the distances.

31. Hence the quantity of light contained in any pencil is continually diminished, the greater the distance from the radiant: and this diminution is more or less, according to the more or less opacity of the medium through which it passes. This makes it impossible to assign exactly the different proportions of the density of light, at different distances from the radiant; but in all cases, the decrease of the density of light is greater than as the squares of the said distances. And this is the cause why objects appear less bright the farther they are from the spectator. For if none of the rays were stopt in their passage, the degree of brightness of the picture of an object upon the retina, would be the same at all distances between the eye and object, supposing the aperture of the eye's pupil to remain the same. For in that case the magnitude of the picture upon the retina, and the density of the light would increase or decrease together in the same proportion, *viz.* reciprocally as the squares of the distances of the eye from the object; and therefore the density of the light upon the retina would be invariable at all distances *.

* For example; when the eye approaches as near again to the object, the picture upon the retina becomes quadruple; and the quantity of light received from the object thro' the same aperture

Luminous bodies ſhining in the dark, as a *lamp* or *torch*, &c. emit to very great diſtances much more light than in that caſe is neceſſary for viſion. And tho' their light ſuffers a continual diminution by the heterogeneity of the medium, the farther it goes; yet there being ſtill more left than is requiſite for viſion, their ſplendour is not ſenſibly increaſed or diminiſhed, by leſſening or increaſing their diſtances.

Light not all reflected.

32. Light is not only diverted out of its courſe by the reflection of bodies it meets with, but alſo a great part of it is frequently ſuffocated, or as it were deſtroyed by them. For it is manifeſt, that black or dark bodies reflect much leſs light than others of darker colours; and it is probable, that no bodies whatever reflect all the light they receive *. Hence the ſurface of the moon having in it ſeveral large dark ſpots, the quantity of light we receive from her muſt be leſs than according to the computation in *Art.* 29.

33. Hence the power of the eye to diſcern objects without inconvenience, by different quantities of light, is vaſtly extenſive. For not only objects of different colours placed in the ſame light, are ſeen with equal eaſe; but even objects ſo ſmall as the letters of a book may be diſtinguiſhed and read by a clear moon-light. And admitting the ſurface of the eye's pupil to be ten times greater in a weak light than in a ſtrong light, yet the proportion of the weakeſt light to the ſtrongeſt by which the eye can conveniently ſee objects, perhaps does not exceed that of 1 to 10,000 †. How exquiſite are our faculties!

Speculum. Plane and ſpherical Speculums.

34. *Defin.* Any ſmooth or poliſhed ſurface, reflecting light, is called a *Speculum*. Reflecting ſurfaces that are *flat*, as a *Looking-glaſs*, are called ‖ *Plane Speculums*; and thoſe that are *Segments of a Sphere*, are accordingly called *Spherical Speculums*; and as they are convex or concave as ECF, *fig.* 3, or *fig.* 4. they are called *convex* or *concave Speculums*.

35. *N. B.* There will be frequent occaſion to conſider the ſines of incidence and refraction; and therefore both for the ſake of brevity and perſpicuity,

aperture of the pupil at half the diſtance, is alſo quadruple: and this being equally ſpread over four times the ſurface of the retina, the light is juſt as denſe as before when the object was at twice the diſtance.

* Light, whatever afterwards become of its component particles, ceaſes to be light as ſoon as its motion is ſtopt. Whether there be a decreaſe in the velocity of light, is impoſſible for us to know from any experiment. But it cannot well be doubted, that ſince light is continually propagated from the ſun and other ſelf-ſhining bodies, ſo it muſt alſo ſuffer a continual waſte and decay, to make room for theſe continual ſupplies.

† By a well-contrived experiment M. *Bouguer* found full moon-light at a medium to be about 300,000 times weaker than the ſun's light. Vide the learned Dr. *Smith*'s Remark 96. upon his compleat ſyſtem of Optics. But it is to be obſerved, that ſky-light, which properly is day-light, is always much weaker than the direct ſun's light, and the degrees of it do not only differ conſiderably, at different times of the year, but alſo frequently in the ſame day; and thoſe different degrees are not eaſily meaſured, ſo as to be reduced to numbers. However, it is manifeſt that the difference of the degrees of light ſuitable for viſion, is very conſiderable.

‖ Theſe are alſo called *Mirrours*, but Opticians generally make uſe only of the word *Speculum*.

perſpicuity, we ſhall often abbreviate theſe expreſſions, by calling the *Sine of Incidence*, I; and the *Sine of Refraction*, R. And when the ratio between theſe ſines is the only thing under conſideration, without regarding which is the ſine of incidence, or which the ſine of refraction, we ſhall call the *greater* of theſe ſines, *m*; and the *leſſer* of them, *n*. Hence, and from *Art.* 14. if the incidence is from a rare medium into a denſe, it will be I : R :: *m* : *n*; and when the incidence is from a denſe medium into a rare, it will be I : R :: *n* : *m*. If the refraction be betwixt air and glaſs, *m* : *n* :: [a]3 : 2; if betwixt air and water, [a] Art. 20.
m : *n* :: [a]4 : 3.

36. *Lemma.* Small arches and their ſines are very near in a ratio of equality *, and therefore have the ſame ratio to each other, as their ſines nearly; that is, if the Arches DO, *d*O, are but ſmall, it will be Fig. 2.
DO : *d*O :: DF : *df* nearly. And the ſmaller are the arches, the leſs will be the error in ſuppoſing them equal to their ſines. Thus, if the arch DO does not exceed 20 *min.* and the radius CD or CO be divided into 10,000,000 of equal parts; the exceſs of the arch DO above its ſine DF will be leſs than one of thoſe parts.

Alſo the verſed ſines OF, O*f* of very ſmall arches OD, O*d* are as the ſquares of the ſines FD, *fd* of thoſe arches nearly. For OF : FD :: FD : FP; and O*f* : *fd* :: *fd* : *f*P; but when the arches OD, O*d* are very ſmall, FP and *f*P will be nearly in a ratio of equality, and therefore OF : O*f* :: FD^2 : *fd*^{2} nearly.

SECTION II.

Of the Reflection and Refraction of a Pencil of Rays at a plane Surface.

§. 1. *Of Reflection at a plane Surface.*

PROPOSITION I.

37. *A Pencil of rays diverging from a point* Q, *will after reflection by a plane ſpeculum* BAC, *diverge in ſuch a manner, that if the reflected rays were produced backwards, they would all meet in a point* q, *taken in the perpendicular* QA *to that plane produced, as far below* A *as* Q *is above it; that is,* QAq *is a ſtrait line, and* qA = AQ. Fig. 5.

* Mathematicians will at firſt ſight underſtand this lemma; and this is not a proper place to prove propoſitions purely geometrical. And tho' the lemma is never ſtrictly true till the arch vaniſhes, yet as by diminiſhing the arch the error may be reduced to a leſs quantity than any aſſignable one, no objection can lie againſt it.

Let QB be any incident ray, and let QA, BF be each perpendicular to the plane or ſtrait line BAC, and make the angle DBF equal to
[a] 11. I. the angle QBF; then will BD be the [a]reflected ray. Produce DB backwards until it meets QA produced in q; I ſay q is the focus of all the rays diverging from Q upon BAC, and that $Aq = AQ$. For BF,
[b] Conſtruct. Qq being [b]parallel, the angle $QBF = {}^{c}\angle BQq = {}^{b}FBD = {}^{c}BqQ$;
[c] 29. I *Eucl.* wherefore the right-angled triangles BAQ, BAq having beſides the right angles at A, the ∠s BQA, BqA equal, and the ſide BA common;
[d] 26. I *Eucl.* the ſides AQ, Aq are alſo [d]equal. The ſame argument holds good with reſpect to any other incident ray, as QC; and therefore all the rays diverging from Q, will after reflection by the plane BC, belong to the focus q. *Q. E. D.*

38. *Corol.* 1. Hence, and from *Art.* 13. if q be the focus of incident rays, Q will be the focus of the reflected ones; that is, if rays DB, EC, *&c.* converge towards the focus q, they will all meet in Q after reflection.

39. *Corol.* 2. If the ſpeculum BC be inclined a little, as into the poſition kCK, ſo that CH ſhall be a perpendicular thereto at the point C; the incident ray QC ſhall be reflected into CI, ſo that the ∠ ECI contained between the two reflected rays CE, CI at both poſitions of the ſpeculum ſhall be double the angle GCH, the inclination given the ſpeculum. And hence, *Any error in the figure of a ſpeculum, as well as any inclination given it, produces a double error in its effect.*

PROP. II.

40. *Images formed by the reflection of a plane ſurface, are equal and ſimilar to the objects; and are at the ſame diſtance from, and have a like ſituation with reſpect to the backſide of the plane; that the objects are from, and have with reſpect to the foreſide.*

Fig. 6, 7, 8. For the images p, q, r of the radiating points P, Q, R [a]being in the perpendiculars PBp, QAq, RCr to the reflecting plane BC, and at
[a] 37. I. the ſame diſtance from one ſide of the plane, [a]that the radiants are from the other; that is, ſo that Bp, Aq, Cr are equal reſpectively to BP, AQ, CR: And this being true of the images of every point of the object PQR, it is manifeſt that the whole image p, q, r is equal and ſimilar to the object PQR; and has a like ſituation with reſpect to one ſide of the plane, that the object has with reſpect to the other. *Q. E. D.*

41. *Corol.* 1. If the poſition of the object PQR, and of the eye e, be given, ſuppoſing the pupil of the eye to be only a point *, the parts of

* In the figures the breadth of the pupil is taken in, and it is manifeſt that it receives light from a larger part of the ſpeculum than it would do if it was no bigger than a point. And in the next

of the ſpeculum *a*,*b* which reflect to the eye the rays proceeding from any points P,R of the object, may be found by drawing from the correſponding points *p*,*r* of the image the ſtrait lines *pae*, *rbe*. For the rays diverging from the points P,R upon the parts *a*,*b*, do after reflection belong to the [a]focus's *p*,*r*.

[a] 37. I.

Or the point *a* of the ſpeculum which reflects to the eye at *e*, the rays diverging from P, may be found without having the place of the image *p*: Thus, upon the ſpeculum let fall the perpendiculars PB, *eu*. Now if *a* be a point from whence the reflected ray comes to the eye, the angles P*a*B, *eau* will be equal. Wherefore PB : B*a* :: *eu* : *au*; and therefore PB + *eu* : B*a* + *au* (B*u*) :: *eu* : *au*. After the ſame manner may be found the point *b*, which reflects to the eye a ray which came from R, *&c.*

42. *Corol.* 2. The eye at *e* will ſee the whole object PQR by rays reflected from the part of the ſpeculum contained between *a*,*b*, and from no other part; all the reſt of the ſpeculum in this caſe being uſeleſs. But if any part of *ab* be covered, ſome part of the image will be accordingly loſt. If either the eye or the object change their poſitions, the rays that come to the eye will be reflected from a different part of the ſpeculum, which part may be found as in the preceding *Article*.

43. *Corol.* 3. The diſtance of any point of the image from the eye, is equal to the incident and reflected rays taken both together; that is, *pe*, or *re*, is equal to P*a*+*ae*, or R*b*+*be*. And tho' with reſpect to the different ſides of the ſpeculum, the object and image have a like ſituation; yet to a ſpectator who firſt looks at one and then at the other, the caſe will be different.

44. *Corol.* 4. If *ab* be the breadth of the ſpeculum, and *e* the place of the eye; the breadth of the images that can be ſeen by the eye at *e*, will be contained between the lines *ea*, *eb* produced indefinitely. And therefore having the diſtances and ſituations of objects before the ſpeculum given, the extent of thoſe that are viſible by the reflection of a ſpeculum having a given breadth, may be found from the extent of the viſible image.

45. *Corol.* 5. The nearer is the eye, and the leſs oblique to the ſpeculum, the greater will be the field of viſible images.

46. *Corol.* 6. If an object PR be placed parallel to a plane ſpeculum BC, and two lines as P*p*, P*r* be drawn from any point P thereof to the extremities of its image *pr*; the part BA of the ſpeculum included

Fig. 6.

next article, the eye will ſee the object by rays reflected from a ſmaller part of the ſpeculum, than it could do if its pupil was only a point. The differences in both caſes is manifeſt from the figures.

cluded between theſe two lines will be equal to half the length of the object or of its image *pr*. For the triangles PBA, P*pr* being ſimilar, and PB being equal to half P*p*, it will be BA $=\frac{1}{2}pr=\frac{1}{2}$ PR. Whence,

47. *Corol.* 7. A perſon viewing himſelf in a plane looking-glaſs placed upright, will ſee his image compleat in a part of the glaſs, whoſe length and breadth is equal to half the length and breadth of the correſponding parts of his own body. And this will be always the caſe at whatever diſtance he ſtands from the glaſs.

48. *Corol.* 8. *A ſpectator will ſee his own image as far beyond the ſpeculum, as he is before it; and as he moves to or from the ſpeculum, his image will at the ſame time, move towards or from him on the other ſide, but apparently with a double velocity, becauſe the two motions are equal, and contrarily. In like manner, if while the ſpectator is at reſt, an object be in motion, its image behind the ſpeculum will be ſeen to move at the ſame rate. And if the ſpectator moves, the images of objects that are at reſt will appear to approach or recede from him, after the ſame manner as when he moves towards real objects.

49. *Corol.* 9. Whatever has been ſaid of the image of any object, or primary radiant, holds alſo true of the image of another image, &c. whence by two † plane ſpeculums duly placed, a multiplication of images may be made; and the diſtance of any image will be equal to the length of a ray propagated from the primary radiant through all the intermediate reflections to the eye.

§. 2. *Of Refraction at a plane Surface.*

PROP. III.

Fig. 9, 10. 50. *If* Q *be the focus of incident rays upon a refracting plane* BAC, *and a point* q *be taken in the perpendicular* QA *(produced if need be) to that plane, ſo that* Aq *ſhall be to* AQ, *as the ſine of incidence to the ſine of refraction; or if it be made* R : I :: AQ : Aq, *the point* q *will be the focus of thoſe rays that are neareſt to the perpendicular* QA *after refraction: that is, if the points of incidence* B, C *of the rays* QB, QC *are not remote from* A; *the refracted rays* Be, Cf *produced backwards would meet in* q *nearly.*

Let QC be any incident ray, and C*f* the refracted ray, which produce backwards until it meets the perpendicular AQ ſomewhere, as in *q*. Through the point of incidence C draw EH parallel to QA; then will [a]29.I.*Eucl.* QCE $\doteq$ [a]AQC be the angle of incidence, and HC*f* $=$ [a]A*q*C be the angle

* It is here preſuppoſed that images formed by plane ſpeculums, appear in their true places; and this ſuppoſition is confirmed by univerſal experience. *Vide* Book II. Sect. 3.

† *Vide* Ibid.

angle of refraction. And in the triangle QqC, it will be *As the ſine of* the $\angle$ CQq (AQC), is *to the ſine of the* $\angle$ CqQ (AqC*) *ſo* Cq *to* CQ: that is, I : R :: [b]Cq : CQ. But when the angle of incidence AQC is very ſmall, it will be Cq : CQ :: †Aq : AQ nearly; that is, I : R :: Aq : AQ nearly. And the nearer any incident ray as Qb, or Qa is to the perpendicular QA, the nearer will the ſides qb, Qb, or qa, Qa of the $\triangle$s Qqb or Qqa approach to a †ratio of equality with qA, QA until at length they coincide. And therefore the point q found as above, is without ſenſible error, the focues of the refracted rays. *Q. E. D.*

[b] Conſtruct.

51. *Corol.* 1. If q be the focus of converging rays (eB, fC, *&c.*) Q will be their focus after refraction.

52. *Corol.* 2. Becauſe Aq : AQ :: [a]I : R; it will be, Qq (Aq ~ AQ) : Aq :: $m-n$ (I ~ R) : m or n (I) according as the incidence is into a rare or denſe medium. And if the point of incidence of a ray QC is ſo near to A, that the angles and their ſines may be reckoned proportional, it will be

[a] 50. I.

$\left\{\begin{matrix} m \\ n \end{matrix}\right\}$ (I) : $m-n$ :: $\angle$ *Incidence* AQC : *refracted* $\angle$ Qcq = bCf.

53. *Corol.* 3. The image q of a ſmall object Q placed under water is one fourth nearer to the ſurface than the object. And hence the bottom of a pond of water is one third part deeper than it appears to a ſpectator above. For Aq being [a] $\frac{3}{4}$ of AQ; qQ is $\frac{1}{3}$ of Aq. This may be an uſeful caution to perſons who are not ſwimmers, to prevent their plunging out of their depths unwarily ‖.

Fig. 10. Water is $\frac{1}{3}$ deeper than it appears.

[a] 20 & 52 I.

Hence we have the reaſon of the common phænomenon of a ſhilling or other object placed in an empty veſſel, appearing to be elevated higher and higher as the veſſel is filling with water. Suppoſe the veſſel empty, Bk its ſide, and Q the object at the bottom: If the eye be at e, the object will be hid by the ſide Bk; but by filling the veſſel with water, it will become viſible, and be ſeen at q, the ray QB being refracted into Be. And if the eye be ſo placed as to ſee the object at Q when the veſſel was empty, while it is filling the object will appear to riſe gradually in the line Qq.

Why an object at the bottom of an empty veſſel appears elevated as the veſſel is filling with water.

54. *Coról.* 4. If the refracting medium is glaſs, and the angles of incidence and refraction as AQC, AqC be ſo ſmall as to be reckoned proportional to their ſines; the refracted angle QCq = fch, that is, the angle contained between the incident ray produced and the refracted ray, will be [a] $\frac{1}{3}$ of the greateſt of thoſe angles, and [a] $\frac{1}{2}$ of the leaſt of them. And hence if the angle of incidence be increaſed or diminiſhed, the refracted angle will be increaſed or diminiſhed by $\frac{1}{3}$ of that augmented

Fig. 9, 10.

[a] 20 & 52 I.

or

* By the known rules of Trigonometry.

† This is manifeſt from an inſpection of the figures.

‖ If a ſtrait ſtick be immerſed perpendicularly into water, until the part immerſed appears of equal length with the part above; theſe two parts being meaſured, will be found to be to one another, about as 4 to 3.

cluded between these two lines will be equal to half the length of the object or of its image *pr*. For the triangles PBA, P*pr* being similar, and PB being equal to half P*p*, it will be BA $=\frac{1}{2}$ *pr* $=\frac{1}{2}$ PR. Whence,

47. *Corol.* 7. A person viewing himself in a plane looking-glass placed upright, will see his image compleat in a part of the glass, whose length and breadth is equal to half the length and breadth of the corresponding parts of his own body. And this will be always the case at whatever distance he stands from the glass.

48. *Corol.* 8. *A spectator will see his own image as far beyond the speculum, as he is before it; and as he moves to or from the speculum, his image will at the same time, move towards or from him on the other side, but apparently with a double velocity, because the two motions are equal, and contrarily. In like manner, if while the spectator is at rest, an object be in motion, its image behind the speculum will be seen to move at the same rate. And if the spectator moves, the images of objects that are at rest will appear to approach or recede from him, after the same manner as when he moves towards real objects.

49. *Corol.* 9. Whatever has been said of the image of any object, or primary radiant, holds also true of the image of another image, *&c.* whence by two † plane specula duly placed, a multiplication of images may be made; and the distance of any image will be equal to the length of a ray propagated from the primary radiant through all the intermediate reflections to the eye.

§. 2. *Of Refraction at a plane Surface.*

PROP. III.

Fig. 9, 10. 50. *If* Q *be the focus of incident rays upon a refracting plane* BAC, *and a point* q *be taken in the perpendicular* QA *(produced if need be) to that plane, so that* Aq *shall be to* AQ, *as the sine of incidence to the sine of refraction; or if it be made* R : I :: AQ : Aq, *the point* q *will be the focus of those rays that are nearest to the perpendicular* QA *after refraction: that is, if the points of incidence* B, C *of the rays* QB, QC *are not remote from* A; *the refracted rays* Be, Cf *produced backwards would meet in* q *nearly.*

Let QC be any incident ray, and C*f* the refracted ray, which produce backwards until it meets the perpendicular AQ somewhere, as in *q*. Through the point of incidence C draw EH parallel to QA; then will
[a]29.I.*Eucl.* QCE $=$[a] AQC be the angle of incidence, and HC*f* $=$[a] A*q*C be the angle

* It is here presuppose that images formed by plane specula, appear in their true places; and this supposition is confirmed by universal experience. *Vide* Book II. Sect. 3.

† *Vide* Ibid.

angle of refraction. And in the triangle QqC, it will be *As the ſine of* the $\angle$ CQq (AQC), is *to the ſine of the* $\angle$ CqQ (AqC*) *ſo* Cq *to* CQ: that is, I : R :: [b]Cq : CQ. But when the angle of incidence AQC is very ſmall, it will be Cq : CQ :: †Aq : AQ nearly; that is, I : R :: Aq : AQ nearly. And the nearer any incident ray as Qb, or Qa is to the perpendicular QA, the nearer will the ſides qb, Qb, or qa, Qa of the $\triangle$s Qqb or Qqa approach to a †ratio of equality with qA, QA until at length they coincide. And therefore the point q found as above, is without ſenſible error, the focues of the refracted rays. *Q. E. D.*

[b] Conſtruct.

51. *Corol.* 1. If q be the focus of converging rays (eB, fC, *&c.*) Q will be their focus after refraction.

52. *Corol.* 2. Becauſe Aq : AQ :: [a]I : R; it will be, Qq (Aq ~ AQ) : Aq :: $m-n$ (I ~ R) : m or n (I) according as the incidence is into a rare or denſe medium. And if the point of incidence of a ray QC is ſo near to A, that the angles and their ſines may be reckoned proportional, it will be

[a] 50. I.

$\left\{\begin{matrix} m \\ n \end{matrix}\right\}$ (I) : $m-n$ :: $\angle$ *Incidence* AQC : *refracted* $\angle$ Qcq$=$*h*C*f*.

53. *Corol.* 3. The image q of a ſmall object Q placed under water is one fourth nearer to the ſurface than the object. And hence the bottom of a pond of water is one third part deeper than it appears to a ſpectator above. For Aq being [a]$\frac{3}{4}$ of AQ; qQ is $\frac{1}{3}$ of Aq. This may be an uſeful caution to perſons who are not ſwimmers, to prevent their plunging out of their depths unwarily ‖.

Fig. 10. Water is $\frac{1}{3}$ deeper than it appears.

[a] 20 & 52 I.

Hence we have the reaſon of the common phænomenon of a ſhilling or other object placed in an empty veſſel, appearing to be elevated higher and higher as the veſſel is filling with water. Suppoſe the veſſel empty, Bk its ſide, and Q the object at the bottom: If the eye be at e, the object will be hid by the ſide Bk; but by filling the veſſel with water, it will become viſible, and be ſeen at q, the ray QB being refracted into Be. And if the eye be ſo placed as to ſee the object at Q when the veſſel was empty, while it is filling the object will appear to riſe gradually in the line Qq.

Why an object at the bottom of an empty veſſel appears elevated as the veſſel is filling with water.

54. *Coról.* 4. If the refracting medium is glaſs, and the angles of incidence and refraction as AQC, AqC be ſo ſmall as to be reckoned proportional to their ſines; the refracted angle QCq $=$ *fch*, that is, the angle contained between the incident ray produced and the refracted ray, will be [a]$\frac{1}{3}$ of the greateſt of thoſe angles, and [a]$\frac{1}{2}$ of the leaſt of them. And hence if the angle of incidence be increaſed or diminiſhed, the refracted angle will be increaſed or diminiſhed by $\frac{1}{3}$ of that augmented

Fig. 9, 10.

[a] 20 & 52 I.

or

* By the known rules of Trigonometry.

† This is manifeſt from an inſpection of the figures.

‖ If a ſtrait ſtick be immerſed perpendicularly into water, until the part immerſed appears of equal length with the part above; theſe two parts being meaſured, will be found to be to one another, about as 4 to 3.

or diminiſhed part, if the incidence is from air to glaſs; and by $\frac{1}{3}$ if the incidence is from glaſs to air.

55. *Corol.* 5. Hence, and from *Art.* 39. the aberration ariſing from a fault in the figure of glaſs, if a ray be refracted, and the refraction be from air to glaſs, is but $\frac{1}{6}$ of the aberration of a reflected ray: And the aberration of refraction from glaſs to air is $\frac{1}{4}$ of the aberration of reflection; the angle of incidence in both the caſes of reflection and refraction being the ſame. And if the medium is water, the aberration of refraction will be $\frac{1}{8}$ and $\frac{1}{6}$ of the aberration of reflection ariſing from a fault in the ſurface.

Fig. 13, 14. 56. Scholium. *When an incident ray as* QC *is oblique to the plane refracting ſurface* AC; *the proper focus* x *of the refracted ray* CD, *will be farther from* Q *than the geometrical focus* q; that is, Qx *is always greater than* Qq. *And the more oblique is the incidence*, that is, *the greater is the angle* AQC, *the greater will be the aberration* qx *of the point* x *from* q, *the focus of thoſe rays that are next to the axis or perpendicular* QA. In each figure ſuppoſe a ſtrait line drawn from *q* to C. Now if *q* be ſuppoſed to be the focus of the refracted ray CD, it ought

[a] 50. I. to be, AQ : A*q* :: [a]CQ : C*q* :: [a]I : R. But it is manifeſt that theſe ratio's are not equal. For if QA, and *q*A be made radius's, CQ, and C*q* will be the ſecants of the unequal angles AQC, A*q*C. And becauſe it muſt be AQ : A*q* :: [a]CQ : C*x*, it is manifeſt that in *fig.* 13. C*x* is greater than C*q*; and in *fig.* 14. C*x* muſt be leſs than C*q*; and in both caſes Q*x* is greater than Q*q*. And it is manifeſt alſo, that the greater is the angle AQC, the greater will be *qx*. Therefore, *&c. Q. E. D.*

PROP. IV.

Fig. 13, 14. 57. *The focus* Q *of any incident ray as* QC *upon a refracting plane* AC, *being given*; *to find the point* x *where the refracted ray* CD *produced backwards would cut the axis or perpendicular* QA *(produced if need be) to the refracting plane.*

In the perpendicular AQ, take the point *q* ſuch that AQ : A*q* :: R : I; and through the point *q* parallel to the refracting plane AC, draw the indefinite line *qt*. Let *t* be the point where CQ (produced if need be) cuts the line *qt*: From the center C with the radius C*t* deſcribe the arch *tx*, and the point *x*, where this arch meets the perpendicular AQ, is the focus of the refracted ray CD. For the right-angled triangles

[a] 4. VI *Euc.* QAC, Q*qt* being ſimilar, it will be CQ : Q*t* :: [a]AQ : Q*q*; and there-

[b] Conſtruct. fore C*t* (CQ ∓ Q*t*) = [b]C*x* : CQ :: A*q* (AQ ∓ Q*q*) : AQ :: [b]I : R, and conſequently the ∠ A*x*C = HCD is the angle of refraction *. Therefore, *&c.* *Q. E. I.*

Or,

* The author of this elegant conſtruction was Dr. *Barrow*.

Or the proper focus x of the refracted ray CD, may be otherwiſe found, thus: ſay,

I : R :: *Sine* ∠ AQC : *Sine* ∠ AxC; or R : I :: *Co-ſecant* AQC : *Coſec.* AxC. then, *Co-tangent* AQC : *Co-tangent* AxC :: AQ : Ax.

58. *Lemma.* The verſed ſines of very ſmall arches of unequal circles, having the ſame right ſine common to them all, are reciprocally proportional to the diameters of theſe circles, very nearly. Fig. 13, 14.

From the centers Q, x deſcribe the arches Ca, Cb; then will Qa = QC, and $x b$ = xC. I ſay the verſed ſines Aa, Ab of the unequal arches Ca, Cb having the ſame right ſine CA, are reciprocally as the radii Qa, $x b$ nearly.

For Aa : AC :: [a]AC : 2AQ + Aa; and, [a] 13. VI. *Eu.*

Ab : AC :: [a]AC : 2Ax + Ab; therefore,

Aa : Ab :: 2Ax + Ab : 2AQ + Aa.

But when Aa and Ab are but ſmall in proportion to the diameters 2Ax + 2Ab, 2AQ + 2Aa, it will be without ſenſible error; Aa : Ab :: 2Ax + 2Ab : 2AQ + 2Aa. And the two laſt terms being divided by 2, it will be Aa : Ab :: $x b$: Qa nearly. *Q. E. D.*

59. *Corol.* Aa : Ab :: I : R nearly; and therefore Aa = $\frac{\text{I}}{\text{R}}$ Ab; and Ab = $\frac{\text{R}}{\text{I}}$ Aa. In *fig.* 13. Aa = $\frac{m}{n}$ Ab; and in *fig.* 14 Aa = $\frac{n}{m}$ Ab.

60. *Schol.* The aberration qx is always in the ſame ratio to Aa the verſed ſine of the angle of incidence nearly. And qx = $\frac{m}{n}$ Aa − $\frac{n}{m}$ Aa nearly. For,

CASE 1. *When the incidence is into a denſe medium; it will be,* Fig. 13.

n : m :: [a]AQ : Aq :: AQ + Aa (CQ) : Aq + [†]$\frac{m}{n}$ Aa = [a]Cx = [a]bx = Ax + Ab. Wherefore Ax = Aq + $\frac{m}{n}$ Aa − Ab; and conſequently qx = $\frac{m}{n}$ Aa − Ab = [b]$\frac{m}{n}$ Aa − $\frac{n}{m}$ Aa. *Q. E. D.*

[a] Conſtruct. & 35. I.

[b] 59. I.

CASE 2. *When the incidence is into a rare medium; it will be,* Fig. 14.

m : n :: [a]AQ : Aq :: AQ + Aa (CQ) : Aq + $\frac{n}{m}$ Aa = [a]Cx = [a]bx = Ax + Ab; wherefore Aq = Ax + Ab − $\frac{n}{m}$ Aa; and therefore qx = Ab − $\frac{n}{m}$ Aa = [b]$\frac{m}{n}$ Aa − $\frac{n}{m}$ Aa. *Q. E. D.*

61. *Corol.* Hence, and from *Art.* 20. when the refraction is between glaſs and air, the aberration qx is $\frac{5}{6}$ of Aa, the verſed ſine of the angle of incidence; for $\frac{m}{n}$ − $\frac{n}{m}$ = $\frac{3}{2}$ − $\frac{4}{3}$ = $\frac{9}{6}$ − $\frac{4}{6}$ = $\frac{5}{6}$. In like manner when the refraction is between air and water, qx = $\frac{7}{12}$ Aa.

† For n : m :: Aa : $\frac{m}{n}$ Aa.

PROP. V.

62. *Images formed by refractions at plane ſurfaces, are erect, or alike ſituated with the objects, and on the ſame ſide of the refracting planes.*

Fig. 11, 12. From each point P, R, &c. of an object PQR radiating upon the refracting plane BC; upon the ſaid plane let fall the perpendiculars PB, RC, &c. Now if in the ſaid perpendiculars, the points *p*, *r*, &c. be ſo taken that R : I :: BP : B*p* :: CR : C*r*, &c. the points *p*, *r*, &c. ſo [a] 50. I. found will be the [a] images of the points P, R, &c. and therefore all theſe points connected together, will give the *image* *p q r* of the whole *object* PQR. Whence the whole of the propoſition is ſelf-evident.

63. *Corol.* 1. If the object be rectilinear, its image will be ſo too; and the correſponding parts of both will be ſimilar. All other objects and their images, will be ſimilar, if they lie parallel to the refracting plane; otherwiſe not. For if they be oblique, the image of a ſquare may be an oblong; and the image of an oblong parallellogram, may be a ſquare; as is manifeſt from the preceding, and the inſpection of the figures.

64. *Corol.* 2. If *e* be the pupil of the eye, the points P, R of the object will be ſeen by rays *ae*, *be* diverging from the focus's *p*, *r* nearly. When the eye is ſo ſituated that the axis of the pencil entering the pupil is perpendicular to the refracting plane; the focus of that pen- [a] 50. I. cil is [a] accurately the correſponding point of the image. But when the rays of the ſaid pencil are oblique to the refracting plane, thoſe rays if [b] 56. I. produced backwards would unite in a point [b] farther from the object than the correſponding point of the image; and the diſtance of that point from the image will be more or leſs, according to the greater or leſſer obliquity of the rays to the refracting ſurface. And hence it is that objects in a different medium from that wherein is the ſpectator, generally appear a little diſtorted.

65. *Corol.* 3. Objects under water, to a ſpectator above, appear one fourth higher than they really are; and alſo ſomewhat diminiſhed, and diſtorted, if they are not parallel to the ſurface. And to a ſpectator under water, objects that are above appear one third higher than they really are: And if they are oblique to the ſurface of the water, they'll appear ſomewhat magnified and diſtorted.

Fig. 10. Why a ſtrait ſtick immerſed in water appears bent.

66. *Corol.* 4. From what has been above delivered, we have alſo the reaſon why a ſtrait ſtick STQ immerſed obliquely into water, appears bent at the ſurface: the end Q appearing in *q*, and every other part by the refraction of the rays at the ſurface, being elevated in proportion, the whole ſtick appears under the form ST*q*; that is, the part under water

water appears alſo *ſtrait, as T*q*, the only bending being at the ſurface T.

§. 3. *Of the Progreſs of Light through a Medium terminated by plane Surfaces.*

PROP. VI.

67. *A ray of light as* QA, *falling obliquely upon a medium terminated by two parallel plane ſurfaces* RS, TV, *will after its ſecond refraction at* B, *emerge in a line* BC *parallel to the incident ray* QA. Fig. 15.

Produce the incident ray QA to D, and the emergent ray BC to E; and through the points A, B draw the perpendiculars PAO, FBG to the ſides RS, TV. Now becauſe the ſides RS, TV are parallel, the ray AB within the medium is equally inclined to them both; that is, the ∠ OAB = [a]FBA; wherefore if theſe be conſidered as angles of incidence, that is, if the ray be conceived to go both ways along AB, the ∠s PAQ, GBC will be [b]equal; and conſequently their oppoſite ∠s OAD, FBE are [c]equal; and taking away from theſe the equal angles OAB, FBA, the alternate angles DAB, EBA will be equal; and therefore QAD is [d]parallel to CBE. *Q. E. D.*

[a] 29. I. *Euc.*
[b] 13 & 16. I.
[c] 15. I. *Euc.*
[d] 27. I. *Euc.*

68. *Schol.* It is manifeſt that the leſs is the thickneſs of the medium RV, or the leſs is the obliquity of the incident ray, the leſs will be the diſtance between the parallel lines AD, BC: And when RV is very thin, or the angle QAP is very ſmall, the ſaid diſtance will be ſo inconſiderable, that the ray QABC may without ſenſible error be reckoned as one ſtrait line.

PROP. VII.

69. *If a ray* QABC *be tranſmitted through a medium* RST *terminated by two plane ſides* RS, TS *inclined to one another in an angle* RST; *the emergent ray* BC, *will incline from the incident ray* QA, *towards the thickeſt part* RT *of the medium: And if the angle* S, *and the angles of incidence and refraction be ſo ſmall, as to be reckoned proportionable to their ſines; the angle of deviation* QEq, *contained between the incident ray* QAE, *and the emergent ray* CBE*q*, *produced till they meet in* E, *will be to the refracting angle* RST, *as the difference of the ſines of incidence and refraction is to the leſſer of them; that is,* ∠ QE*q* : ∠ RST :: *m*—*n* : *n*. Fig. 16.

Let A*a*, B*b* be perpendicular to the ſides RS, TS reſpectively, and produce them till they meet in D: Alſo produce the incident and emergent rays QA, CB till they meet in E. Now if it be made, *m* : *n* :: Sine ∠ *a*AE : Sine ∠ *a*AB; and *n* : *m* :: Sine ∠ *b*BA : *b*B*q*; *q*B produced will be the emergent ray: whence it is manifeſt,

* For if perpendiculars be drawn from every point of the ſtrait object TQ to the plane ſurface TA, the images of theſe points will divide all the ſaid perpendiculars proportionably. Whence from the 2 of the VI. of *Euclid*, it is eaſy to ſhew, that all theſe images lie in a ſtrait line.

1. That the emergent ray BC, is more inclined towards the thickeſt part RT of the medium, than the incident ray QA: that is, the emergent ray CB produced backwards, lies between QA and S.

[a] Hyp. 2. Becauſe $\angle bBA : \angle bBq :: {}^{a}n : m :: {}^{a}\angle DAB : \angle DAE$; it will alſo be, $\angle bBA : \angle ABq\ (\angle bBq - \angle bBA) :: n : m-n :: \angle DAB : \angle BAE$ [b] 32. I. *Euc.* $(\angle DAE - \angle DAB)$. Wherefore $\angle bBA \doteq \angle DAB\ (= {}^{b}bDA =$ [c] Conſtruct. $^{c}RST) : \angle ABq \doteq \angle BAE\ (= {}^{b}AEq) ::$ [d] 12. V. *Euc.* $^{d}n : m-n$; that is,

$$AEq : RST :: m-n : n. \quad Q.E.D.$$

70. *Corol.* 1. Hence the angle of deviation AEq is invariable in all poſitions of the ray; that is, whether the incident and emergent rays are equally inclined to the refracting ſurfaces, as in *Examp.* 1. or, whether one of them is perpendicular to one of the refracting ſurfaces, and therefore the whole refraction is at the other ſurface, as in *Examp.* 2. or, whether they incline to different ſides of the perpendicular, as in *Examp.* 3. The angle of deviation AEq in all theſe caſes will be the ſame, if the angles of incidence and refraction are ſo ſmall, as to be reckoned proportionable to their ſines.

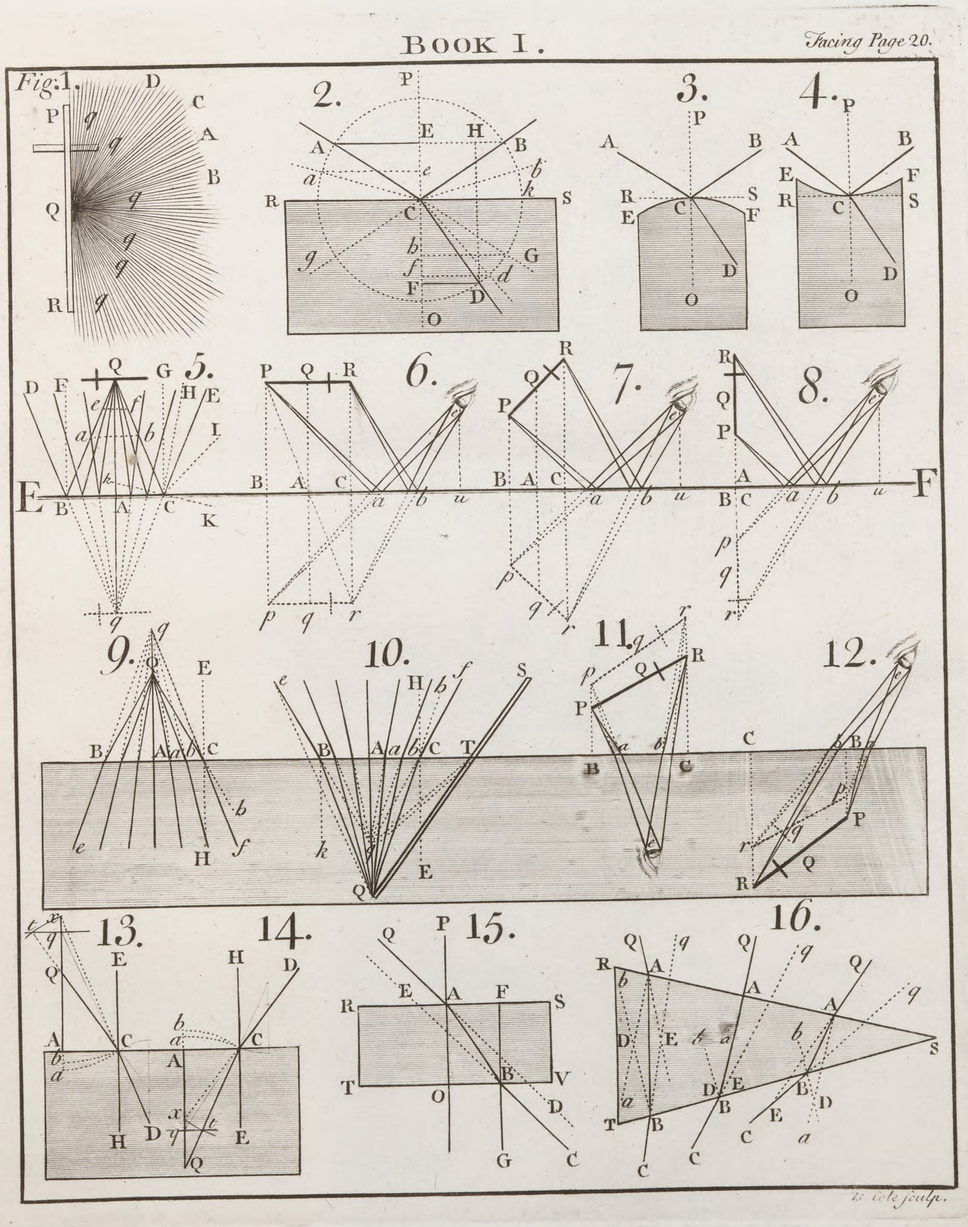
Fig. 16. 71. *Schol.* Becauſe the angle AEq is leſs than S, and arches have a leſs proportion to their ſines, the ſmaller they are; the error ariſing from the ſuppoſition of the arches and their ſines being proportional, will make the angle AEq have a greater ratio to the angle S, than that of $m-n$ to n. And the leſs is the greateſt obliquity of the ray to either of the refracting ſides of the medium, as in *Examp.* 1. where the ray is equally inclined to each of the ſides RS, TS; the nearer will the ratio of the angle AEq to S, be to an equality with the ratio of $m-n$ to n. And the converſe. But in moſt caſes the inequality of theſe ratio's need not be conſidered.

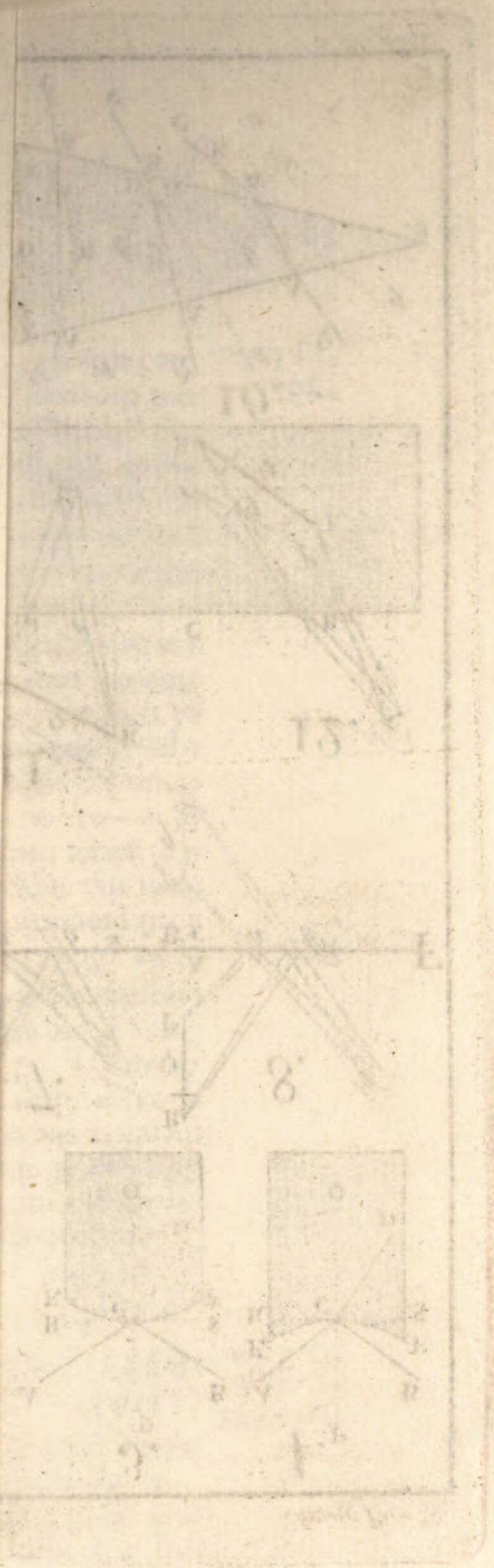
Fig. 18. 72. *Corol.* 2. When the angles of incidence and refraction are ſo ſmall, as to be reckoned proportional to their ſines, the angle Q contained between two incident rays QA, QD upon a triangular priſm RST, will be equal to the angle q contained between the emergent rays BC, EF produced backwards till they meet in q. For in the triangles Qob, qoe, the angles Qbq, qeQ contained between the incident rays QA, QD produced, and the emergent rays BC, EF produced backwards, being [a] equal, and the vertical angles at o being [b] equal, the angles Q, q muſt be equal.

[a] 70. I.
[b] 15. I. *Euc.*

PROP. VIII.

Fig. 17. 73. *If* Q *be the focus of rays diverging upon a medium* AE, *terminated by two plane and parallel ſides* AD, BE; *the focus* q *of the emergent rays will be between* q *and the medium: and* Qq *the diſtance between*


Fig. 1.
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tween the two focus's will be to DE *the thickneſs of the medium, as* m—n *to* m.

Let QDE be perpendicular to the refracting ſurfaces AD, BE, and let QA be any incident ray not very remote from the axis QD. Now, if in the axis DQ produced, the point *a* be taken ſuch, that QD : D*a* :: [a]*n* : *m*; *a* will be the focus of the refracted ray AB. And if the point *q* be taken ſuch, that *a*E : *q*E :: [a]*m* : *n*; *q* will be the focus of the emergent ray BC. Wherefore, *a*Q (*a*D—QD) : *a*D :: *m*—*n* : *m* :: *aq* (*a*E—*q*E) : *a*E :: *a*Q+Q*q* : *a*D+DE (*a*E.) Whence *aq* is greater than *a*Q; and Q*q* : DE :: [b]*m*—*n* : *m*. *Q. E. D.*

[a] 35 & 50. I.

[b] 19. V. *Euc.*

74. *Corol.* When the refracting medium is glaſs, it will be Q*q* = $\frac{1}{3}$DE nearly.

75. Schol. *The aberrations at the different ſurfaces, are contrary ways, and almoſt deſtroy each other.*

Let *x* be the proper focus of the refracted ray AB; *y* the geometrical focus of rays within the medium, whoſe focus of incidence is *x*, after their emerſion at the ſide BE; that is, *x*E : *y*E :: *m* : *n*; and *r* the proper focus of the emergent ray BC. Now the aberrations *xa*, *yr* are in the ratio of the ſines nearly; that is, *xa* : *yr* :: [a]*m* : *n* nearly. But *m* : *n* :: [b]*x*E : *y*E. Wherefore,

*x*E—*xa* (*a*E) : *y*E—*yr* (*r*E) :: *m* : *n* :: [b]*a*E : *q*E. And therefore the point *r* coincides with *q* nearly. *Q. E. D.*

[a] 58, 59, & 60. I.

[b] Conſtruct.

Otherwiſe thus: produce QA to *s*, and draw A*t* parallel to QE, and *tu* parallel to QA; then will Q*u* = A*t* = DE. Now the ray CB produced backwards to *q* is [c]parallel to AQ; and *st* : B*t* :: [d] *m* : *n*, nearly. But Q*u* : *qu* :: [e] *st* : B*t*; and therefore Q*q* : Q*u* :: *m*—*n* : *m*; that is, the point *q* found as in the *Prop.* is without ſenſible error the focus of the emergent ray BC. *Q. E. D.*

[c] 67. I.

[d] 35. I.

[e] 2. VI. *Euc.*

PROP. IX.

76. *Having the focus of incident rays, upon a medium terminated by two plane ſides inclined to one another in a given angle, to find the focus of the emergent rays.*

Let Q be the focus of incident rays, QA, QD upon the triangular priſm RST. Upon the ſide SR (produced if need be) let fall the perpendicular QG, and in this perpendicular take the point *a* ſuch that G*a* : G*a* :: *n* : *m*, then will *a* [a]be the geometrical focus of the pencil of refracted rays AB, DE within the priſm. Again in the perpendicular *a*H to the ſide TS (produced if need be) take the point *q* ſuch, that H*a* : H*q* :: *m* : *n*; and [a] *q* will be the geometrical focus of the pencil of emergent rays BC, EF. *Q. E. I.*

Fig. 18.

[a] 50 & 35. I.

77.

77. *Schol.* The aberrations of the rays AB, DE; and of BC, EF from the focus's *a*, *q* may be found, as in *Art.* 57: The proper foci of the ſaid rays, will be in lines without, and parallel to, the line *a*H; and the more oblique is the incidence of any ray QA or QD, the more remote will the focus's of the refracted rays be from the line *a*H.

78. *Corol.* The line Q*q* joining the focus's Q, *q* is parallel to the line GH, joining the points G, H where the perpendiculars meet the refracting ſides, RS, TS.

PROP. X.

Fig. 19, 20. 79. *The image* pqr *of an object* PQR, *formed by the refraction of a medium terminated by two plane and parallel ſides* BC, bc; *is erect, equal and parallel to the object, and on the ſame ſide of the medium, but nearer to it by* $\frac{m-n}{m}$ *part of the thickneſs of the medium.*

Draw the perpendiculars PB*b*, QA*a*, RC*c*, &c. from each point of the object to the refracting ſurfaces, then will the diſtances of the ſeveral correſponding focus's P*p*, Q*q*, R*r*, &c. be each to the thickneſs
[a] 73. I. of the medium A*a* or B*b*, &c. as [a] *m* — *n* to *m*; that is Q*q* = $\frac{Aa \times m-n}{m}$. *Q. E. D.*

80. *Corol.* When the refracting medium is glaſs, the diſtance between the object and the image will be $\frac{1}{3}$ of the glaſs's thickneſs.

Fig. 21. 81. *Schol.* 1. The image *pq* of an object PQ formed by rays refracted by a triangular priſm RST, may be determined by finding the images of each point of the object, after the manner ſhewed in *Art.* 76. Thus, from each point Q, P, &c. of the object upon the refracting ſide SR (produced if need be) let fall the perpendiculars QG, PI, &c. and in theſe perpendiculars take the points *a*, *b*, &c. ſuch that *n* : *m* :: GQ : G*a* :: IP : I*b*, &c. From the points *a*, *b*, &c. thus found, upon the ſide TS produced, let fall the perpendiculars *a*H, *b*K, &c. and in theſe perpendiculars take the points *q*, *p*, &c. ſuch, that *m* : *n* :: H*a* : H*q* :: K*b* : K*p*, &c. and the points *q*, *p*, &c. thus found will be the places of the images of the points Q, P, &c.

82. *Schol.* 2. From the conſtruction of the figure, it appears that the image is not exactly parallel to the object; nor are the object and image exactly equal as to length, and conſequently if they have any breadth they will not be exactly ſimilar; for there will be no inequality betwixt thoſe ſides and their images, that are parallel to the axis of the priſm. But becauſe of the great obliquity of the rays to the refracting ſides, the apparent images, or the images of objects ſeen by rays tranſmitted through two ſides of a triangular priſm, will, in many caſes, be very obſcure and imperfect; and differ greatly as to likeneſs, ſituation, &c. from thoſe found by theory.

SECTION

SECTION III.

Of the Reflection and Refraction of Light at a Spherical Surface.

§. 1. *Of Reflected Rays.*

83. *Defin.* LET O be the center of the ſpherical reflecting ſurface or ſpeculum BAC, then will any line as OA drawn from O to the ſpeculum be perpendicular thereto; and therefore be the axis of a pencil of rays whoſe focus is any where in the ſaid perpendicular. Any line OA drawn from the center to the ſpeculum, is alſo called the *Axis of the Speculum*; but more particularly that line, as OA, is called the *Axis* which is drawn from the center to the middle of the ſpeculum. Let F be the geometrical focus of rays parallel to the axis, after reflection, or the focus of thoſe rays that are neareſt to it; then is F called the *Focus of the Speculum*; alſo the *principal Focus*, to diſtinguiſh it from the focus of diverging or converging rays; alſo F in *fig.* 22. is called the *burning Focus*, becauſe it is the focus of the ſun's rays. Fig. 22, 23.

Axis, Focus of a Speculum.

PROP. XI.

84. *When parallel rays are incident upon a ſpherical ſpeculum* BAC, *the geometrical focus* F, *of the reflected rays, will biſect that radius* OA, *which is parallel to the incident rays.* Fig. 22, 23.

Let DB be an incident ray parallel to the radius OA, and from the center O to the point of incidence B, draw the perpendicular OB, and produce it *fig.* 23. to *o*; alſo in that figure produce DB to *d*, and let *f*BF be a ſtrait line. Now the angle of reflection OBF or *o*B*f* = [a] angle of incidence DBO or DB*o* = [b] angle BOF, therefore is OF = [c] FB. But when the triangle OFB is vaniſhing, that is when the point B is very near to A, FB will be = FA nearly, and therefore OF will be equal FA nearly. Therefore, *&c.* *Q. E. D.*

[a] 11. I.
[b] 29. I. *Euc.*
[c] 6. I. *Euc.*

85. *Corol.* 1. If F be the focus of incident rays, the geometrical focus of the reflected rays will be at an infinite diſtance; that is, rays diverging from the principal focus of a concave ſpeculum, or converging towards the ſaid focus of a convex ſpeculum, will have the reflected rays next the axis, parallel thereto.

86. *Corol.* 2. When the angle of incidence DBO (= BOA) is conſiderable, that is, when the point of incidence B is at a ſenſible diſtance from A, the reflected ray will interſect the axis in ſome point *x*, between Fig. 24.

tween F and the vertex A of the ſpeculum; and the greater is the angle BOA, the greater will be the *aberration* Fx from the geometrical focus
[a] 84. I. F. For Ox = [a] xB, and Ox + xB is greater than OB or OA; and therefore Ox is greater than ($\frac{1}{2}$OA) OF. And it is manifeſt that the greater is the angle O, the more will Ox exceed OF.

When the ∠ O is 45°, the ∠ OxB will be 90°; that is, the ray will be reflected along the right ſine BV of the angle O. If the angle O is 60°, the triangle OxB will be equilateral, and the point x will coincide with A; that is, the ray will be reflected into the vertex A of the ſpeculum.

Fig.22,23. 87. *Corol.* 3. Hence, if F be the focus of incident rays, the rays that are remote from the axis, will after reflection by a concave ſpeculum *converge*; and *diverge* after reflection by a convex ſpeculum.

Fig. 24. Longitudinal and lateral Aberration. 88. *Defin.* Let FA be the axis of a pencil to which the ray Bx belongs; F the geometrical focus of that pencil, and x the proper focus of the ray Bx. At the point F upon the axis FA raiſe the perpendicular FG, and produce it till it meets Bx produced in G. Now Fx is called the *longitudinal Aberration*, and FG the *lateral Aberration*, of the ray Bx.

PROP. XII.

Fig. 24. 89. *Let* DB *be an incident ray parallel to the axis* OA; BT *a tangent at* B *to* AB; *then will* OT *be the ſecant of the angle* O = DBO *the angle of incidence. I ſay, the longitudinal aberration* Fx *of the reflected ray* Bx, *is equal to half* AT *the exceſs of the ſecant* OT *of the angle of incidence above the radius* OA *or* OB.

[a] Conſtruct. For the ∠ xBT = [a] 90° − ∠O(=OBx) and alſo the ∠T = [b] 90° − ∠O; and therefore xT = [c] xB = xO. Wherefore (becauſe OT = 2Ox = 2OF + AT) Ox = OF + $\frac{1}{2}$AT; that is, Fx = $\frac{1}{2}$ AT. *Q. E. D.*

[b] 32. I. *Eucl.* [c] 6. I. *Eucl.*

And thus having the angle of incidence given, by a table of ſecants we have Fx; or, in the triangle OxB, we have OB and all the angles, to find Ox.

90. *Corol.* 1. When the angle of incidence is not very large, the lateral aberration FG = $\frac{VB^3}{2OA^2}$ nearly. For VA or AT = [d] $\frac{VB^2}{2OA}$ nearly,
[d] 13. VI. *Eu.* and ſo Fx = $\frac{VB^2}{4OA}$ nearly. But FG : Fx $\left(\frac{VB^2}{4OA}\right)$ :: VB : xV = $\frac{1}{2}$ OA nearly. Therefore, *&c.*

91. *Schol.* 1. When the angles of incidence are not very large, the *longitudinal aberrations* will be *as the ſquares*, and *the lateral ones as the cubes, of the ſines of theſe angles*, nearly.

Draw

Draw the right line BV. Now the right-angled triangles OVB,BVT being similar, it will be,

OV (OA—VA) : VB :: VB : VT; But when the arch AB is very small, VA will be incomparably less than the invariable quantity OA; and therefore a small augmentation or diminution of the arch AB, will not sensibly affect the first term OV; whence VT will be as VB^2, nearly. Also in this case, VA is nearly equal to AT; for VA (OB—OV) : AT (OT — OB) :: OV : OA; and therefore Fx ($= \frac{1}{4}$VT nearly) will be as VB^2 nearly. *Q. E. D.*

Again, If whilst the $\angle$O is increasing or diminishing, Fx be supposed to remain invariable; FG will be as VB, for the $\triangle$s xFG, xVB are similar: And if during the augmentation of the $\angle$ O, the $\angle x$ be supposed to remain invariable, or that BG moves parallel to itself; FG will be as Fx, that is, as VB^2. Whence it is manifest, that since Fx and the angle x are together both increased or both diminished, FG will be as $VB \times VB^2$; that is, as VB^3 nearly. *Q. E. D.*

92. *Schol.* 2. When the arch AB has a sensible magnitude, the *increase* or *diminution* of VT will be greater than in the ratio of VB^2, and *that* of VA will be less than *that* of AT (which is always as Fx), and therefore *that* of VT will be less than *that* of Fx: And consequently the ratio of Fx is still greater than that of VB^2; and therefore also the ratio of FG (which with respect to Fx increases or diminishes as VB^3 to VB^2) is greater than that of VB^3. However in most cases, VB^2 and VB^3 are sufficiently accurate for estimating the ratio's of the aberrations, of rays reflected from different distances from the axis.

93. *Corol.* 2. The rays that are nearest the axis, are reflected much closer together, than those that are remote: And therefore the burning focus of the speculum, or the point in the axis where most rays are collected together, will be in F, or very near to F.

It is also easy to conceive, that the reflected rays are denser or closer together between F and the speculum, than they are at the same distance from F on the contrary side.

PROP. XIII.

94. *Let* QA *be perpendicular to the spherical reflecting surface* BA, *whose center is* O, *and principal focus* F; *and let* Q *be the focus of incident rays: Now if it be made* FQ : FO :: FO : Fq; *and the point* q *be placed the same way from* F *that* Q *is from* F, *the point* q *will be the geometrical focus of the reflected rays.* Fig. 25 *to* 28.

Let B be a point of incidence not remote from A, to which from the center O draw the perpendicular OB, and parallel to the incident and reflected rays QB, B*q*, draw the lines O*a*, O*b*; then will the tri-

[a] 29. I *Eucl.* [b] 4. VI. *Euc.* [c] 11. I. angles QO*b*, O*qa* be [a] ſimilar, and therefore Q*b* : O*b* :: [b] O*a* : *qa*. Now the triangles O*b*B, O*a*B having the common baſe OB, are equicrural and alſo equal to each other; (for the ∠ *b*BO = [c] OB*a*, and the alternate ∠ *a*OB = [a] *b*BO, alſo *b*OB = [a] *a*BO) and therefore when theſe triangles are vaniſhing, that is, when B comes to A, the ſides O*b*, O*a* will be each = ½ OB = OF, and the points *b*, *a* will coincide with F: Wherefore it will be QF : OF :: OF : *q*F. *Q. E. D.*

The figures ſerve for the caſes of a *convex* as well as for a concave ſurface, by ſuppoſing the incident and reflected rays to be produced backwards through that ſurface.

[94.] *Corol.* 1. QO (QF ∓ OF or OF ∓ QF) : OF :: *q*O (OF ∓ *q*F or *q*F ∓ OF) : *q*F. Alſo QO : Q*q* (QO ± *q*O or *q*O ± QO) :: OF (AF) : A*q* (AF ± *q*F or *q*F ± AF.)

95. *Corol.* 2. If *q* be the focus of incident rays, Q will be the focus of the reflected ones. Compare together *fig.* 25, 27; and alſo *fig.* 26, 28.

96. *Corol.* 3. Concave and convex ſpeculums (that are ſegments of the ſame ſphere) are juſt oppoſite in their effects; that is, rays falling upon them the juſt contrary ways, will have the ſame focus after reflection.

Fig. 25, 27. 97. *Corol.* 4. Both the focus's are united in O; that is, when one is in O the other will be alſo in O, and their motion from O is the contrary way, but with a very different velocity; the one being continually retarded or accelerated, as the other moves uniformly from or towards F: So that when one is in F, the other will be at an infinite diſtance (for when QF or *q*F is infinitely little, the other muſt be infinitely great.) And when *(fig. 25.)* QO = OF, O*q* will be = *q*F (for then QF = 2 OF; and therefore *q*F = ½ OF). So that an image moves as far, whilſt the object moves back from the center of a ſpeculum half the length of the radius of the ſpeculum, as it would afterwards, if the object ſhould proceed to an infinite diſtance.

Fig. 26, 28. 98. *Corol.* 5. When one of the focus's is between F and A, the other will be on the contrary ſide of the ſpeculum: Alſo their motions will be contrary, as before; that is, as one moves towards or from the ſpeculum on the one ſide, the other will alſo move towards or from the ſpeculum on the other ſide; but with a different velocity: For when one is in F, the other will be at an infinite diſtance, and yet they'll both arrive at A, which is the place where they ultimately unite, in the ſame time. When one is half way between F and A, the other will be within twice that diſtance of A on the other ſide. The motions of the two focus's on each ſide of A, being exactly like to their motions on each ſide of O, in the preceding Corollary.

99.

99. *Corol.* 6. If the focus of incident rays, diverging upon a concave ſpeculum, or converging upon a convex ſpeculum, be farther from the ſpeculum than its principal focus F; the focus of the reflected rays will be alſo on that ſame ſide of the ſpeculum; that is, the rays reflected by the concave ſpeculum will have a real focus, and thoſe reflected by the convex ſpeculum will diverge from an imaginary focus: And the contrary happens, when the focus of incident rays is between F and the ſpeculum; that is, the focus of the reflected rays will be on the other ſide of the ſpeculum; the rays reflected from the concave ſpeculum diverging from, and thoſe reflected by the convex ſpeculum converging towards, that ſaid focus. Fig.25,27. Fig.26,28.

100. *Corol.* 7. When the focus's Q, *q*, lie on the ſame ſide of the reflecting ſurface; if the incident rays diverge from Q, the reflected ones will converge towards *q*; and if the incident rays converge towards Q, the reflected ones will diverge from *q*. But when the focus's Q, *q*, are on contrary ſides of the ſurface: If the incident rays diverge from, or converge towards Q, the reflected rays will accordingly diverge from or converge towards *q*. Fig.25,27. Fig.26,28.

101. *Schol.* When Q is in O, all the rays reflected by the ſpeculum will either accurately converge to O, or diverge from O, without any aberration. But in all other caſes, the focus *q*, found as by the Propoſition, will be the focus of thoſe rays only that are neareſt the axis; all others after reflection, interſecting the axis at a leſſer or greater diſtance from *q*, as their incidence is leſs or more oblique.

PROP. XIV.

102. *Having the focus* q *and the point of incidence* B, *of an incident ray* qB, *together with the geometrical focus* Q *of the reflected rays; to find the proper focus* Y, *of the ray* qB *after reflection, it will be* Fig.27,28.

$\overline{qO \mp \frac{1}{2}}$ *ſecant of* AB $\times \overline{\frac{1}{2}}$ *Radius* OA : $qO \mp \frac{1}{2}$ *Rad.* $\times \overline{\frac{1}{2}}$ *ſec. of* AB :: OQ : OY.

Parallel to the incident and reflected rays *q*B, BY, draw the lines O*b*, O*a*: draw alſo the tangent B*t*, and parallel thereto through the point *a* or *b*, draw the line *ade*; then will *ad* biſect OB at right angles, For the △ O*a*B or O*b*B [a] is equicrural; and therefore $Oe = [b] et =$ $\frac{1}{2}$ ſecant of the arch AB. Now,

[a] Conſtruct.
[b] 2. VI *Euc.*

*q*Y : OY :: [b] *q*B : (O*b*) *a*B :: [b] *qt* : *et*; and therefore,
qe (*qt* $\mp$ *et*) : O*e* ($=$ *et*) :: *q*O (*q*Y $\mp$ OY) : OY; Alſo
OF : *q*F :: [c] QO : *q*O; Wherefore, [c] [94.] I.
qe $\times$ OF : O*e* $\times$ *q*F :: (*q*O $\times$ QO : OY $\times$ *q*O ::) OQ : OY; that is,
$\overline{qO \mp \frac{1}{2}}$ *ſecant* $\times \overline{\frac{1}{2}}$ *Rad.* : $\overline{qO \mp \frac{1}{2}}$ *Rad.* $\times \overline{\frac{1}{2}}$ *ſecant* :: OQ : OY. *Q.E.D.*

Or,

Or, the point Y may be found by Trigonometry, thus: In the △ qOB, are given the ſides qO, OB, and the ∠ included qOB, to find the ∠ qBO: then in the △ OBY will be given the ſide OB, and all the angles, to find OY.

103. *Corol.* 1. OQ is to QY, as the firſt term is to the difference of the ſecond and firſt; that is, OQ : QY :: $qe \times$ OF : O$e \times q$F — $qe \times$ OF.

104. *Corol.* 2. If YB is an incident ray, q will be its focus after reflection. And if QB is an incident ray, its focus y after reflection will be ſuch, that the ∠ qBy = QBY: and therefore the aberration QY being found, the aberration qy is alſo eaſily found. For in the △ qBy, we may have the ſide qB and all the ∠s, to find qy.

105. *Corol.* 3. OQ is leſs than OY. And therefore when both the
Fig. 27. focus's Q,q, are on the ſame ſide of the ſpeculum, the proper focus Y or y, of a ray qB or QB after reflection, will be nearer the ſpeculum than the geometrical focus Q or q. Alſo when the focus's Q,q are on different ſides; if the focus q of the incident ray be of the convex-ſide of the ſpeculum, the proper focus Y of the reflected ray, will be between the
Fig. 28. geometrical focus Q and the ſpeculum: But if the focus Q of the incident ray, be between F and the ſpeculum; the proper focus y of the reflected ray, will be farther from the ſpeculum than the geometrical focus q.

106. *Corol.* 4. When q (the focus of incidence) is in O, the points Q,Y, will both coincide with O: And the arch AB remaining the ſame; the greater is Oq, if the focus's are both on the ſame ſide, otherwiſe the leſſer is Oq, the greater will be the aberration QY; and when OQ is infinite, QY becomes = Fx (*fig.* 24.) the aberration of parallel rays. And Oq remaining the ſame, the greater is the arch AB, the greater will be the aberration QY.

107. *Corol.* 5. The arch AB remaining the ſame, the aberration QY continually increaſes, as the point Q or Y is farther removed from O towards A: And when it is greateſt of all, that is, when Q, and conſequently q is in A, it will be equal to twice the exceſs of the ſecant of the arch AB above the radius, nearly.

Fig. 28. Draw the chord AB, then will the ∠ ABt be the complement of the angle of incidence, and conſequently the reflected ray will belong to the focus S; ſo that the ∠ SBt = ABt. And therefore AS = 2At nearly, is the aberration.

[a] 102. I. 108. *Corol.* 6. Becauſe $qe \times$ OF : qF $\times$ Oe :: [a]OQ : OY; And

[b] [94.] I. qF : OF :: [b]qO : OQ; it will alſo be, qe : Oe :: qO : OY; which is a very eaſy rule for finding the point Y.

Y. That is, $qO \doteq \frac{1}{2} sec. : \frac{1}{2} sec. :: qO : OY$. Alſo $qO \doteq \frac{1}{2} Rad. : \frac{1}{2} Rad. :: {}^{b}qO : OQ$.

109. *Corol.* 7. To find the lateral aberration QG; having drawn the ſine BV, it will be YV : VB :: QY : QG. Whence, the nearer the point Y is to V, the greater is the proportion of QG to QY. And when Y is in V, QG will be infinite. In like manner when the focus of the reflected rays, is farther from the ſpeculum than the center O, the farther it is from O, the leſs proportion will the lateral aberration bear to the longitudinal.

§. 2. *Of Images formed by rays reflected at a ſpherical Surface.*

PROP. XV.

110. *Having the poſition of an object* PQR, *with reſpect to a given ſpherical ſpeculum* BAC; *to find the image* pqr. Fig. 29 *to* 32.

Through the center O of the ſpeculum, and as many points P, Q, R, *&c.* of the object as are neceſſary, draw the lines POB, QOA, ROC, *&c.* and biſect the perpendiculars OB, OA, OC, *&c.* in E, F, G, *&c.* Then in theſe perpendiculars (produced if need be) find the points *p*, *q*, *r*, *&c.* by ſaying PE : OE :: OE : E*p*; and QF : OF :: OF : F*q*; alſo RG : OG :: OG : G*r*, *&c.* and lay theſe points on the ſame ſide of the arch EFG, that the object is of; then will the points *p*, *q*, *r*, *&c.* thus found [a] be the *geometrical Foci* after reflection, of thoſe pencils whoſe foci of incidence were the points P, Q, R, *&c.* And by connecting theſe points orderly together, we ſhall have the *Image* pqr *of the whole Object* PQR. *Q. E. I.*

[a] 94. I.

After the ſame manner as the images of phyſical lines are found, by dividing them into phyſical points; ſo alſo may the images of ſurfaces be found by dividing them into phyſical lines, and connecting together the images of thoſe lines.

In the figures, to prevent too great a confuſion of lines, there are drawn lines only from the extreams and middle part of the object; but every other point of the object as well as theſe is underſtood to propagate rays of light to the ſpeculum. Alſo in the pencils beſides the axes, POB, QOA, ROC there are drawn only two rays P*b*, P*b*; Q*a*, Q*a*; R*c*, R*c*; one on each ſide the reſpective axes at a ſmall diſtance from them, the rays that are between theſe contributing moſtly towards forming the image; the others, whoſe incidence are more oblique, having their proper foci after reflection, at [b] ſome diſtance from the geometrical *Foci* *p*, *q*, *r*.

[b] 105. I.

111. *Corol.* 1. The lengths of the object and image are to each other in the ratio of their diſtances from the center O of the ſpeculum; that is, PQR : *pqr* :: OQ : O*q*. And ∠ POR = ∠ PO*r*; that is, the object and its image form both the ſame angle, at the center O of the ſpeculum.

112. *Corol.* 2. When the object and its image are on different ſides of the center, as in *fig.* 29, 30. the image will be inverted: But when they are on the ſame ſide, as in *fig.* 31, 32. the image will be erect.

113. *Corol.* 3. A convex ſpeculum, *fig.* 32. cannot form a real image; nor a concave one, when the object is betwixt it and its principal focus, as in *fig.* 31. the reflected rays repreſented by *bb*, *bb*; *aa*, *aa*; *cc*, *cc*; in theſe two caſes diverging as from the points *p*, *q*, *r* on the other ſide of the ſpeculum: And theſe alſo are the only two caſes where the image is erect. A real image, as in *fig.* 29, 30. being always inverted.

114. *Corol.* 4. If the object PQR be an arch of a circle, concentric to the ſpherical ſurface BAC, its image *pqr* will be alſo a ſimilar concentric arch. And ſo will alſo a ſpherical ſegment, formed by the rotation of the arch PQR round the axis OA, have a ſimilar concentric ſegment for its image.

115. *Corol.* 5. In all other caſes, excepting as in the laſt article, the object and its image will not be exactly ſimilar. But as a ſmall arch differs inſenſibly from a ſtrait line; ſo a ſmall ſtrait object at a good diſtance from the ſpeculum, may be reckoned to have a ſtrait image very nearly.

116. *Schol.* 1. The image of a ſtrait object is curvilineal, and its curvity is the ſame way with that of the ſpeculum *. And the nearer the object is to the principal focus F, *fig.* 30, 31. the greater will be the curvity of its image.

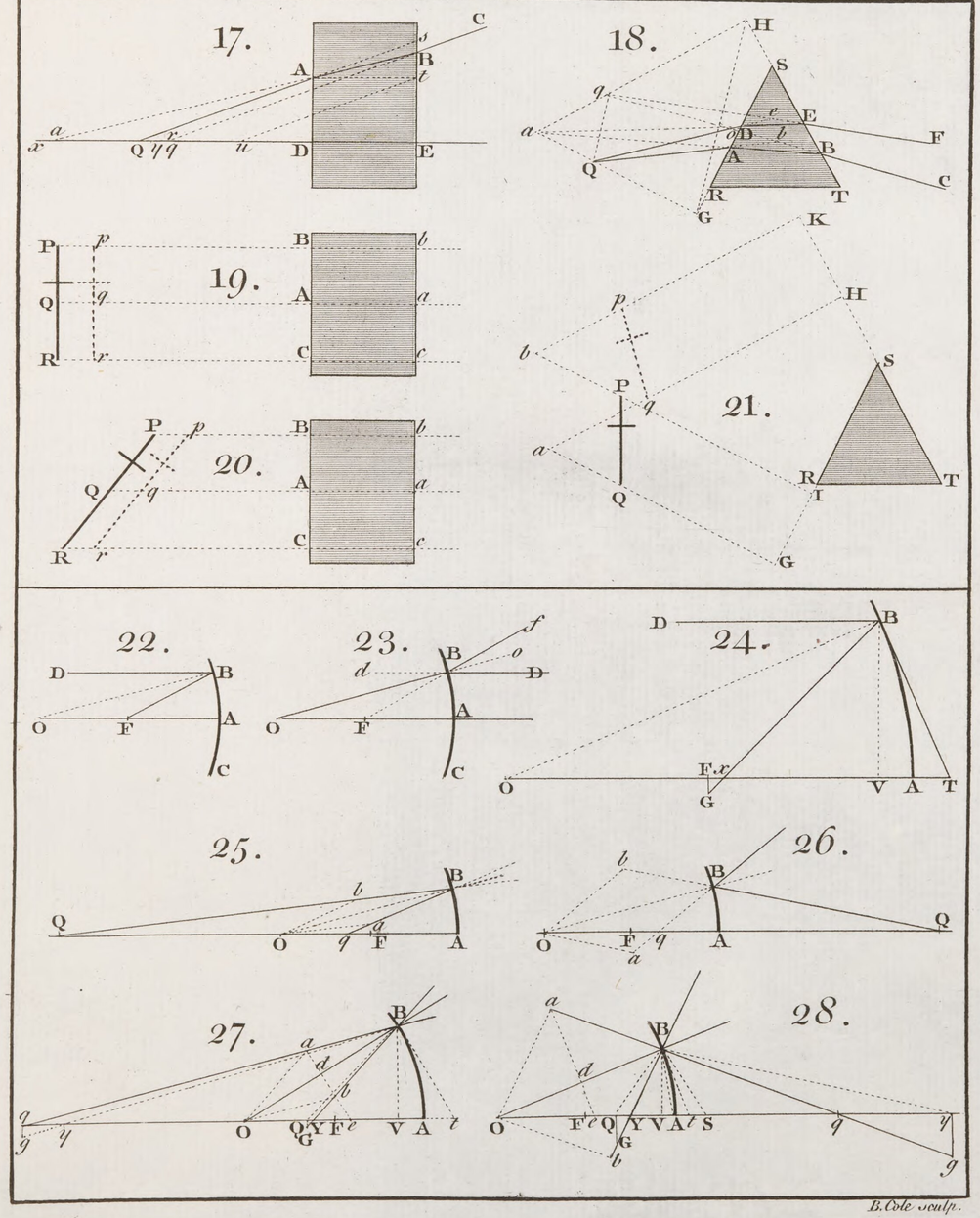
Fig. 29 *to* 32.

Becauſe QF : OF :: [a]OF : F*q*; it will alſo be,
QF : QO (QF ∓ OF) :: OF : O*q* (OF ∓ F*q*).
Alſo PE : PO :: OE : O*p*. Wherefore,
OF : QF :: O*q* : OQ; and OF (OE) : PE :: O*p* : OP. Whence it is manifeſt, that

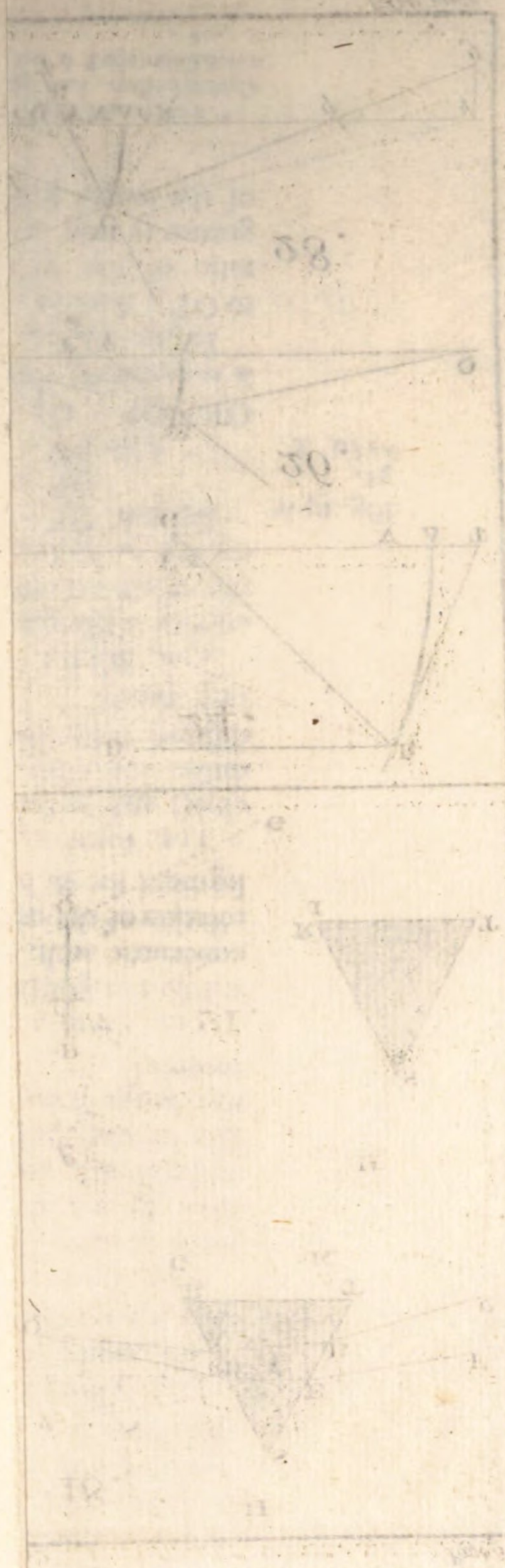
[a] 110. I.

In *fig.* 29, 31. the ratio of O*q* to OQ is greater than the ratio of O*p* to OP; And in *fig.* 30, 32. the ratio of O*q* to OQ is leſs than the ratio of O*p* to OP. And ſo will alſo the ratio of O*q* to OQ be greater or leſs, as the caſe is, than the ratio of the diſtance of any point of the image from O, to the diſtance of the correſponding point of the

* The learned Dr. *Barrow*, in his XVIIth Optical Lecture, hath ſhewed that *the image of a rectilinear object is one or other of the Conic Sections* nearly. But it would be needleſs here to purſue the ſpeculation of images any farther.



B. Cole sculp.



the object from O: And conſequently *the image* qp *is a curve, and its curvity is the ſame way with that of the ſpeculum* AB. And it is manifeſt alſo, that *the nearer the point* Q *is to* F, fig. 30, 31. *the greater will be the curvity of the image.* Therefore, *&c. Q. E. D.*

117. *Schol.* 2. Since an object placed at any diſtance, and its image, form both [a] the ſame angle at the center of a ſpeculum; the angle under which any very remote object appears, being given, it will be eaſy to find the diameter of its image. [a] 111. I.

Let O be the center, and *q* the principal focus of the concave ſpeculum A; and let the angle POR = *por* be that under which the ſun, or any other very remote object, appears to the naked eye. Biſect the angle *p*O*r* with the line O*q*; then in the right-angled triangle O*qr*, or O*qp*, we ſhall have the ſide O*q* and all the angles, to find *qr* or *qp* = the ſemi-diameter of the image. Fig. 33.

118. *Schol.* 3. The theory of the focus's of rays reflected by a ſpherical ſpeculum may be thus illuſtrated. Through the center O of the ſpeculum A, draw the indefinite line BD, and biſect OA in F; from the points O and A, divide the lines OB, AD into parts, each equal to OF or FA, marked by the figures 1, 2, 3, 4, 5, *&c.* And from F take on each ſide F 1, F $\frac{1}{2}$, F $\frac{1}{3}$, F $\frac{1}{4}$, F $\frac{1}{5}$, *&c.* each equal to the whole or $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, *&c.* of FO or FA. Now if in the line OB, the point 1 or 2, 3, 4, 5, *&c.* be the focus of incident rays; the correſponding point 1 or $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, *&c.* in the line OF will accordingly be the focus of the reflected rays. And, *vice verſâ*, if theſe laſt be the foci of incidence, the others will be the foci after reflection. In like manner, if 1, 2, 3, 4, 5, *&c.* in the line AD be the foci of incidence; 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, *&c.* in the line AF will be the foci of the reflected rays. And, *vice verſâ*. So that the foci of incidence and reflection unite in O and A: And their motions from O and A towards B and F in the one caſe, and towards D and F in the other caſe, are exactly ſimilar; F being the focus of parallel rays in both caſes. Fig. 34.

119. If in a dark room a lighted candle be held farther from a concave ſpeculum than its principal focus F, as ſuppoſe at 2; its image will be ſeen diſtinct, but inverted, upon a white paper held at the correſponding point $\frac{1}{2}$. And if as the candle is moved towards or from the ſpeculum, the paper be alſo moved ſo as to retain the diſtinct image of the candle, we ſhall have an ocular proof of the theory of the *Foci* of incident and reflected rays. When the candle is at O, the paper will be there alſo. But becauſe the image of an object placed between F and a concave ſpeculum, or any where before a convex ſpeculum, is on the other ſide of the ſpeculum; in either caſe the experiment of the candle and paper cannot take place.

PROP.

PROP. XVI.

120. *The ſituation of an object, and the place of the eye, being given; to find the viſible area, or how much of the object can be ſeen at one view, by rays reflected from a given portion or aperture of a given ſpeculum.*

Fig. 35 *to* 38. Let PR be an indefinite object, E the place of the eye; BC the given aperture of the ſpeculum, and O its center. From E to the extremities B, C of the aperture of the ſpeculum, draw the lines EB, EC; draw alſo the perpendiculars OB, OC; and to the indefinite object PR, draw the lines BP, CR by making the angles OBP, OCR equal reſpectively to the angles OBE, OCE; then will PR be the viſible area required. For it is manifeſt that the rays PB, RC will be reflected to E; and alſo that no rays diverging upon the ſpeculum BC, from points that are farther from the axis than P,R will be reflected to E; and conſequently that PR is the largeſt object that can be ſeen in the aperture BC from the point E. *Q. E. I.*

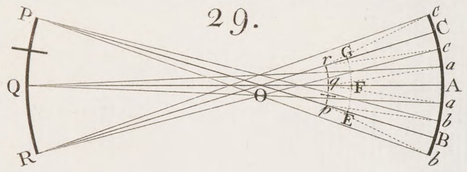
Or, PR may be otherwiſe thus found: Draw the indefinite image *pr*, and having drawn the lines EB, EC, produce them, if need be, till they cut the image in the points *p*,*r*; then lines drawn from O to the points *p*,*r*, and produced as the caſe requires, will give the limits of PR.

Fig. 35,36. 121. *Corol.* 1. When the object is farther from a concave ſpeculum than its principal focus; if E is farther from the ſpeculum than the image *pr*, the farther E is from *pr*, the greater will be PR; for then the angles OBP, OCR will be greater; and the object will appear inverted. And when E is in *pr*, PR will be diminiſhed to nothing. If E be between *pr* and the ſpeculum; the nearer E is to the ſpeculum, the greater will be PR; and PR will appear erect.

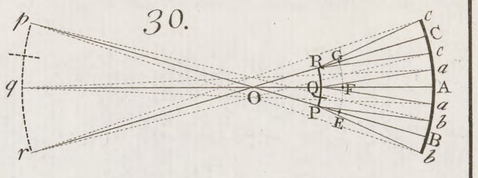
Fig. 37,38. 122. *Corol.* 2. When the object is nearer a concave ſpeculum than its principal focus, or any where before a convex ſpeculum, it will appear erect, wherever E is placed: And the nearer E is to the ſpeculum, the greater will be PR.

123. *Schol.* In finding PR, no allowance was made for the breadth of the eye's pupil; but that is eaſily done by drawing lines from the extreams of the pupil to the points B,C; and this would make PR ſomewhat greater. But that nicety would be ſuperfluous; for in ſome caſes the phenomena do not exactly agree with this theory, for reaſons that will be given in the next Book.

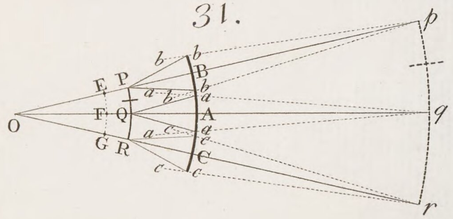
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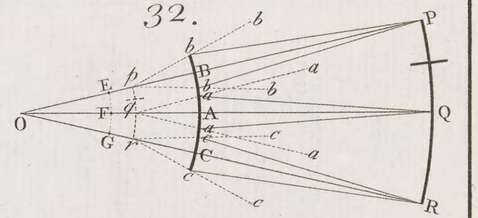
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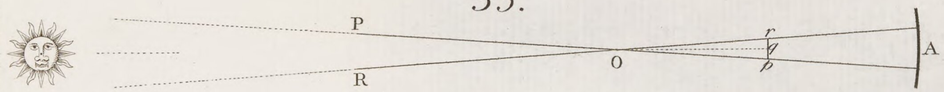
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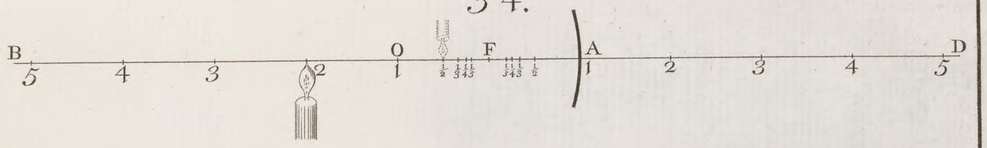
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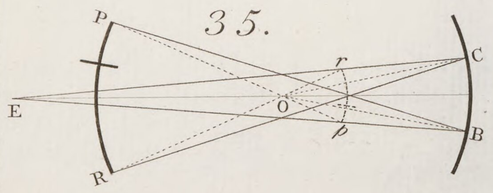
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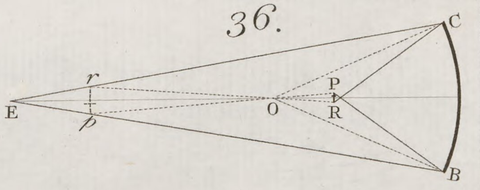
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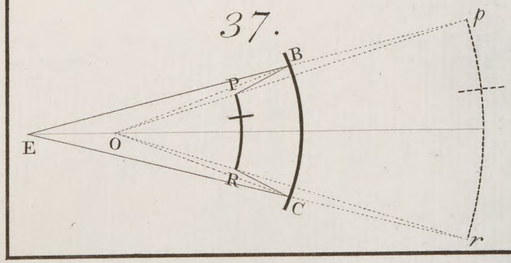
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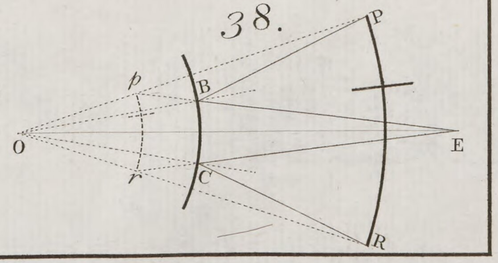
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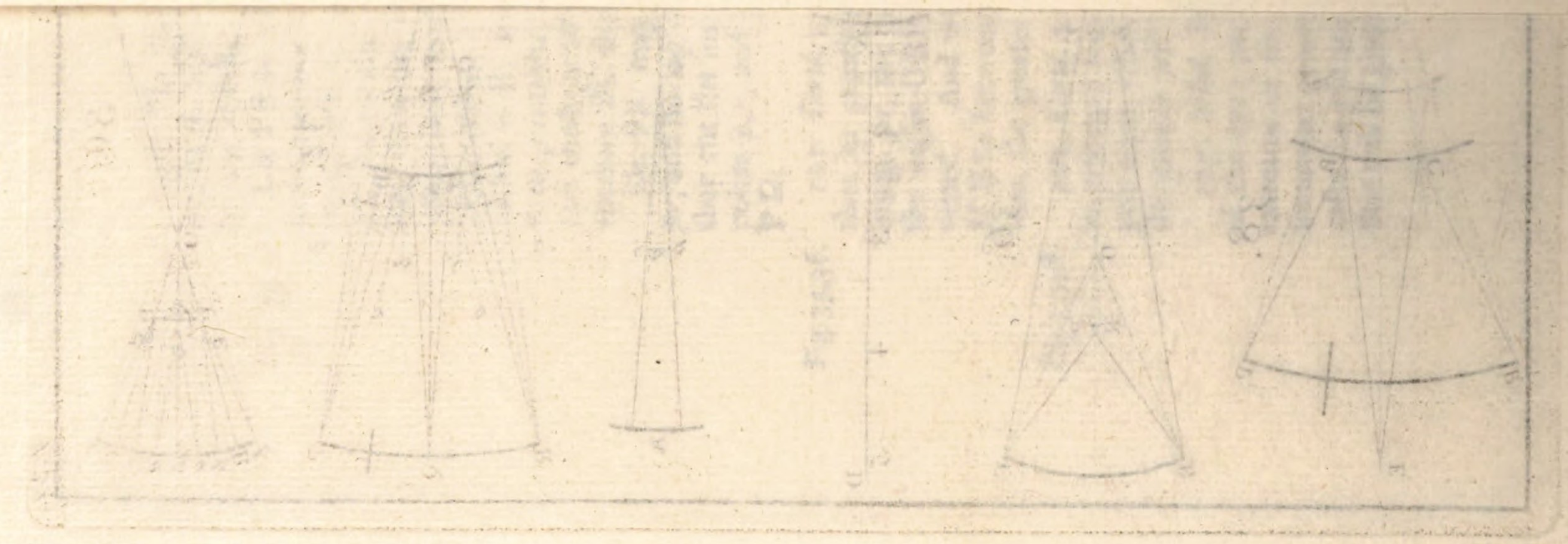


37.



38.





§. 3. *Of the Refraction of Light at a Spherical Surface.*

PROP. XVII.

124. *To find the geometrical focus of parallel rays, after refraction at a ſpherical ſurface, of any given refracting medium.*

Let BAC be a ſpherical refracting ſurface, whoſe center is O, and let the perpendicular KAOF to the refracting ſurface, be the axis of a pencil of parallel rays. Now if in the ſaid axis the point F be taken ſuch, that AF : OF :: I : R; and the point F be taken on the ſame ſide of A that O is of, if the incidence is out of a rare medium; otherwiſe on the contrary ſide, the point F will be the focus required. Fig. 39 to 42.

Let DB be any incident ray, parallel to the axis KOF, and through the point of incidence B, draw the perpendicular OBE; then will the ∠ DBE, *fig.* 39, 42. or its ſupplement DBO, *fig.* 40, 41. be the angle of incidence: And if BF (or its continuation BL, *fig.* 40, 42.) be the refracted ray; the ∠ OBF, *fig.* 39, 40. or its ſupplement EBF, *fig.* 41, 42. will be the angle of refraction. Wherefore in the triangle OBF, it will be BF : OF ::[a] I : R; And AF : OF ::[b] I : R. But when the arch AB is vaniſhing, BF will be equal to AF, and therefore the point F is the geometrical focus of the refracted rays. *Q. E. I.*

[a] Trigonom.
[b] Hypoth.

125. *Corol.* 1. AO : AF :: *m*—*n* : I; and AO : OF :: *m*—*n* : R.

126. *Corol.* 2. When the incidence is into a denſe medium, AO : OF :: *m*—*n* : *n*; and AO : AF :: *m*—*n* : *m*. Alſo OF : AF :: *n* : *m*. Fig. 39, 40.

127. *Corol.* 3. When the incidence is into a rare medium; AO : OF :: *m*—*n* : *m*; and AO : AF :: *m*—*n* : *n*. Alſo OF : AF :: *m* : *n*. Fig. 41, 42.

From *Art.* 13, 20, and 124, we have alſo the following Corollaries.

128. *Corol.* 4. Rays in Air, falling upon the { convex / concave } ſurface of glaſs BAC, parallel to the axis KOF; will after refraction { converge to / diverge from } a focus F, whoſe diſtance FA from the vertex A, is three times the radius OA, of the arch BAC. Fig. 39, 40.

129. *Corol.* 5. Rays in Glaſs, { diverging from / converging towards } a focus F, whoſe diſtance from the vertex A, is three times the radius OA; do after refraction by the { concave / convex } ſurface of Air BAC, become parallel to the axis FOK. Fig. 39, 40.

Fig.41,42. 130. *Corol.* 6. Rays in glaſs, parallel to the axis OAF; do after refraction by the {concave / convex} ſurface of air BAC, {converge to / diverge from} a focus F, whoſe diſtance from the vertex A is twice the radius OA.

Fig.41,42. 131. *Corol.* 7. Rays in air, {diverging from / converging towards} a focus F, whoſe diſtance from the vertex A, is twice the radius OA; do after refraction by the {convex / concave} ſurface of glaſs BAC, become parallel to the axis OAF.

132. *Corol.* 8. Convex and concave ſurfaces, are juſt oppoſite in their effects: that is a pencil of parallel rays refracted by either, will have its focus at the ſame diſtance from the vertex, but on contrary ſides; as appears by comparing the figures 39, 40; and 41,42. together.

133. *Corol.* 9. Parallel rays, refracted by a {convex / concave} ſurface of a {denſe / rare} medium, will have a real focus, as in *fig.* 39, 41: But being refracted by a {concave / convex} ſurface of a {denſe / rare} medium, as in *fig.* 40, 42. the focus will be only imaginary.

Fig. 39 *to* 42. 134. *Schol.* The point F is the focus of thoſe rays only, that are neareſt to the axis AOF; all other rays as *db*, whoſe incidence are more oblique, will have their focus at ſome point *x*, between F and A; and the more oblique is the incidence, the greater will be the aberration F*x*.

[a] 124. I. [b] Trigonom. For AF : OF ::[a] I : R ::[b] *bx* : O*x*; Now in *fig.* 39, 40. AF (= OF + O*b*) is greater than *b*F; And in *fig.* 41, 42. AF (=OF—O*b*) is leſs than *b*F; and therefore in both caſes, OF is greater than O*x*; that is, the point *x* is ſomewhere between F and A. And it is manifeſt, that the greater is the arch A*b*, the greater will be the aberration F*x*. Therefore, *&c. Q. E. D.*

PROP. XVIII.

Fig. 39 *to* 42. 135. *The ratio of the ſines of incidence and refraction, and the angle of incidence* = AOb, *of any ray* db *upon the given ſurface* BAC, *being given; to find the point* x, *where the refracted ray cuts the axis* AOF; *And conſequently to find the aberration* Fx.

Make, Sine, ∠ AO*b* : Sine ∠ O*bx* (or *ebx*) :: I : R; and draw *bx*: then in the △ O*bx*, will be given the ſide O*b* and all the angles, to find O*x*. And OF—O*x*=F*x*. *Q. E. I.*

PROP.

PROP. XIX.

136. *The longitudinal aberration* Fx, *of a parallel ray incident upon the ſpherical ſurface of a denſe medium*, fig. 39, 40. *is to* vA *the the verſed ſine of the angle of incidence*; *as* nn *to* mm—mn *nearly. And when the incidence of the parallel ray is upon a ſpherical ſurface of a rare medium*, fig. 41, 42. Fx : vA :: mm : mn—nn *nearly.*

From the center *x* deſcribe the arch *ba*, and let fall the perpendicular *bv*.

Now, 1. $Ox : bx ::$ [a]$OF : AF$. Wherefore, [a] Conſtruc.

2. $Fx\ (OF - Ox) : OF :: (AF \mp bx \text{ or } ax)\ Fx \pm Aa : AF$; therefore

3. $Fx : Aa\ (\overline{Fx \pm Aa} \backsim Fx) ::$ [b] $OF : OA\ (AF \backsim OF) :: \left\{\begin{matrix} n \\ m \end{matrix}\right\} : m - n.$ [b] 126 & 127. I.

4. $va : vA ::$ [c]$OA : xa$ nearly: But in *fig.* 39, 40. xa is nearly † equal to xA; and in *fig.* 41, 42. xa is nearly equal to †FA. Wherefore ſubſtituting theſe equals, it will be [c] 58 I.

5. $Aa\ (vA \mp va) : vA :: \begin{matrix} Ox \\ OF \end{matrix}\left(\begin{matrix} Ax - OA \\ FA + OA \end{matrix}\right) : \begin{matrix} xb \\ AF \end{matrix} :: \begin{matrix} [b]n \\ m \end{matrix} : \begin{matrix} m. \\ n. \end{matrix}$

Wherefore compounding the 3d and 5th ſteps, we ſhall have,

Caſe 1. $Fx : vA :: nn : mm - mn$ nearly; and Fig. 39, 40.

Caſe 2. $Fx : vA :: mm : mn - nn$ nearly. *Q. E. D.* Fig. 41, 42.

137. *Corol.* 1. The angle of incidence being the ſame, the aberration Fx in *fig.* 39, 40. will be the ſame; and ſo it will alſo be in *fig.* 41, 42. but the aberration in the two laſt figures will be greater than in the former.

138. *Corol.* 2. If inſtead of *m*, *n*, we put the numbers 3, 2, which expreſs their ratio; we ſhall have in *Caſe* 1. $Fx = \frac{4}{3} vA$; and in *Caſe* 2. $Fx = \frac{2}{3} vA$: And therefore Fx in *Caſe* 1. is to Fx in *Caſe* 2. as 8 to 27.

139. *Corol.* 3. Hence and from *Art.* 89, 91. the longitudinal aberration of a parallel ray incident upon a ſpherical reflecting ſurface, is to the longitudinal aberration of a parallel ray of a like incidence upon a ſpherical refracting ſurface, whoſe radius is the ſame with that of the reflecting ſurface; in *Caſe* 1, as 3 to 8; and in *Caſe* 2, as 1 to 9; nearly.

† When the arch AB is very ſmall xa, xA, *fig.* 39, 40. and xa, FA, *fig.* 41, 42. are pretty near in a ratio of equality; but as the arch AB increaſes, the quantities aſſumed for xa are greater than it, and therefore Fx bears a greater proportion to vA than is expreſſed above.

PROP. XX.

140. *The longitudinal aberrations are as the ſquares, and the lateral ones as the cubes, of the ſines of the angles of incidence nearly.*

[a] 136. I. [b] 91. I. For, 1. The longitudinal aberration Fx is almoſt in a conſtant [a]ratio to the verſed ſine vA; and vA is as [b]bv^2 nearly.

Again, 2. The lateral aberration FG, whilſt the angle of incidence remains the ſame, is in ſome certain ratio to Fx, that is, in a certain ratio to bv^2; but as the angle of incidence increaſes or diminiſhes, FG does moreover increaſe or diminiſh as the ſine of the $\angle$ FxG, that is, as bv; and therefore the whole increaſe or decreaſe of FG is as bv^3 nearly. Therefore, *&c.* *Q. E. D.*

PROP. XXI.

141. *The focus of incident rays, either diverging or converging upon a ſpherical refracting ſurface, being given; to find the geometrical focus of the refracted rays.*

Fig. 43 *to* 48. *Conſtruction.* Let BAC be a ſpherical refracting ſurface whoſe center is O, and let Q be the focus of incident rays, (either diverging as QB, *fig.* 43, 45, 48. or converging as MB, *fig.* 44, 46, 47): through the points Q, O, draw the indefinite line Q O q, and let K B k be a ſtrait line parallel thereto. In the line Q O produced, let f be the geometrical focus of rays parallel to the axis Q O B, as kB; (that is, let it be m—n : n :: A O : Of); and parallel to the incident ray Q B or MB, through the center O draw the perpendicular aOb, and take Ob = Of. Through the point b draw the line Bb, and produce it till it meets the axis Q O produced in q; and q will be the focus required.

The focus muſt be ſomewhere in the line Q Oq, for this line being perpendicular to the ſurface, is the axis of the pencil whoſe focus of incidence is Q; the refracted rays alſo paſs through b, for b is the focus of rays parallel to the axis Ob. Therefore, *&c.* *Q. E. D.*

142. *To find the focus* q *by calculation:* Take AF = Of, (or let F be the geometrical focus of rays parallel to the axis Q Oq, as K B), and ſay Q F : Q A :: Q O : Qq; and lay the point q on the ſame ſide of Q that F is of; and q will be the geometrical focus required.

[a] 4. VI. *Eu.* [b] Conſtruct. For Ob being parallel to Q B, the triangles Q Bq, Obq are ſimilar, and therefore Q B : Ob :: [a]Qq : qO. But when theſe triangles are vaniſhing, Q B will be = Q A, and Ob = Of = [b]AF. Wherefore ſubſtituting theſe equals, it will be

QA : AF :: Qq : qO; and therefore

Fig. { 43, 44; QA—AF : QA :: Qq—qO : Qq;
45, 46; AF—QA : QA :: qO—Qq : Qq;
47, 48; QA+AF : QA :: Qq+qO : Qq; that is, in each case,

QF : QA :: QO : Qq. *Q. E. D.*

143. *Corol.* 1. QF : AF (QA ± QF) :: QO : Oq (Qq ± QO).

144. *Corol.* 2. QF : OF or Af (QO ± QF) :: QA : Aq (Qq ± QA) :: Of or AF (QA ± QF) : fq (Aq ± Af. i.e. QF : OF (Af) :: QA : Aq :: Of (AF) : fq.

145. *Corol.* 3. If q be the focus of incident rays, Q will be the focus of the refracted ones.

146. *Schol.* 1. This propoſition may be diſtinguiſhed into the ſix following caſes.

Caſe 1. When the focus {Q, q}, of a ray {QB, qB} diverging upon a {convex, concave} ſurface of a {denſe, rare} medium, is beyond {F, f}. Fig. 43.

Caſe 2. When the focus {Q, q}, of a ray {MB, NB} converging upon a {concave, convex} ſurface of a {denſe, rare} medium, is beyond {F, f}. Fig. 44.

Caſe {3, 4} When the focus Q, of a ray {QB diverging, MB converging} upon a {convex, concave} ſurface of a denſe medium, is between F and A: Or, when a ray {bB converges, qB diverges} upon a {concave, convex} ſurface, of a rare medium. Fig. 45, 46.

Caſe {5, 6} When a ray {MB converges, QB diverges} upon a {convex, concave} ſurface of a denſe medium: Or, when the focus q, of a ray {qB diverging, NB converging} upon a {concave, convex} ſurface of a rare medium, is between f and A. Fig. 47, 48.

147. *Schol.* 2. The ſituations of the focus's Q, q, in *Caſe* 1, 2. in *Caſe* 3, 4. and in *Caſe* 5, 6. are alike; that is,

148. *Caſe* 1, and 2. The focus's Q, q are on oppoſite ſides of the ſurface BAC; and when Q is in F, q is at an infinite diſtance beyond f; and as Q recedes beyond F, q approaches towards f, and at laſt coincides with f, when Q is at an infinite diſtance. Fig. 43, 44.

149.

Fig.45,46. 149. *Caſe* 3, 4. Q and *q* are both on the ſame ſide, and the whole motion of Q is from A to F; but *q* may be either between A and F, or beyond F. If Q is in A, *q* will be alſo in A; and as Q recedes from A, *q* alſo recedes from it the ſame way, but with an accelerated velocity; ſo that when Q is got very near to F, *q* will be got at almoſt an infinite diſtance beyond F.

Fig.47,48. 150. *Caſe* 5,6. Q and *q* are here alſo both on the ſame ſide, and the whole motion of *q* is properly from O to *f*; but Q may be either between O and *f*, or beyond *f*: When Q is in O, *q* will be alſo in O. And if Q be between O and A, *q* will be between O and Q.

151. *Schol.* 3. In all theſe caſes, the focus's Q,*q* move both the ſame way. And the ſurfaces of the mediums remaining the ſame, *Caſe* 1, 3, 5, and alſo *Caſe* 2, 4, 6. paſs from one into the other.

152. *Schol.* 4. Convex and concave ſurfaces of like mediums; as alſo denſe and rare mediums with like ſurfaces; are juſt oppoſite in their effects.

153. *Schol.* 5. Whatever has been ſhewed as to the focus's Q,*q*; is alſo applicable to objects and their images. And ſince the axis's of all pencils do evidently interſect in O, it is manifeſt alſo, that the breadth of the object will be to that of its image, * as OQ to O*q*.

[153.] *Schol.* 6. If an object be an arch of a circle concentric to the ſpherical refracting ſurface, its image will be alſo a ſimilar concentric arch. And ſo will alſo a concentric ſpherical ſegment, have a ſimilar concentric ſegment for its image.

Fig.43,45. 48. But a ſtrait object PQ, will have a curvilineal image *pq*, whoſe curvity is the contrary way to that of the refracting ſurface BAC; as will be manifeſt by comparing the † rules for finding the focus's *q*,*p*.

Alſo a ſtrait object (*pq*) within the denſe medium, *fig.* 43, 46, 47. will have a curvilineal image (PQ); but its curvity will be the ſame way with that of the refracting ſurface.

PROP. XXII.

Fig. 43 *to* 48. 154. *Having the focus* Q, *of any incident ray* QB *or* MB, *and the diſtance of the point of incidence* B *from the vertex* A, *or* ∠ AOB; *to find the proper focus* y, *of the refracted ray.*

Having drawn QB, and the perpendicular OBG; with the ſides QO, OB, and the angle QOB, find the angle of incidence QBG, or QBO: Then ſay,

I : R :: Sine ∠ GBQ or QBO : Sine ∠ OB*y* or GB*y*; and draw B*y*, which will be the refracted ray.

Now

* Vide Art. 110, 111. I. † Vid. 116, & 143. I.

Now in the triangle OBy will be given the ſide OB, and all the angles; to find Oy, which gives the point y. *Q. E. I.*

155. *Schol.* Let b be the geometrical focus after refraction of rays parallel to the axis aOB, and x the point where the refracted ray By cuts that axis: Upon the axis's Ob, Oq at the points b, q raiſe the perpendiculars bs, qt. Now bs is the lateral aberration, of the parallel ray QB after refraction, from the geometrical focus b; and qt is the lateral aberration, of the diverging or converging ray QB or MB, from the geometrical focus q. I ſay $bs : qt :: Af : Aq$ nearly.

For in the △s Bbs, Bqt, having the ſame very acute angle at B, the lines bs, qt do not ſenſibly differ from perpendiculars at the points b, q upon the line bq; in which caſe the △s Bbs, Bqt, would be exactly ſimilar; and then it would be accurately, $Bb : Bq :: bs : qt$. But as the point B approaches nearer to A, the △s Bbs, Bqt approach nearer to a ſimilarity; and the lines Bb, Bq approach alſo nearer to an equality with the lines Af, Aq; and therefore it will be nearly $Af : Aq :: bs : qt$. *Q. E. D.*

156. *Corol.* 1. Draw the right line Bv. And it will be, $Bv : qt :: vy : yq$. Wherefore alſo $Bv \mp qt : qt :: vq\ (vy \mp yq) : yq$. Alſo $Bv : Bv \mp qt :: vy : vq\ (vy \mp yq)$.

157. *Corol.* 2. Hence, having either the lateral or longitudinal aberration of any ray, the other is eaſily found. And from *Art.* 155. having the lateral aberration of a parallel ray, the lateral aberration of any other ray of a like incidence, is alſo readily found ſomewhat nearly. But both are accurately found by *Art.* 142, 154.

158. *Corol.* 3. When AQ is $\left\{\begin{matrix}\text{greater}\\\text{leſſer}\end{matrix}\right\}$ than AF, Aq is $\left\{\begin{matrix}\text{greater}\\\text{leſſer}\end{matrix}\right\}$ than Ay. And *cæteris paribus* the more oblique is the incidence, the more will the ray be refracted, or the greater is qy. And hence an object may be ſo placed, that the rays next the axis ſhall diverge after refraction by a convex ſurface, when the more remote rays ſhall have a real focus.

I ſhall conclude this ſection, with the two following conſtructions, for finding the proper focus of a ray refracted by a ſingle ſurface *.

159. *To find the proper focus, of a ray parallel to the axis of a refracting ſurface, and at a given diſtance from it.*

Let O be the center of the ſpherical refracting ſurface BAC, and DB an incident ray parallel to the axis AO; to find the proper focus x of the refracted ray. In AO produced, find the geometrical focus F, by taking $AF : OF :: {}^{a}I : R$; and divide OF in E, by taking $EF : EO :: I : R$. From the center E with the radius EF deſcribe a circle EFG; and through the given points B, O, draw a line BOa cutting this circle Fig. 49, 50.

[a] 124. I.

* Dr. *Barrow*, I think, was the Author of theſe Conſtructions. Vid. *Bar.* Lect. XI. XIII.

circle in *a*; then in the axis OF take O*x* = O*a*, and drawing B*x* it will be the refracted ray. Join E*a* and A*a*, and ſince AF : OF :: [b](I : R ::[b]) EF : EO; it will be,

[b] Conſtruct.

AE (AF—EF) : EF :: EF (OF—EO) : EO. But EF = [b]E*a*; and therefore ſince the legs of the △s AE*a*, *a*EO having the common ∠ E, are proportional, it will alſo be, A*a* : *a*E :: [c]*a*O : OE; and therefore A*a* : *a*O :: EF (*a*E) : OE :: [b]I : R. But in the △s AO*a*, BO*x*, we have AO = [b]BO, O*a* = [b]O*x* and the ∠ AO*a* = [d]∠ BO*x*; and conſequently B*x* = [c]A*a*: Therefore B*x* : O*x* :: (A*a* : *a*O ::) I : R. But the ∠ BO*x* is equal to the angle of incidence or to its ſupplement, and B*x* is as the ſine of that angle; and therefore OB*x* is the angle of refraction or its ſupplement; that is, B*x* is the refracted ray. *Q. E. I.*

[c] 6. VI. *Euc.*

[d] 15. I. *Euc.*

[e] 4. I. *Euc.*

160. *Corol.* As the point B approaches to A, the point *a* approaches to F; and therefore every two contiguous rays will interſect each other before they croſs the axis OF.

Fig. 51 *to* 55.

161. *Schol. Fig.* 49. correſponds to *fig.* 39, and the ſame conſtruction will ſerve to find the point *x*, *fig.* 40. *Fig.* 50. correſponds to 41, and will alſo ſerve to find *x*, *fig.* 42.

162. *Having the focus* Q, *and the point of incidence* B, *of a ray either diverging or converging upon a given ſpherical ſurface* BAC; *to find the proper focus* y *of the refracted ray.*

From Q through the center O of the refracting ſurface BAC, draw the axis QO, and produce it both ways, if need be; and to the point of incidence B draw the perpendicular OB which produce as the caſe requires. In the axis QO take the point R ſuch, that OQ : OR :: I : R; and with the radius QB, from the center R deſcribe an arch interſecting OB in D; parallel to RD draw B*y*, and B*y* will be the refracted ray.

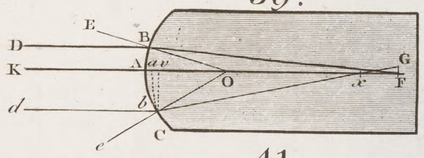
Upon the axis QO, the incident ray QB (produced if need be), and the refracted ray B*y*, let fall the perpendiculars B*v*, O*a*, O*b*. Now becauſe B*y* is parallel to RD, the triangles BO*y*, DOR are ſimilar; and ſo are alſo the right-angled △s *yb*O, *yvb*; as are likewiſe the right-angled △s Q*v*B, Q*a*O. Wherefore OR : QB (RD) :: O*y* : B*y* :: O*b* : B*v*. Therefore OR : O*b* :: QB : B*v* :: QO : O*a*; and conſequently OQ : OR (:: I : R) :: O*a* : O*b*. But O*a* is the ſine of incidence, and therefore O*b* is the ſine of refraction; that is, B*y* is the refracted ray. *Q. E. I.*

The figures here referred to, correſpond to thoſe in the preceding plate; as *fig.* 51, to 43. and the ſame kind of conſtruction will ſerve for 44. *Fig.* 52. correſponds to 45, as doth alſo 53 to 46, only changing the places of Q, *y*. *Fig.* 54, 55. correſpond to 47, 48.

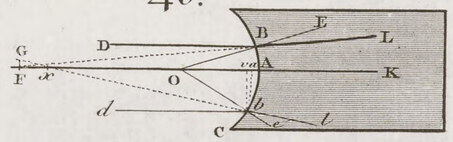
163. *Corol.* 1. If QB be equal to OR, the point D will coincide with O; and then the refracted ray will be parallel to the axis.

164.

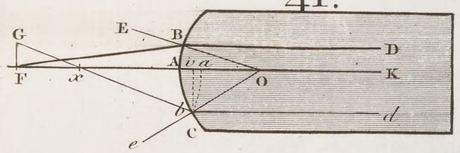
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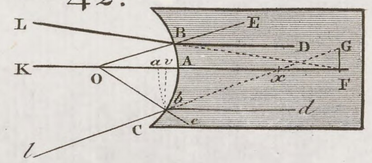
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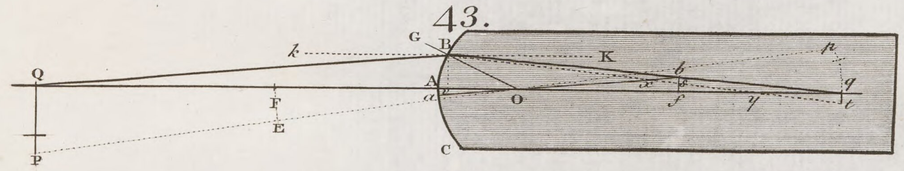
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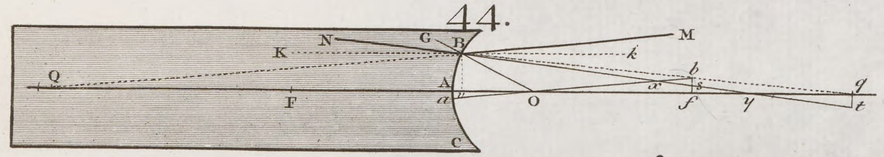
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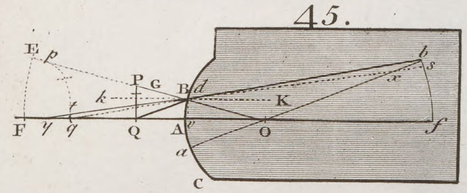
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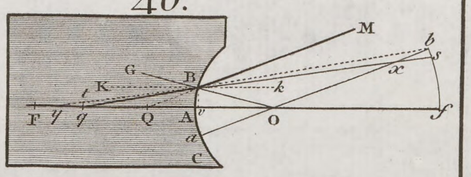
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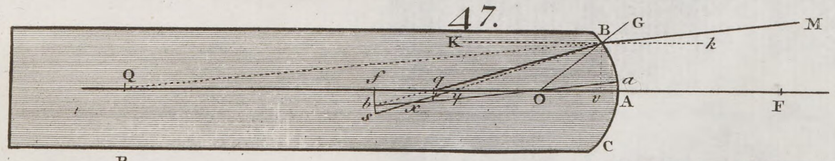
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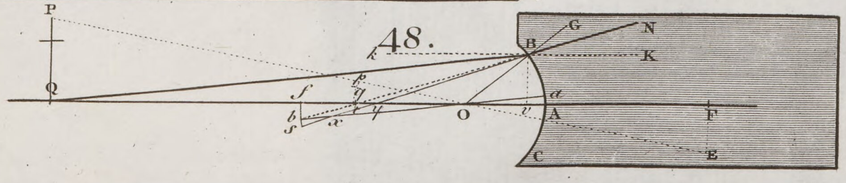
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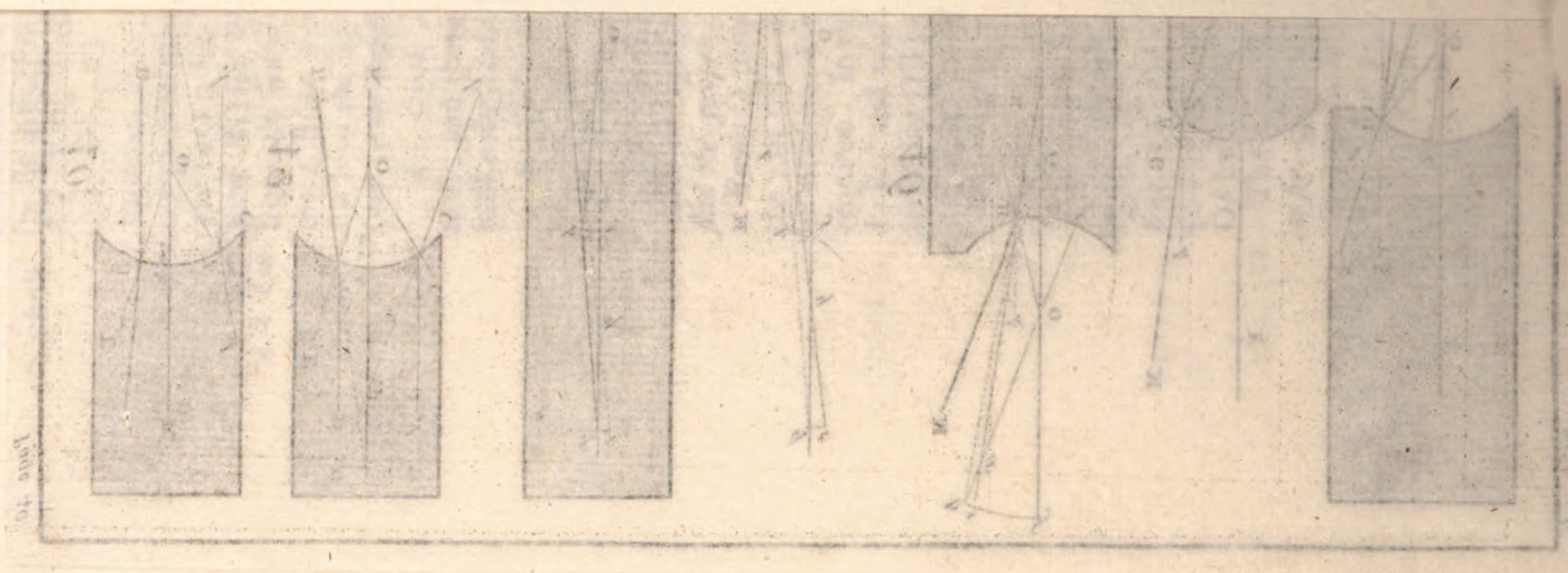
47.



48.



B. Cole sculp.



164. *Corol.* 2. If QB be greater than OR (*fig.* 51, 54, 55.) the focus *y* will fall on the ſame ſide of the ſurface BAC, as the center O is of: And the focus *y* will fall on the contrary ſide, if QB be leſs than OR, (*fig.* 52, 53.)

165. *Corol.* 3. If the point D falls on the ſame ſide of the axis as the point B, (*fig.* 52 to 55); the focus's Q, *y* will be alſo on the ſame ſide of the ſurface BAC: And the ſaid focus's Q, *y* will be on contrary ſides, when the points D, B are on different ſides of the axis, (*fig.* 51.)

166. *Corol.* 4. DO : OB :: OR : O*y* :: RD or QB : B*y*. Whence to find the geometrical focus *q*, becauſe in that caſe QA = QB = RD, it will be

167. *Corol.* 5. QA ∽ OR (OD) : OA :: OR : O*q*; but OR = [a] $\frac{R}{I}$QO. [a] Conſtruct.

Therefore QA ∽ $\frac{R}{I}$QO : OA :: $\frac{R}{I}$QO : O*q* :: (RD : B*q* ::) QA : A*q*.

And thus the focus *q* may be found without having the point R.

SECTION IV.

Of Lens's.

168. *Defin.* 1. A LENS in Optics (a ſection whereof is repreſented by the ſhaded figures), is a tranſparent body of a different denſity from the ambient medium, and terminated by two ſurfaces *, either both ſpherical, or one plain and the other ſpherical. And becauſe lens's for optical uſes are made of glaſs, it is uſual to call them *glaſſes*, with the addition of the uſe that they are intended for; as a *magnifying-glaſs*, a *ſpectacle-glaſs*; *object-glaſs*, or *eye-glaſs of a Teleſcope*, &c. Fig. 56 to 72. Lens.

169. *Defin.* 2. A lens having one ſide plain and the other convex, as in *fig.* 57, 59. is called a *plano-convex lens*: So the lens, *fig.* 58, 60. is called a *plano-concave*. A lens terminated by two convex ſides or two concave ſides, is accordingly called a *double convex*, as *fig.* 61. or *double concave lens*, as *fig.* 62. A lens having one ſide concave and the other convex, as *fig.* 63, or 64. is called a *concavo-convex lens*. The lens *fig.* 63. is called a *meniſcus*; as is alſo very often, tho' improperly, the lens, *fig.* 64. Plano-convex, plano-concave, double convex, double concave, concavo-convex, meniſcus, lens's.

170. *Defin.* 3. When the particular figure is not conſidered, a lens that is thickeſt in the middle, is called a *convex-lens*; and that which is Convex and concave lens's.

* This definition might be made more general; for any tranſparent body ſo figured as to divert the courſe of the rays refracted by it, towards or from a certain focus, is properly an optical lens; but we ſhall have occaſion to conſider no other but thoſe above defined. A tranſparent ſphere may be alſo called a lens.

is thinneſt in the middle, is called a *concave lens*, without farther diſtinction.

Axis of a lens.

171. *Defin.* 4. A ſtrait line drawn perpendicular to both the ſurfaces of any lens, is called the Axis of *the lens*. The axis therefore paſſes through the centers of the ſpherical ſides: And it is manifeſt that no lens, excepting the ſphere, can have but one axis; and therefore all pencils are oblique, excepting thoſe whoſe *foci* are in the axis of the lens.

Focus, or principal, ſolar, or burning focus of a lens.

Focal length of a lens.

172. *Defin.* 5. The geometrical focus of a direct pencil of parallel rays, (or that point in the axis where rays that are parallel and neareſt thereto, are united after refraction, if the lens is thickeſt in the middle; or from whence they diverge, if the lens is thinneſt in the middle) is called the *focus of the lens:* Sometimes the *principal focus*, by way of diſtinction from the focus after refraction, of diverging or converging rays. It is alſo called the *ſolar* or *burning focus*, if the lens is convex, becauſe there the ſun's rays are cloſeſt united. The diſtance of the principal focus from the lens, is called the *focal length of the lens.*

§. 1. *To find the principal focus of any given Lens.*

PROP. XXIII.

173. *To find the principal focus of a ſphere.*

Fig. 56. Let DB be an incident ray parallel and very near to the axis ARE*b*, and let *b* be the geometrical focus of the pencil whoſe axis is AR*b*, after the firſt refraction by the ſurface AB. Biſect E*b* in F, and F will be the focus required.

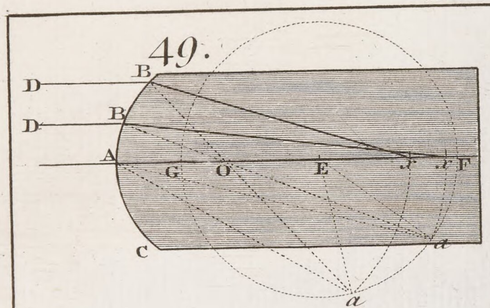
Produce the incident and emergent rays DB, F*a* till they meet in K, and from the center R draw RB, R*a*G. Now becauſe the ∠ *ba*G = [a]R*a*B = [b]RB*a*; and Sine ∠RB*a* : Sine ∠KB*a* :: [c]Sine ∠ *ba*G : Sine ∠F*ab* (:: *n* : *m*—*n*). Therefore the ∠F*ab* = KB*a* = [d]F*ba*; and conſequently F*a* = [e]F*b*. But when *a* coincides with E, FE will be = F*a*; and therefore the point F, which biſects E*b*, is the focus of thoſe rays that are neareſt to the axis AE. *Q. E. I.*

[a] 15. I. *Eucl.*
[b] 5. I. *Eucl.*
[c] Laws of refraction.
[d] 29. I. *Euc.*
[e] 6. I. *Euc.*

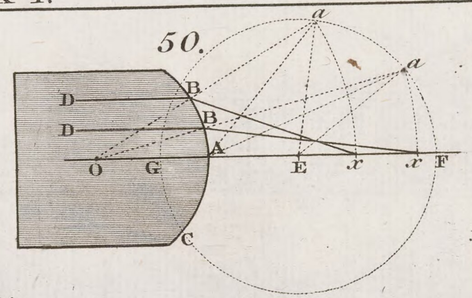
174. *Corol.* Hence the focus F of a glaſs ſphere, is one fourth of its diameter diſtant from the vertex. For AR (RE) = [f] ½ R*b*, and therefore FE = ½AR = ¼AE.

[f] 128. I.

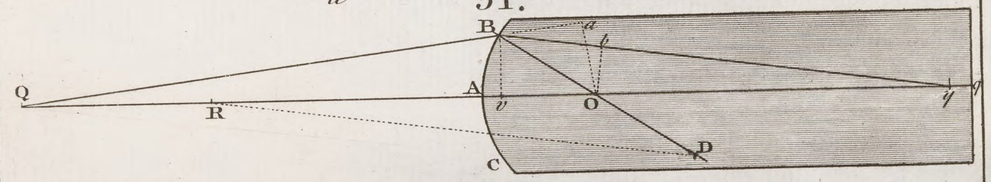
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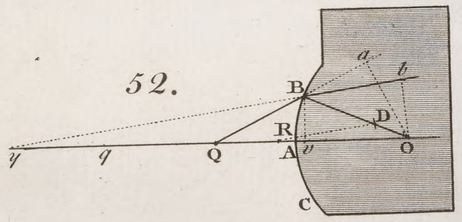
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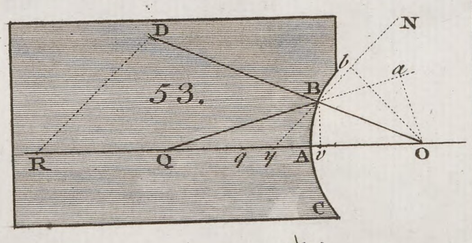
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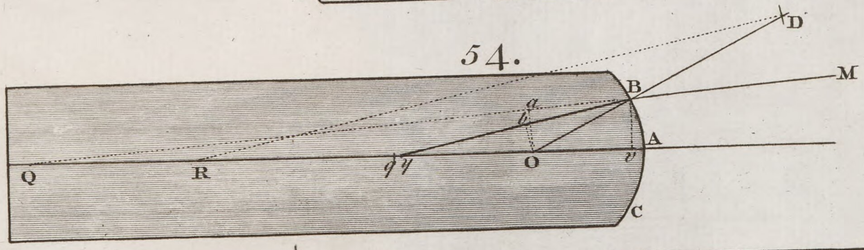
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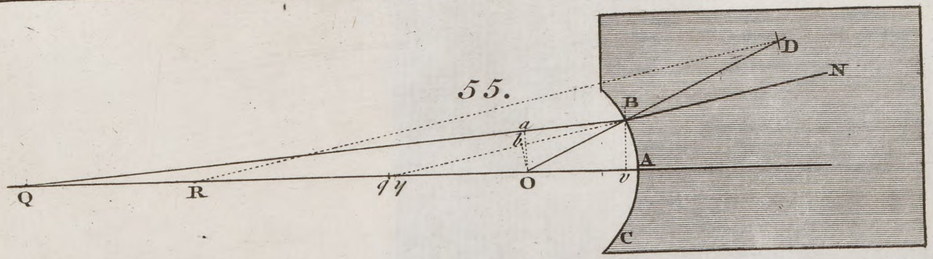
53.



54.



55.



B. Cole sculp.

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Axis of a lens.

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Focal length of a lens.

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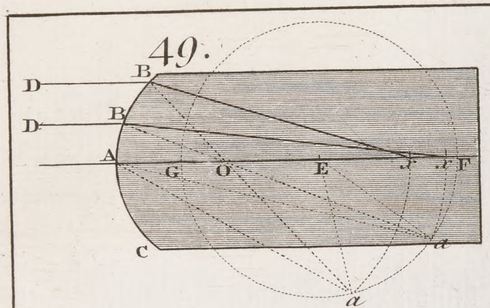
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[a] 15. I. *Eucl.*
[b] 5. I. *Eucl.*
[c] Laws of refraction.
[d] 29. I. *Euc.*
[e] 6. I. *Euc.*

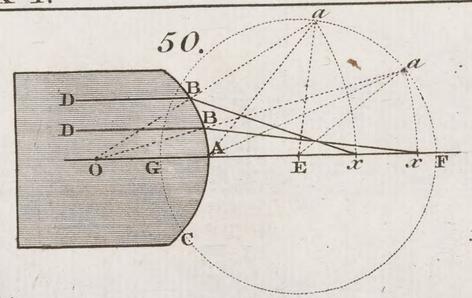
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[f] 128. I.

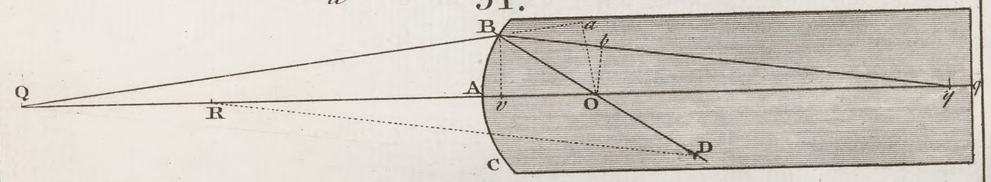
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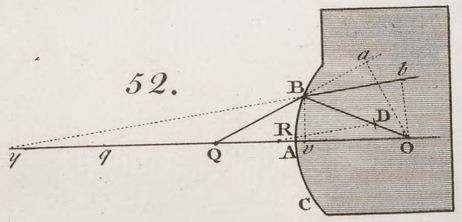
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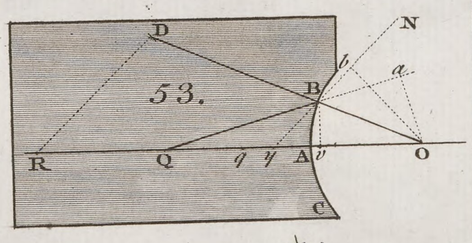
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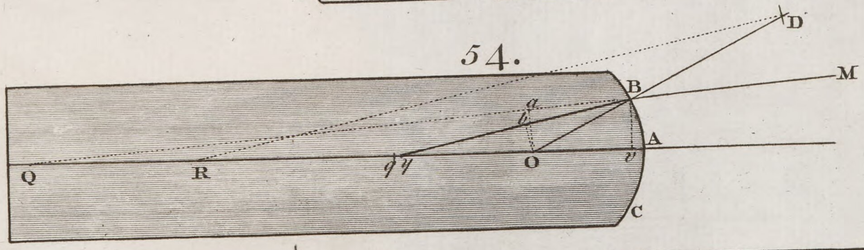
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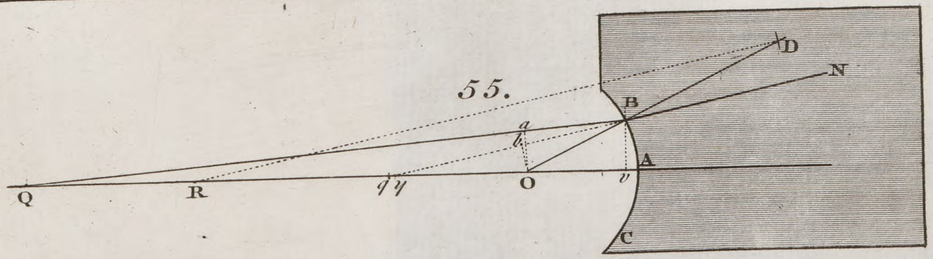
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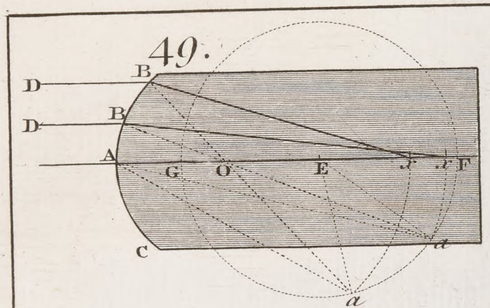
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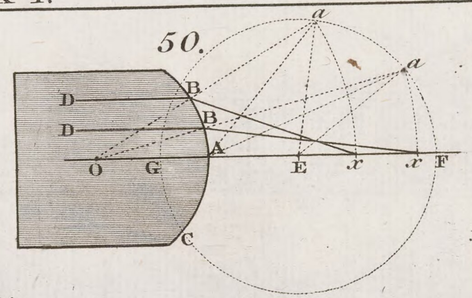
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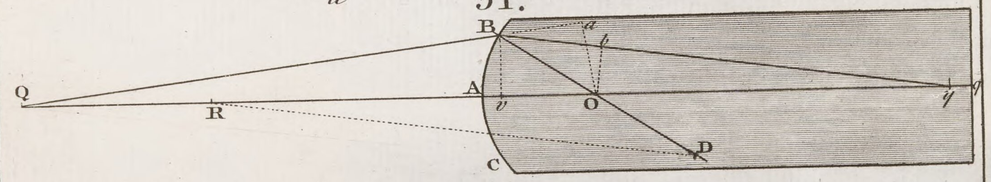
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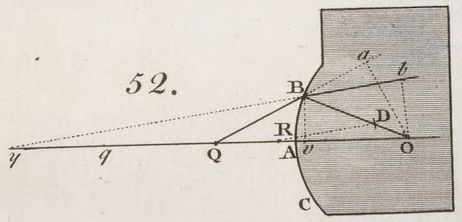
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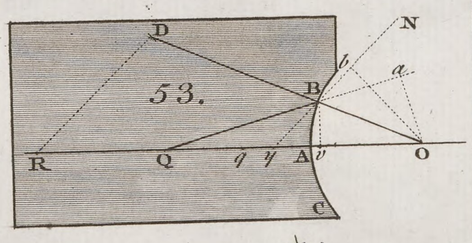
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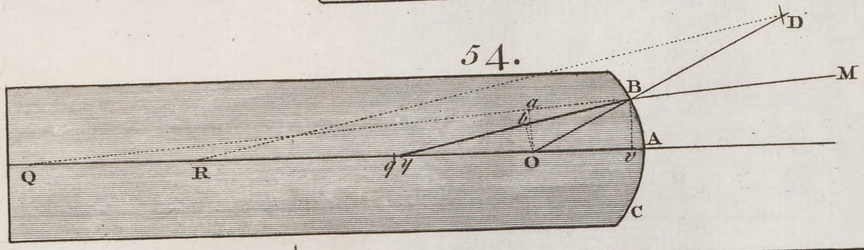
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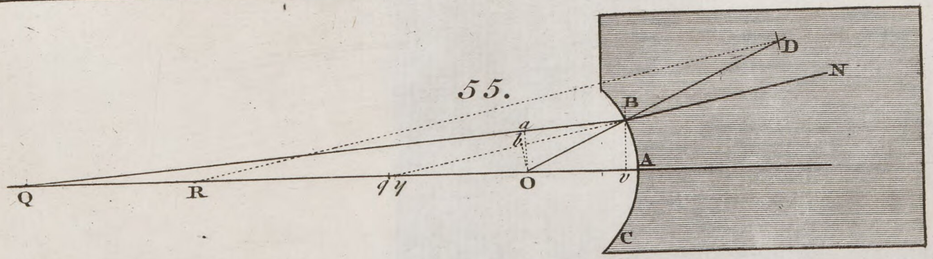
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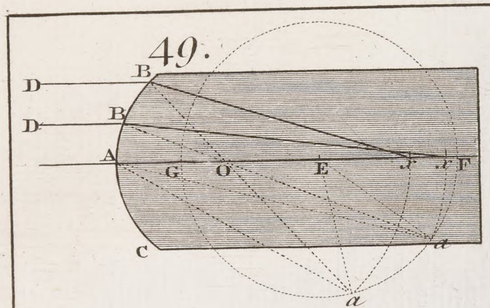
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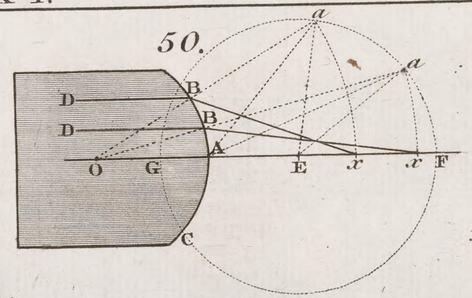
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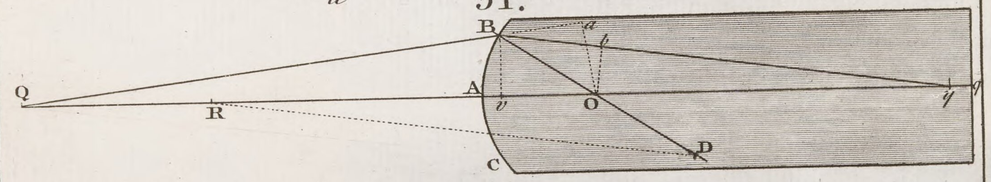
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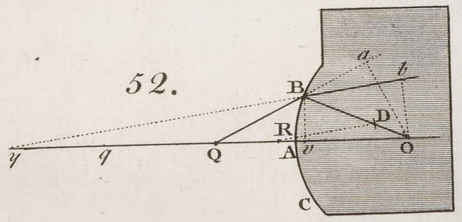
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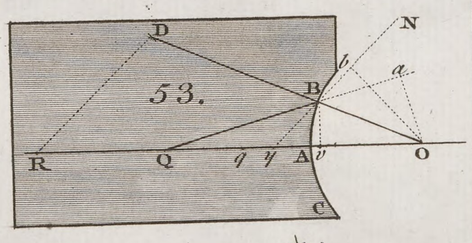
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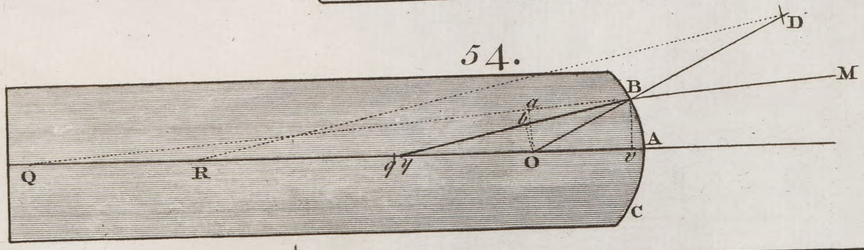
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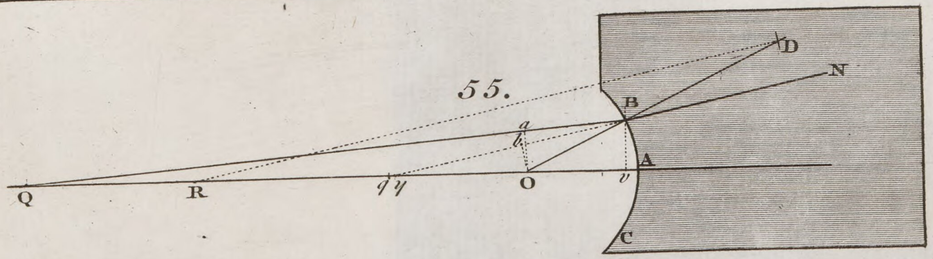
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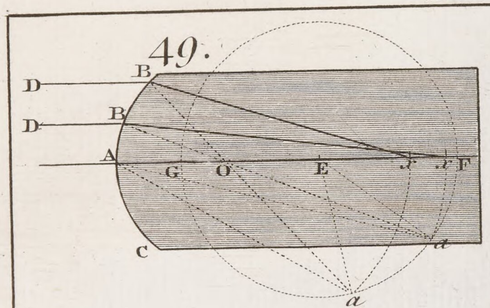
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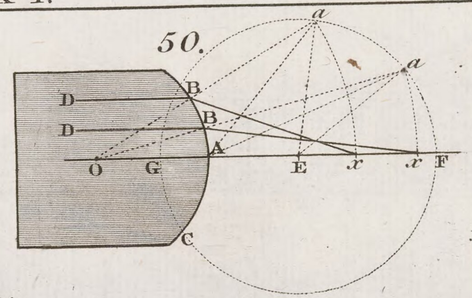
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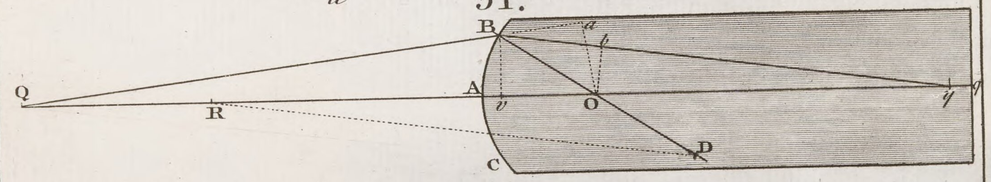
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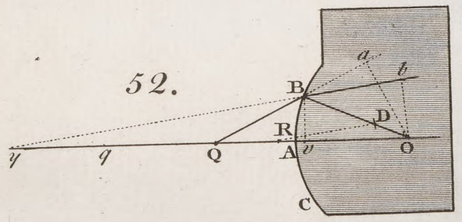
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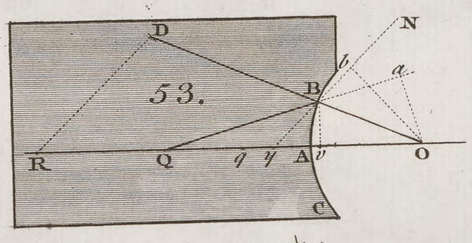
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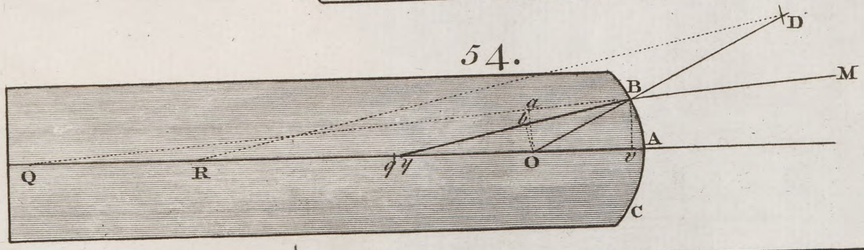
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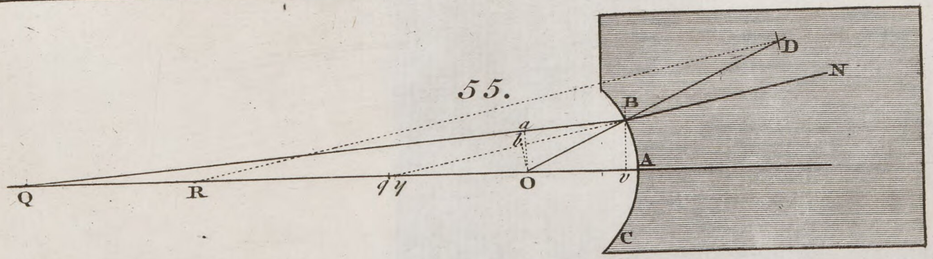
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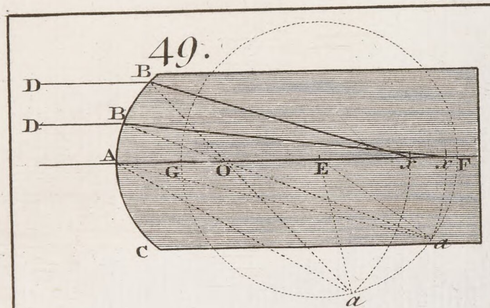
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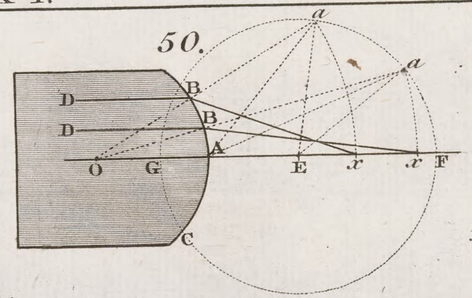
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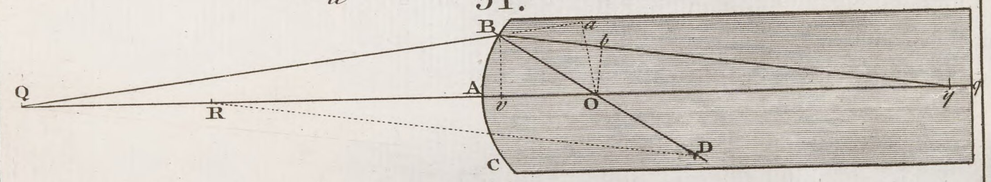
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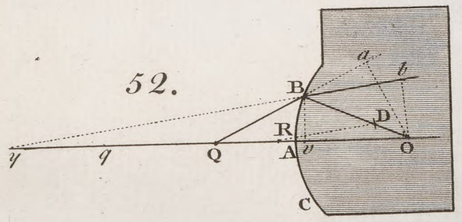
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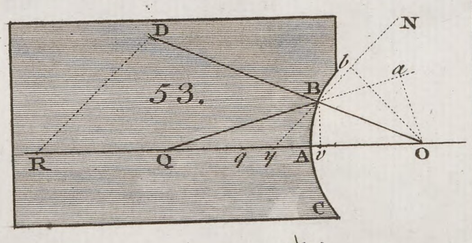
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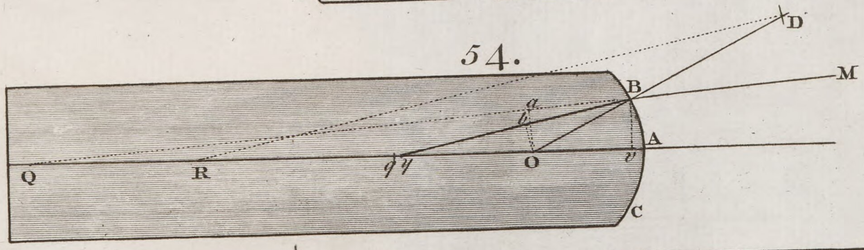
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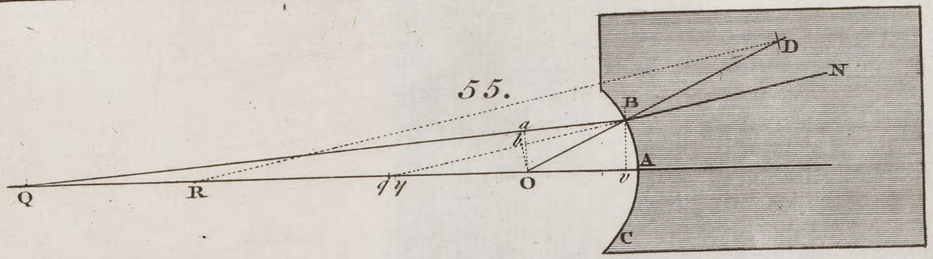
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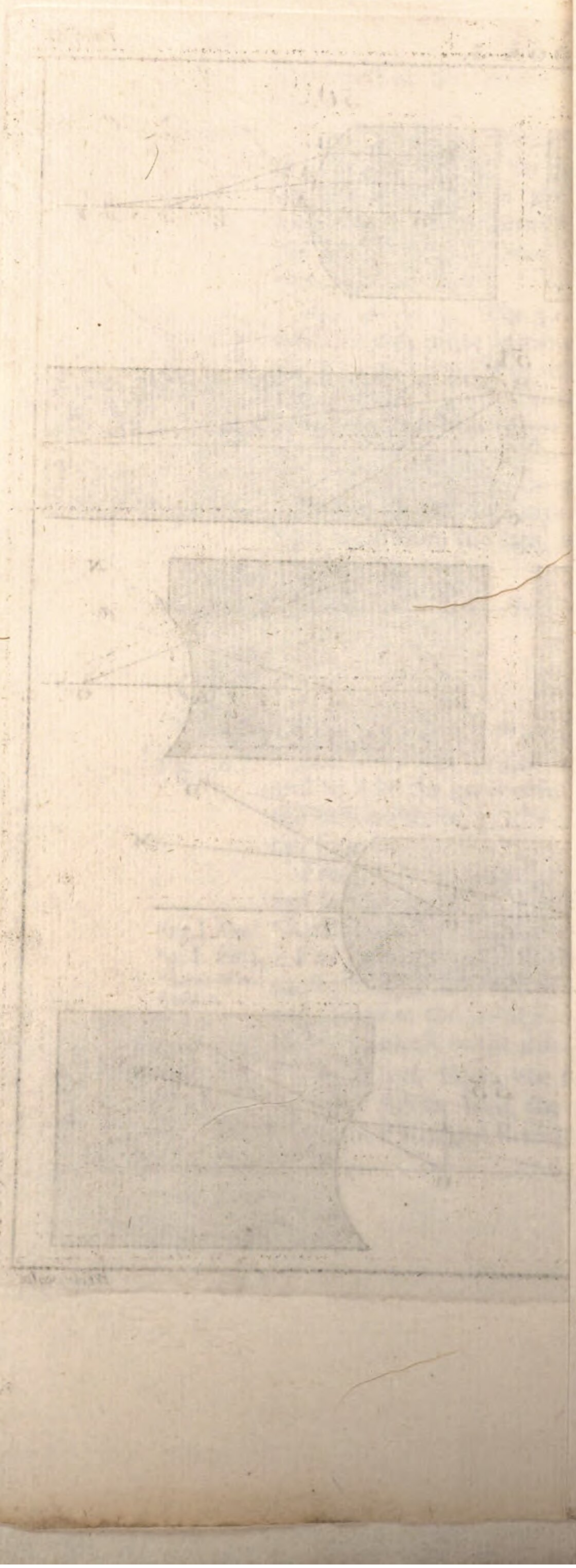
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B. Cole sculp.



PROP. XXIV.

To find the principal focus of a plano-convex, or a plano-concave lens.

Let RAF be the axis of the lens (or a line drawn perpendicular to both its surfaces), R the center of the spherical surface BAC, and let DB be an incident ray parallel and very near to the axis. Now this proposition admits of two cases, according to which side of the lens is turned to the incident rays. Fig. 57 to 60.

175. *Case* 1. When the incident ray falls upon the plane side, it will pass on directly to B unrefracted. Wherefore, when the point B is very near to A, if it be made RA : AF :: [a] $m-n : n$; the point F so found will be the focus. *Q. E. I.* Fig. 57, 58. [a] 127. I.

[175.] *Case* 2. When the incidence of the parallel rays is upon the spherical side of the lens. Fig. 59, 60.

Let AR : Rb :: $m-n : n$; and b [a] will be the point where the refracted ray Ba produced would meet the axis. [a] 126. I.

Again, take Eb : EF (:: R : I) :: $m : n$; and F [b] will be the focus. *Q. E. I.* [b] 50. I.

176. *Corol.* 1. $AF = \frac{n}{m}Eb \pm AE = \frac{n}{m}Ab \pm \frac{m-n}{m}AE$. Fig 59, 60.

For $\frac{n}{m}Ab = \frac{n}{m}Eb \pm \frac{n}{m}AE$; and $\pm \frac{n}{m}AE$ together with $\pm \frac{m-n}{m}AE = \pm AE$.

177. *Corol.* 2. AF (*fig.* 57.) = EF (*fig.* 59.) $+\frac{n}{m}AE$. And EF (*fig.* 58.) = AF (*fig.* 60.) $-\frac{n}{m}AE$. And in *fig.* 60. $AF = \frac{n}{m-n}AR - \frac{m-n}{m}AE$.

178. *Schol.* 1. A plano-convex glass, with its plain side towards the incident rays, (*fig.* 57.) hath its focus at the distance of twice the radius of convexity from the glass; that is $AF = 2RA$. And a plano-concave exposed in like manner, (*fig.* 58.) hath its focus at twice the radius from the concave side. But a plano-convex glass with its convex-side towards the incident rays, (*fig.* 59.) hath its focus nearer to the glass than a diameter of the convexity, in the ratio of bE to bA; or $EF (= \frac{2}{3}Fb) = \frac{2}{3}Ab - \frac{2}{3}AE = 2AR - \frac{2}{3}AE$. And in *fig.* 60. the focal length $FA = 2RA - \frac{1}{3}AE$; and consequently $EF = 2RA + \frac{2}{3}AE$.

179. *Schol.* 2. If the thickness AE, is very small in comparison of the radius RA, then the focal lengths of each of these lens's, may be considered as being equal one to the other; and in glass equal to twice the radius RA.

180. *Schol.* 3. The focus of a glaſs hemiſphere, with its convex ſide expoſed to parallel rays, is about $\frac{1}{3}$ of the diameter, diſtant from the glaſs.

Let *fig.* 59. be ſuppoſed a glaſs hemiſphere; then will E coincide with R, and the focus *b* of the rays after their firſt refraction, will be in the ſame place as before; ſo that $Rb = 2RA$. And to find F the focus of the emergent rays, it will be,

$Rb : RF :: m : n :: 3 : 2$; therefore $RF = \frac{2}{3}Rb = \frac{4}{3}RA$. *Q. E. D.*

But if the parallel rays fall upon the plane ſide of the hemiſphere, as in *fig.* 57. the focal length will be the ſame with that of a thin plano-convex lens in the ſame poſition; for the ray DB will paſs to B unrefracted by the plane ſide.

181. *Schol.* 4. Plano-convex and plano-concave lens's, that are unequal ſegments of the ſame ſphere, with their ſpherical ſides expoſed to parallel rays; the ſmaller the ſegment, or the thinner is the lens, the greater will be the focal diſtance from, the neareſt ſide of, the lens: And the contrary, reckoning from the fartheſt ſide of the lens. But if parallel rays are incident upon the plain ſides, their focal lengths will be the ſame, how unequal ſoever the lens's are in thickneſs.

PROP. XXV.

182. *To find the principal focus of any lens, whoſe ſurfaces are both ſpherical.*

Fig. 61 *to* 64. 1. Let R be the center of the ſpherical ſide AB, and *r* the center of the ſide *ab*; draw the axis R*r*, and produce it both ways. Let K*b* be an incident ray parallel, and very near, to the axis R*r*. Now take $ar : rq :: m - n : n$; and place *q* the ſame way from *r*, that *r* is from *a*;

[a] 126. I. then will *q* [a]be the geometrical focus of the parallel rays incident upon *ab*; or *q* may be conſidered as the focus of the incident ray *b*B upon the ſecond ſurface AB.

2. Let *f*,F be the geometrical foci of rays coming both ways, parallel to the axis, upon the ſurface AB; (*f* the focus of parallel rays coming from without, and F the focus of parallel rays within the lens.) Now to find the focus Q of the emergent rays, ſay,

[b] 144. I. $fq : Rf\,(AF) ::$ [b]$Aq : AQ$; or $fq : Rf\,(AF) ::$ [b]$RF\,(Af) : FQ$; and laying the point Q on the ſame ſide of A that *q* is of, or on the ſame ſide of F that R is of; [b]the point Q thus found will be the focus required *.

183. *Corol.* 1. $fq : Rq\,(fq \sim fR) :: Aq : Qq\,(Aq \sim AQ) :: RF : RQ\,(RF \sim FQ)$. Alſo $fq : fA\,(fq \mp Aq) :: AF : FQ\,(AF \mp AQ)$.

184.

* Compare *fig.* 61 with *fig.* 45; 62 with 46; 63 with 48; and *fig.* 64 with *fig.* 47.

184. *Schol.* 1. If the incident ray comes the contrary way, and it be made $qf : qr :: af : ax$; the point x will be the focus. This will be manifeſt by ſuppoſing f and q to change places, then it will be 1. $m—n : n ::$ [a] $AR : Rq$; 2. $fq : fr ::$ [b] $aq : ax$; that is, tranſpoſing the letters q,f back again, as they ſtand in the figures; $qf : qr :: af : ax$. *Q. E. D.*

[a] 126. I.
[b] 144. I.

185. *Schol.* 2. The rules $fq : Rf :: Aq : AQ$; and $qf : qr :: af : ax$; for finding the geometrical foci Q,x; contain the following. Let the radii RA, ra be called R,r, and the thickneſs Aa of the lens's be called t.

1. When the lens is a double convex, it will be, Fig. 61.

$$\frac{mR+mr}{m-n} - t : \frac{nR}{m-n} :: \frac{mr}{m-n} - t : AQ;$$

And $\frac{mR+mr}{m-n} - t : \frac{nr}{m-n} :: \frac{mR}{m-n} - t : ax.$

2. When the lens is a double concave; Fig. 62.

$$\frac{mR+mr}{m-n} + t : \frac{nR}{m-n} :: \frac{mr}{m-n} + t : AQ;$$

And $\frac{mR+mr}{m-n} + t : \frac{nr}{m-n} :: \frac{mR}{m-n} + t : ax.$

3. When the lens is a meniſcus, or a convex concavo-convex. Fig. 63.

$$\frac{mR-mr}{m-n} + t : \frac{nR}{m-n} :: \frac{mr}{m-n} - t : AQ;$$

And, $\frac{mR-mr}{m-n} + t : \frac{nr}{m-n} :: \frac{mR}{m-n} + t : ax.$

4. When the lens is a concave concavo-convex; Fig. 64.

$$\frac{mR-mr}{m-n} - t : \frac{nR}{m-n} :: \frac{mr}{m-n} + t : AQ;$$

And $\frac{mR-mr}{m-n} - t : \frac{nr}{m-n} :: \frac{mR}{m-n} - t : ax.$

For $m—n : m :: R : Af = \frac{mR}{m-n}$; and $m—n : m :: r : aq = \frac{mr}{m-n}$; Alſo $m—n : n :: R : Rf = \frac{nR}{m-n}$; and $m—n : n :: r : rq = \frac{nr}{m-n}$. Whence by adding or ſubtracting t, as the caſe requires; the four above caſes are ſufficiently manifeſt.

186. *Corol.* 2. In all caſes, the leſs is the ratio of R to r, the ſum of both being the ſame, the greater will be AQ and ax: And AQ and ax are greateſt of all, when R and r are equal. For in the above caſes, the firſt term will be the ſame whatever be the ratio of R to r; and it is manifeſt that the rectangle under the two middlemoſt terms,

The leſs is $\frac{R}{r}$ the greater is AQ and ax.

is greateſt of all, when R and *r* are equal; and that theſe rectangles will be the leſs, the greater is the ratio of R to *r*.

In a convex lens, the leſs is *t*, the greater is AQ and *ax*. And in a concave lens, the greater is *t*, the greater is AQ and *ax*.

187. *Corol.* 3. All other things being the ſame: In a convex lens (*fig.* 61, 63.) the leſs is *t*, the greater will be AQ and *ax*. In a concave lens (*fig.* 62, 64.) the greater is *t*, the greater alſo will be AQ and *ax*; this will be manifeſt by comparing together the terms wherein *t* is (*Art.* 185). In *fig.* 61. the firſt term is greater than the third; and in both *t* is ſubtracted. In *fig.* 63. *t* is added to the firſt, and *t* is either ſubtracted from the third, or the third term is greater than the firſt; whence in both figures, the greater is *t*, the leſs will be the fourth term; or the greater will be the ratio of the ſecond term, which is an invariable quantity, to the fourth *. In like manner, in *fig.* 62, 64. by comparing the firſt and third terms together, it will manifeſtly appear, that the greater is *t*, the greater alſo will be AQ and *ax*. But in a convex lens, the greater is *t*, the greater will be *a*Q and A*x*; and in a concave lens, the contrary.

188. *Schol.* 3. Rays refracted by a ſingle ſurface of what kind ſoever, or by a lens of what figure ſoever; the leſs is the ratio of the greater of the ſines of incidence and refraction to the leſſer; that is, the leſs is the ratio of *m* to *n*, or the nearer they approach to an equality, the greater will be the diſtance of the focus of the refracted rays, from the refracting ſurface or lens.

PROP. XXVI.

189. *If rays come both ways, upon a lens whoſe ſides have unequal curvities, parallel to the axis; the focus, after refraction by the lens, of thoſe rays that fall upon the flatteſt ſide, will be the fartheſt from a convex lens, and the neareſt to a concave lens.*

This has been already ſhewed of plano-convex and plano-concave lens's in *Art.* 177. And becauſe in *Art.* 185, in each couplet of proportionals, the firſt terms, and the diviſors to the two middle terms are the ſame; AQ and *ax* will be as the rectangles under the correſponding middle terms, (multiplied by the common rectangles $\overline{m-n} \times \overline{m-n}$). that is,

In *Fig.* 61. $mn\mathrm{R}r - n\mathrm{R}t : mn\mathrm{R}r - nrt :: \mathrm{AQ} : ax.$

Fig. 62. $mn\mathrm{R}r + n\mathrm{R}t : mn\mathrm{R}r + nrt :: \mathrm{AQ} : ax.$

Fig. 63. $mn\mathrm{R}r - n\mathrm{R}t : mn\mathrm{R}r + nrt :: \mathrm{AQ} : ax.$

Fig. 64. $mn\mathrm{R}r + n\mathrm{R}t : mn\mathrm{R}r - nrt :: \mathrm{AQ} : ax.$

Therefore in *Fig.* 61, 63. *ax* is greater than AQ; and in *Fig.* 62, 64. *ax* is leſs than AQ. *Q. E. D.* 190.

* If to or from two unequal quantities, the ſame quantity be added or ſubtracted, the leaſt quantity will be thereby moſt affected; that is, it will accordingly have a greater or leſs proportion to the greateſt, than it had before. Let the two quantities be 3, 4. Now the ratio of 3 ± 1 $\left\{\begin{matrix}4\\2\end{matrix}\right\}$ to 4 ± 1 $\left\{\begin{matrix}5\\3\end{matrix}\right\}$ is $\left\{\begin{matrix}\text{greater}\\\text{leſſer}\end{matrix}\right\}$ than the ratio of 3 to 4.

190. *Schol.* If the lens's be glaſs, the rules in *Art.* 185. are convertible into the following;

1. $3R+3r-t : 2R :: 3r-t : AQ$; Fig. 61.
And $3R+3r-t : 2r :: 3R-t : ax.$

2. $3R+3r+t : 2R :: 3r+t : AQ$; Fig. 62.
And $3R+3r+t : 2r :: 3R+t : ax.$

3. $3R-3r+t : 2R :: 3r-t : AQ$; Fig. 63.
And $3R-3r+t : 2r :: 3R+t : ax.$

4. $3R-3r-t : 2R :: 3r+t : AQ$; Fig. 64.
And $3R-3r-t : 2r :: 3R-t : ax.$

191. *Corol.* 1. If the lens's be glaſs, and their thickneſs be neglected as inconſiderable, the above rules will become $3R \pm 3r : \begin{matrix}2R\\2r\end{matrix} :: \begin{matrix}3r\\3R\end{matrix} : \begin{matrix}AQ.\\ax.\end{matrix}$

And dividing the antecedents by 3, we ſhall have the following:

192. *Corol.* 2. $R \pm r : \begin{matrix}2R\\2r\end{matrix} :: \begin{matrix}r\\R\end{matrix} : \begin{matrix}AQ;\\ax:\end{matrix}$ that is, in a double convex or double concave glaſs, whoſe thickneſs is inconſiderable, it will be

As the ſum of the radii, is to one of them;

But in a concavo-convex glaſs, it will be

As the difference of the radii, is to one of them; ſo is double the other to the focal diſtance.

193. *Corol.* 3. Hence in a double convex or double concave glaſs, whoſe thickneſs is inconſiderable, and ſides are ſegments of the ſame ſphere, the focal length is equal to the radius of the ſphere.

194. *Corol.* 4. In a concavo-convex glaſs whoſe thickneſs is inconſiderable, if the radius RA (R) be $= 3ra$ $(3r)$; the focus Q will fall in the center R of the flatteſt ſide. For it will be $2r(R-r) : r :: 3r(2R) : br = AR$.

And if the proportion of RA to ra, is greater than that of 3 to 1; Q will be between the center R and the lens; but if the ſaid proportion is leſs than that of 3 to 1, Q will be farther than R from the lens.

PROP. XXVII.

195. *If the diſtance* Rr *of the centers* R, r *of the ſides* BA, ba *of a concavo-convex medium, be to* Aa *the thickneſs of the medium, as* n *to* m; *and* r *be to the ſame ſide of* R *that* a *is of* A; *the incident and emergent parts* KB, bM, *of a ray* KB *parallel and very near to the axis* Ar, *will be parallel.* Fig. 65.

Let

Let q be the principal focus of the ſide A, and f of the ſide a.

[a] 124. I. [b] Hypoth. Becauſe $Aq : Rq :: {}^{a}m : n :: {}^{a}af : rf :: {}^{b}Aa : Rr$; it will be $aq\,(Aq - Aa) : rq\,(Rq - Rr) :: af : rf$; and $ar\,(aq - rq) : rq :: ar\,(ar - rf) : rf$. Therefore the points q, f coincide; and conſequently the focus of the refracted ray is at an infinite diſtance, that is, bM is parallel to KB. *Q. E. D.*

196. *Corol.* 1. If the centers R, r, be moved farther aſunder, the points f, q will ſeparate, and the lens will have a real focus, as in *fig.* 63. So that a concavo-convex lens, whoſe ſurfaces are ſegments of the ſame ſphere, has a real focus.

197. *Corol.* 2. If the centers R, r be brought nearer together, f will be nearer to the lens than q, and the lens will have a virtual (and not a real) focus. And this will happen when the centers R, r unite: So that a lens with concentric ſides is to be deemed a concave lens; but when the thickneſs Aa is but ſmall, the divergency of the refracted rays will not be ſenſible. If the centers R, r change ſides, that is, if r gets neareſt to the lens, the caſe will be, as is exemplified in *fig.* 64.

Fig. 66.

Fig. 67, 69, 71.

198. *Defin.* The thickneſs of a convex lens is a part of the axis (AV, *fig.* 67. or Aa, *fig.* 69, 71.) intercepted between the ſides; and its breadth is a line (Ss) drawn between the extreams of the ſides (perpendicular to the axis), the ſides being continued to a ſharp edge.

Fig. 68, 70, 72.

199. *Defin.* A concave lens is underſtood to have its ſides touch in the middle, and its breadth is terminated by two lines (SS, ss) drawn parallel to the axis, and at the ſame diſtance from it on either ſide. The length of (either of) theſe lines ($SS = ss$) is reckoned the thickneſs of the lens; and the diſtance between them ($Ss = Ss$) is the breadth of the lens.

Or, If the concave lens has ſome thickneſs in the middle, the difference between that and the thickneſs at the ends, is to be reckoned the thickneſs of the lens.

200. *Lemma.* The thickneſs, half the breadth, and focal length of a plano-convex or plano-concave glaſs, are continual proportionals, nearly.

Fig. 67, 68.

[a] 179. I. Let the axis Aa be twice the radius of the convexity SAs, and of the concavity Sas; then will [a] a be one of the principal focus's of the plano-convex glaſs, *fig.* 67 *. and [a] A the focus of the plano-concave glaſs, *fig.* 68*. And becauſe SV and Sv are each perpendicular to the diameter Aa of the ſemicircle $ASSa$, it will be

[b] *Vide* 8. VI. *Euc.* $AV : SV :: {}^{b}SV : Va$; and $av : Sv :: {}^{b}Sv : vA$. Therefore, *&c. Q. E. D.*

201.

* This is never ſtrictly true till the thickneſs of the lens has vaniſhed. Vid. *Art.* 178.

201. *Corol.* Let the radius of the convex ſide SA*s* be called R, and of the concave ſide S*as*, *r*; and let SV and S*v* be called S: Now if the thickneſs of the lens be neglected as inconſiderable, in proportion to the focal length; it will be AV : S :: S : 2R; and *av* : S :: S : 2*r*; that is, $AV = \frac{S^2}{2R}$; and $av = \frac{S^2}{2r}$.

PROP. XXVIII.

202. *Half the breadth of any very thin glaſs lens, is a mean proportional betwixt its thickneſs and focal length, nearly.*

This has been already ſhewed of plano-convex and plano-concave glaſſes, in the preceding lemma.

Let half the breadth of the lens, SV or S*v*, be as before called S, and the thickneſs of the lens AV $\pm$ *av* be called *t*. Now AV $=^a \frac{S^2}{2R}$; and *av* $=^a \frac{S^2}{2r}$. Wherefore, Fig. 69 *to* 72. [a] 201. I.

Caſe 1. $t\ (= AV + av) = \frac{S^2}{2R} + \frac{S^2}{2r}$; and by reducing theſe fractions to a common denominator, and dividing by 2, we ſhall have $t = \frac{R+r}{2Rr} S^2$; Wherefore, multiplying each Side by 2R*r*, and dividing by R+*r*, we ſhall have $S^2 = \frac{2Rr}{R+r} t$. Fig.69,70.

Caſe 2. $t\ (= AV - av = \frac{S^2}{2R} \backsim \frac{S^2}{2r}) = \frac{R \backsim r}{2Rr} S^2$; and therefore $S^2 = \frac{2Rr}{R \backsim r} t$. Fig.71,72.

Let the focal length be called *f*, then in *Caſe* 1. $f =^b \frac{2Rr}{R+r}$; and in [b] 192. I. *Caſe* 2. $f =^b \frac{2Rr}{R \backsim r}$; and therefore in both caſes $ft = SS$; that is, $t : S :: S : f$. *Q. E. D* *.

203. *Corol.* Hence, in all ſorts of lens's that are very thin in proportion to their focal lengths: If of theſe three, *viz.* breadth, thickneſs, focal length, any two be the ſame, the third will be the ſame alſo; or any two being given, the third may be found.

§. 2.

* This propoſition is not to be truſted to, when applied to glaſſes of very ſhort focus's; becauſe the thickneſs of ſuch glaſſes uſually bear too great a proportion to their focal lengths.

§. 2. *Of oblique pencils of rays.*

204. The axis of any pencil of rays whatſoever incident upon a refracting ſphere, will paſs thro' the center unrefracted. This is ſelf-evident; for a ray that enters a ſphere perpendicular to the ſurface, emerges out of it alſo perpendicular to the ſurface. But the caſe is different, with reſpect to a lens; for the ſides of it not being concentric, no ray can be perpendicular to both the ſides, but that which coincides with the axis of the lens: And hence all pencils of rays refracted by a lens, excepting that one whoſe axis is perpendicular to both the ſurfaces, are ſaid to be oblique, and called *oblique pencils.*

PROP. XXIX.

205. *In every oblique pencil of rays whatſoever, refracted by any lens, there is one ray whoſe incident and emergent parts are parallel; and this ray is the axis of that pencil, or that ray in it which undergoes the leaſt refraction.*

Fig. 73, *to* 78. In *fig.* 75, to 78. let R,*r* be the centers of the ſpherical ſides A,*a*; and in *fig.* 73, 74. let *r* be alſo the center of the ſpherical ſide *a*. In the* flatteſt ſide of any lens whatſoever aſſume any point B, at which ſuppoſe a ray of light to be refracted, either at its enterance into the lens, or at its emerſion out of it: Draw the perpendicular RB to that ſide, and parallel thereto draw alſo the perpendicular *rb* to the other ſide *a*; join B*b*, then will B*b* be a ray whoſe parts PB, *pb*, without the lens, will be parallel. For the ray B*b* drawn between the two parallels RB, *rb* is [a]equally inclined to them both, and conſequently PB, *bp* are alſo [b]equally inclined to RB, *rb*, and therefore parallel to one another. And after the ſame manner, wherever the point B is aſſumed, we can find a ray B*b*, that ſhall be equally inclined to both the ſurfaces; and therefore in every oblique pencil of rays, there is one ray whoſe incident and emergent parts are parallel. And it is manifeſt alſo, that every other ray paſſing through B, will be more refracted than the ray B*b*; for a plane touching the lens in any other point beſides *b*, will be inclined to a plane touching it in B; and therefore a ray paſſing through B and any other point beſides *b*, [c]will be bent out of its courſe, more or leſs, according as theſe planes are more or leſs inclined. Therefore, *&c. Q. E. D.*

[a] 29. I. *Eucl.*

[b] Laws of refraction.

[c] 69. I.

Qq is the axis of an oblique pencil on the contrary ſide of the axis of the lens.

206.

* The only reaſon for chuſing B in the flatteſt ſide, is to prevent *b* falling out of the lens, which otherwiſe might happen, becauſe the ſide A is a leſſer ſegment than the ſide *a*.

206. *Corol.* The axis of any oblique pencil whatſoever, is equally inclined to each ſide of the lens: And the leſs oblique is the pencil, the nearer will the point B be to the vertex A; and [a] the leſs will be the diſtance between the parallel rays PB, *bp*. [a] 68. I.

PROP. XXX.

207. *In every oblique pencil of rays, the part* Bb *of the axis within the lens, produced if need be, will interſect the axis of the lens in the very ſame point* O; *and the point* O *divides the axis of the lens in ſuch a manner, that* AO : aO :: R : r; *that is,* Fig. 73 *to* 78.

1. *The point* O *is in the vertex of a plano-convex and plano-concave.* Fig. 73, 74.

2. *In the double convex and double concave, the point* O *divides that part of the axis which is within the lens, in the ratio of the radii of the ſides, the ſhorteſt part being next that ſide which has the greateſt curvity.* Fig. 75, 76.

3. *The line* Bb *muſt be produced without a concavo-convex lens, before it will interſect the axis: And the point* O *in the axis, where the ſaid line* Bb *produced interſects it, lies next to that ſide which has the greateſt curvity; and its diſtance from the ſides, is as the radii of theſe ſides reſpectively.* Fig. 77, 78.

Let the radius of the flatteſt ſide A be called R, and the radius of the other ſide, *r*; and becauſe the axis B*b* of any oblique pencil, [a] is equally inclined to both the ſides of the lens, it neceſſarily follows: [a] 206. I.

Caſe 1. In a plano-convex and plano-concave lens, the axis B*b* of any oblique pencil paſſes through the vertex *a*, and therefore the point O is alſo in *a*. For no perpendicular to the ſpherical ſide, can be parallel to a perpendicular to the plane ſide, but that only which paſſes thro' the vertex *a*. Again, becauſe R in this caſe is infinite, and O coincides with *a*; it will be, R : *r* :: AO : *a*O. Therefore, *&c.* *Q. E. D.* Fig. 73, 74.

Caſe 2, and 3. Becauſe the radii RB, *rb* are [b] parallel, the △s RBO, *rb*O are ſimilar; and therefore RB : *rb* :: RO : *r*O. Alſo RB ∓ *rb* : RB or *rb* :: RO ∓ *r*O (R*r*) : RO or *r*O. But the three firſt terms are invariable quantities, and therefore ſo is the fourth likewiſe; that is, in the ſame lens the point O is invariable. And from the poſition of the parallel radii RB, *rb*, it is manifeſt that the point O muſt be within the double convex and double concave lens, and without the concavo-convex. Fig. 75 *to* 78. [b] Hypoth.

Again, becauſe RO : RA (RB) :: *r*O : *ra* (*rb*); it will alſo be,

AO (RO ∽ RA) : RA :: *a*O (*r*O ∽ *ra*) : *ra*; that is,

AO : *a*O :: R : *r*. Therefore, *&c.* *Q. E. D.*

208. *Corol.* The point O is in the middle of a double convex and double concave, whoſe ſides are ſegments of the ſame ſphere. And in every lens whatſoever, the greater proportion the greater radius bears to the leſſer, the nearer will the point O be to the vertex *a* of that ſide which has the greateſt curvity.

PROP. XXXI.

Fig. 73 *to* 78. 209. *If the axis* Bb *of a pencil of rays, is not very oblique to the axis of the lens; the points* s, v *where the parts without the lens* PB, pb *produced, of the axis of the oblique pencil, cut the axis of the lens, divide that axis in ſuch a manner, that*

AO : A*s* :: *m* : *n* :: *a*O : *av*; *nearly.*

[a] Conſtruc. The angles RB*s*, RBO or their ſupplements, [a]are the angles of incidence and refraction at the firſt ſide A; and therefore their ſines are as [b]15&35. I. [b]*m* to *n*. In *fig.* 73, 74. the ∠ RB*s* = [c]A*s*B; and RBO = [c]AOB. [c]29. I. *Euc.* And in *fig.* 75 to 78, the difference between the ∠s RB*s*, RBO, or their ſupplements, and the reſpective ∠s A*s*B, AOB is equal to the ∠ ARB; as will be manifeſt by drawing *d*B parallel to the axis RA. And therefore the ſines of theſe reſpective angles, are pretty nearly in the ſame ratio; that is, Sine ∠ A*s*B : Sine ∠ AOB :: *m* : *n*; nearly. [d] Trigonom. But the ſines of the ∠s A*s*B, AOB [d]are as the ſides BO, B*s*; and when the point B is not very remote from A, BO and B*s* will be as AO and A*s*, nearly. And therefore when the point B is the neareſt of all to A, it will be, AO : A*s* :: *m* : *n*. *Q. E. D.*

In *fig.* 73, 74. the points *a*, *v* and O do all coincide: And in *fig.* 75, to 78, it will be proved as above, that when the point *b* is very near to *a*, it will be, *a*O : *av* :: *m* : *n*. Therefore, *&c.* *Q. E. D.*

210. *Corol.* 1. The farther is the point B from A, and *b* from *a*; [c]56. I. [c]the greater will be O*s* and O*v*.

Let *s* and *v* be points belonging to pencils, that have the leaſt obliquity to the axis of the lens. Then will,

211. *Corol.* 2. $As = \frac{n}{m} AO$; and $av = \frac{n}{m} aO$. Let the thickneſs A*a* of any lens be called *t*: Then in a plano-convex and plano-concave lens, $As = \frac{n}{m} t$. And in a double convex and double concave, whoſe ſides are ſegments of the ſame ſphere, $As = av = \frac{n}{2m} t$.

212. *Corol.* 3. Hence, in a plano-convex and plano-concave glaſs $As = \frac{1}{3} t$. And in a double convex glaſs of equal convexities, and in a double concave glaſs of equal concavities, $As = av = \frac{1}{3} t$.

213.

213. *Schol.* 1. The places of the points s, v are affected only by the different inclinations of the axis of the pencils, refracted by any lens, and not at all by the different diſtances from the lens, of the focus's of the ſaid pencils. The point v will be always without a concavo-convex lens; but the point s may be either without or within that lens, according as the ratio of AO to Aa, is greater or leſs than the ratio of m to n.

214. *Schol.* 2. As the angles ſubtended by objects and their images, are properly thoſe which are contained between the axis's of the pencils belonging to their extreme parts; ſo theſe angles, ſtrictly ſpeaking, are not formed at any common point in or out of a lens, but at the two diſtinct points s, v according to the reſpective ſide of the lens, the object and image are of.

215. *Schol.* 3. The focal length of any lens, is to be reckoned from the point v, if the flatteſt ſide is expoſed to parallel rays; and from the point s, if the parallel rays are incident upon the moſt convex or concave ſide. The point v in all caſes, being the moſt remote from the flatteſt ſide (A) of the lens. *Note*, the points s, v we ſhall ſometimes call the *focal centers* of the lens. Focal centers of a lens.

[215.] *Corol.* Hence, the focal length of a convex lens is, properly, the diſtance between the principal focus and the next focal center. And the focal length of a concave lens, is to be reckoned from its principal focus, to the fartheſt focal center. See *fig.* 57, to 64. and *fig.* 73, to 84.

216. *Schol.* 3. As the axis's of the ſeveral pencils that are not very oblique, are refracted orderly from the ſame point s or v nearly; ſo the refraction of theſe axis's cauſe no ſenſible confuſion in the image.

217. *Schol.* 4. Becauſe $As = {}^{a}\frac{n}{m}AO$; and $av = {}^{a}\frac{n}{m}aO$. [a] 211. I.

Alſo becauſe $R : r :: {}^{b}AO : aO$; it will be, [b] 207. I.

$$R \pm r : \frac{R}{r} :: t\,(AO \pm aO) : \frac{AO}{aO};$$ therefore,

$$AO = \frac{Rt}{R \pm r}; \text{ and } aO = \frac{rt}{R \pm r}.$$ Wherefore,

$$As = \frac{n}{m} \times \frac{Rt}{R \pm r}; \text{ and } av = \frac{n}{m} \times \frac{rt}{R \pm r};$$ and multiplying one ſide of *each* equation by $n \times \overline{R \pm r}$; it will be,

$$As : av :: nRt : nrt.$$

PROP.

PROP. XXXII.

218. *If rays come both ways upon any lens, parallel to the axis; the diſtances of the geometrical foci on each ſide, from the reſpective points* s, v *will be equal.*

Let as before the thickneſs of the lens be called t, and the radii of the ſides AB, ab be called R, r.

Case 1. *When one ſide of the lens is plain.*

Fig. 57 *to* 60. Becauſe AF (*fig.* 57.) $=$ [a]EF $+ \frac{n}{m}t$ (*fig.* 59.) $=$ [b]EF $+$ Es. And

[a] 177. I. EF (*fig.* 58.) $=$ [a]AF $- \frac{n}{m}t$ (*fig.* 60.) $=$ [b]AF $-$ Es. Therefore,

[b] 211. I. AF (*fig.* 57.) $=$ sF, *fig.* 59. And in *fig.* 58. $\left\{\begin{matrix} s\text{F} \\ \text{AF} \end{matrix}\right\} = \left\{\begin{matrix} \text{AF} \\ s\text{F} \end{matrix}\right\}$ *fig.* 60.

Therefore, *&c. Q. E. D.*

Fig. 61 *to* 64. Case 2. *When both ſides of the lens are ſpherical.*

Becauſe in *fig.* $\begin{matrix} 61 \\ 62 \end{matrix}$; $mn\text{R}r \mp n\text{R}t : mn\text{R}r \mp nrt$ :: [c]AQ : ax.

[c] 189. I. By adding or ſubtracting the like proportionals $n\text{R}t : nrt$ :: [d]As : av;

[d] 217. I. it will be, $mn\text{R}r : mn\text{R}r$ :: AQ $\pm$ As (sQ) : $ax \pm av$ (vx).

Alſo becauſe in *fig.* $\begin{matrix} 63 \\ 64 \end{matrix}$; $mn\text{R}r \mp n\text{R}t : mn\text{R}r \pm nrt$ :: [a]AQ : ax;

it will alſo be, $mn\text{R}r : mn\text{R}r$ :: AQ $\pm$ As (sQ) : $ax \mp av$ (vx).

Therefore in all caſes $s\text{Q} = vx$. *Q. E. D.*

219. *Schol.* 1. The radius of a plano-convex or plano-concave lens being the ſame, the focal length (AF, *fig.* 57, 58. and sF, *fig.* 59, 60.) will be the ſame, whatever be the thickneſs of the lens.

220. *Schol.* 2. All other things being the ſame. In a double convex lens, and in the concavo-convex, *fig.* 64. the greater is t, the greater will be sQ or vx; and in the double concave and the meniſcus, *fig.* 63. the greater is t, the leſs will be sQ or vx. For,

[e] 185 & 217. I.

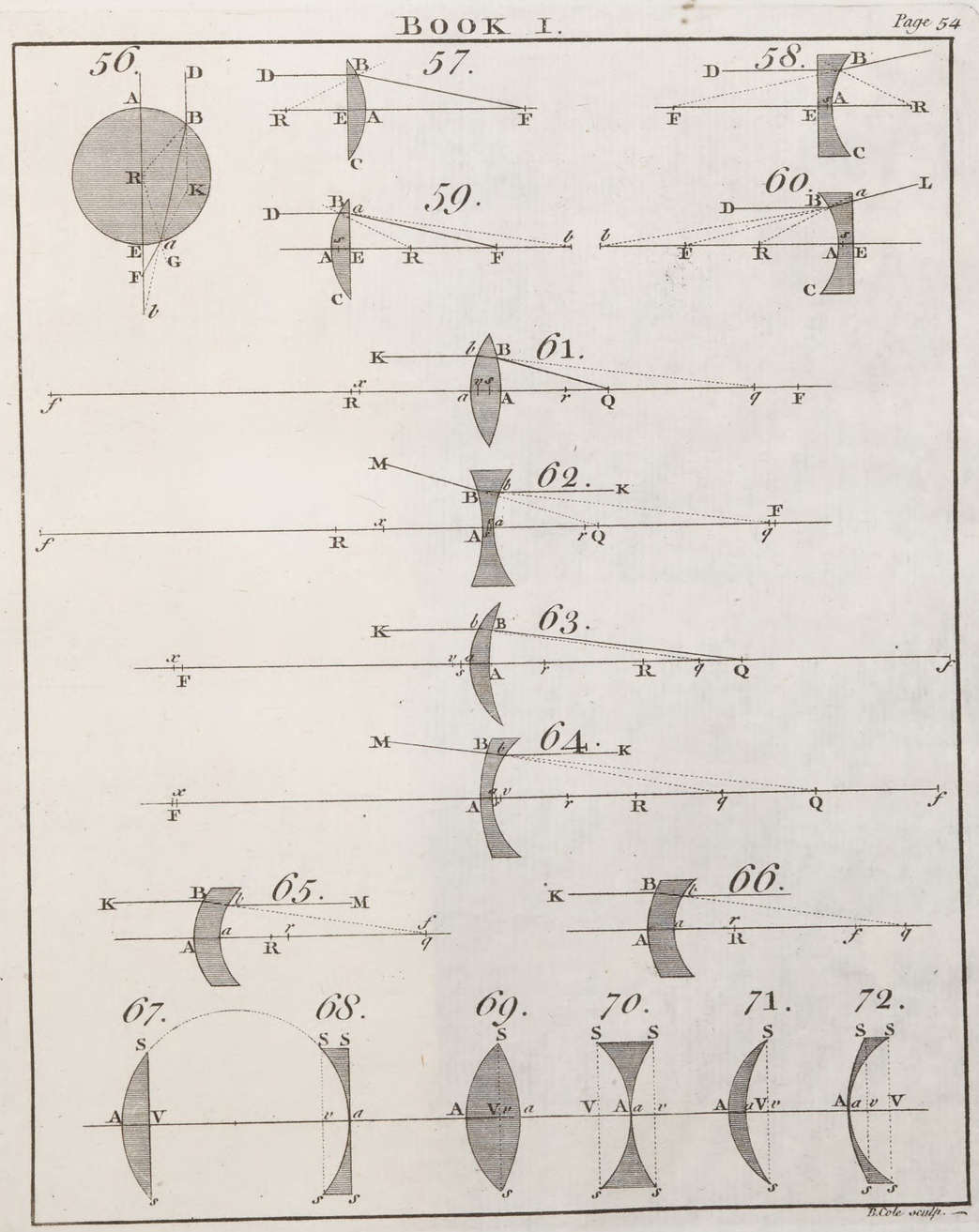
Fig. 61. $\dfrac{\frac{mn\text{R}r}{m-n} - n\text{R}t}{m\text{R} + mr - mt + nt}\,(\text{AQ}) + \dfrac{n\text{R}t}{m\text{R}+mr} = {}^{e}s\text{Q}.$

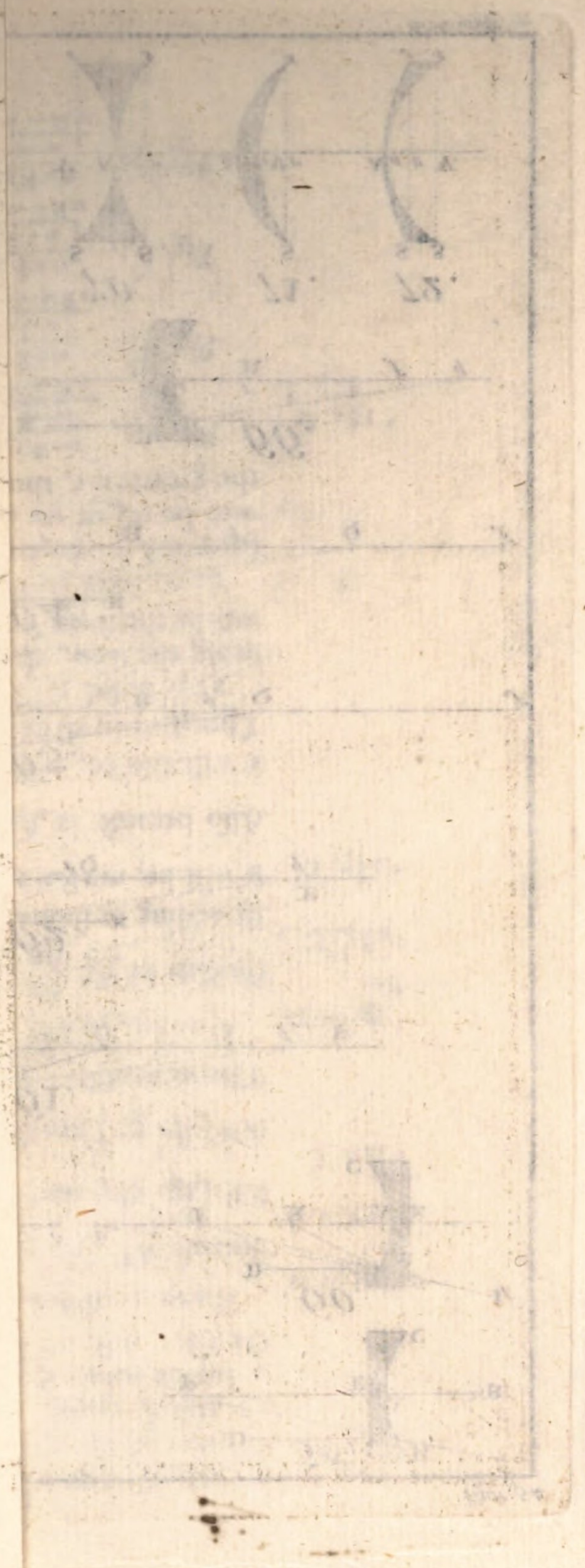
Fig. 62. $\dfrac{\frac{mn\text{R}r}{m-n} + n\text{R}t}{m\text{R} + mr + mt - nt}\,(\text{AQ}) - \dfrac{n\text{R}t}{m\text{R}+mr} = {}^{e}s\text{Q}.$

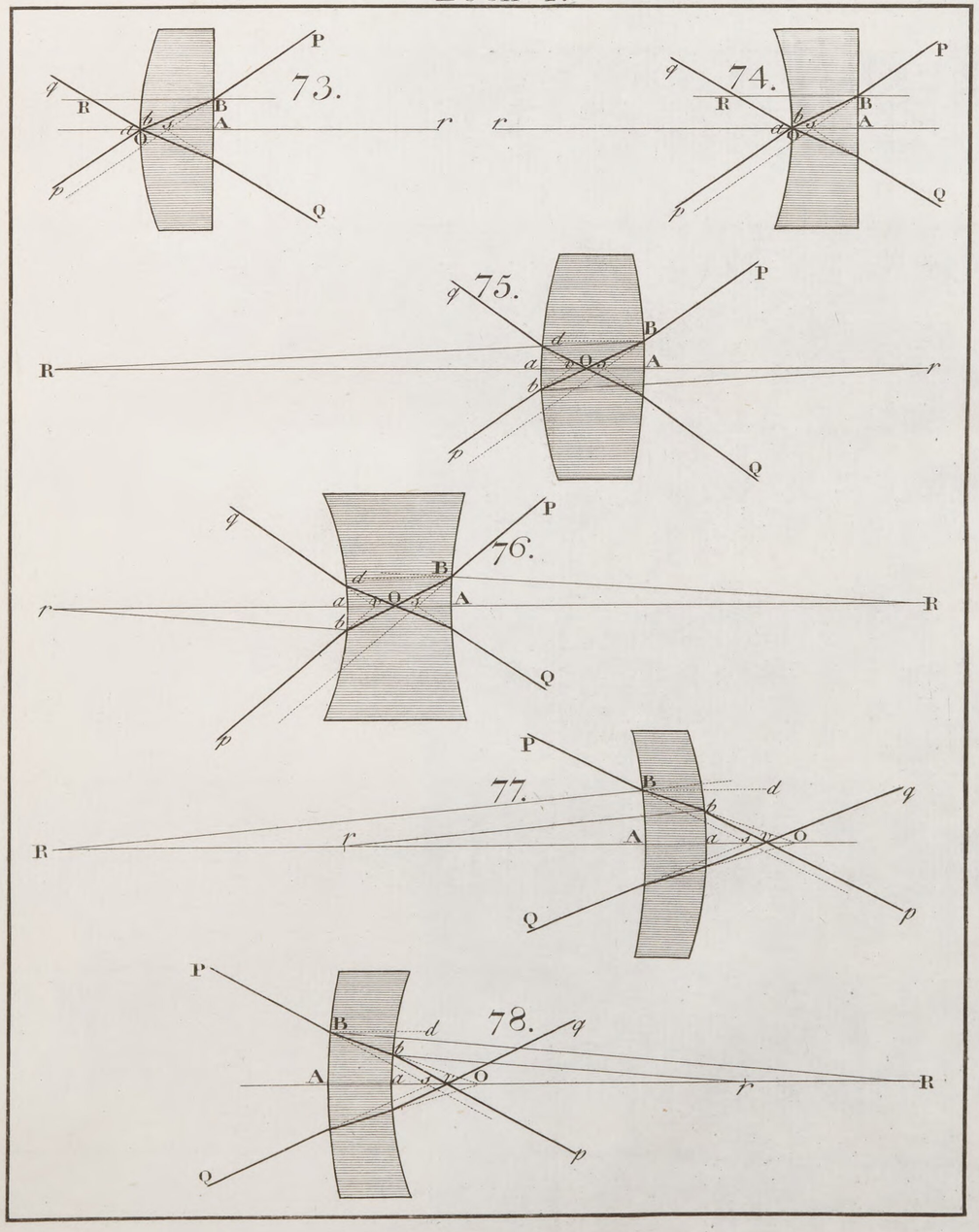
Fig. 63. $\dfrac{\frac{mn\text{R}r}{m-n} - n\text{R}t}{m\text{R} - mr + mt - nt}\,(\text{AQ}) + \dfrac{n\text{R}t}{m\text{R}-mr} = {}^{e}s\text{Q}.$

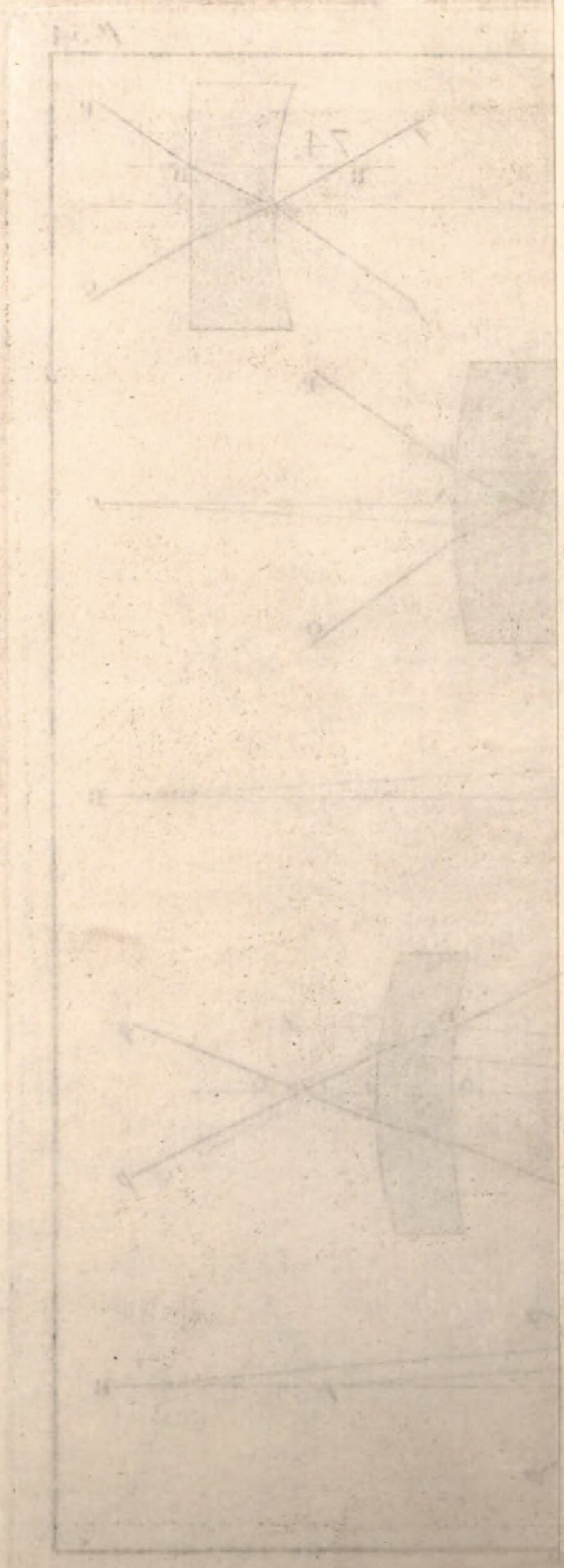
Fig. 64. $\dfrac{\frac{mn\text{R}r}{m-n} + n\text{R}t}{m\text{R} - mr - mt + nt}\,(\text{AQ}) - \dfrac{n\text{R}t}{m\text{R}-mr} = {}^{e}s\text{Q}.$

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In each caſe, the variable quantities $-nRt$, $+nRt$ in the numerators have contrary ſigns; whence it is manifeſt that in *fig.* 61, 64. the greater is t, the greater will be sQ; and in *fig.* 62, 63. the greater is t, the leſs will be sQ.

PROP. XXXIII.

221. *The foci of both direct and oblique pencils of parallel rays, are nearly at the ſame diſtance from the focal center* (s *or* v, *as the caſe is*) *of any lens.*

Let ArQ be the axis of the lens, r the center of the firſt ſurface ab, Q the principal focus of that ſurface, and F the focus of the lens. Let pbBP be the axis of any oblique pencil; and let the emergent part BP, produced backwards cut the axis of the lens in s. Of all the rays parallel to the axis pbBP, there ſhall be one as Dd which will fall perpendicular upon the firſt ſurface ad; and therefore if the medium was continued, this ray (produced backwards from a concave lens, but continued through a convex lens) would paſs through the center r, and the geometrical focus of rays parallel to it would be at q, ſo that $rq = rQ$. This is manifeſt, becauſe drq is perpendicular to the ſurface a, as well as arQ. Fig. 79 *to* 84.

Again, of all the rays parallel to the axis pb, there ſhall be one as Ee (if the lens is broad enough) which ſhall be refracted into eG perpendicular to the ſecond ſurface AB; and therefore the focus of this ray ſhall be in sP produced, and in eG produced; that is, in the point of interſection f. Neglecting the aberration of the firſt ſurface ab; eG produced would unite with the axis drq in q. We are now to ſhew that $sf = sF$ nearly.

CASE 1. *When one ſide of the lens is plain.* Fig. 79, 80.

Becauſe sr, fq; and sf, rq [a] are parallel; $sf = {}^{b}rq = {}^{c}rQ = {}^{d}sF$.

[a] Hypoth. [b] 34. I. *Euc.* [c] Conſtruct. [d] 218. I.

CASE 2. *When both ſides of the lens are ſpherical.* Fig. 81 *to* 84.

Let R be the center of the ſecond ſurface AB, and becauſe eG is perpendicular to this ſurface, it will paſs through the center R. And becauſe rq is [a] parallel to sf, it will be $Rr : Rs$ [e] $:: rq : sf$. Now the two firſt terms (Rr, Rs) are invariable quantities, and admitting rq to remain invariable, sf will be alſo invariable; that is, sF will deſcribe the ſector sFf, and sf will be $= sF$.

[e] 4. VI. *Euc.*

But becauſe of the aberration by the firſt ſurface de, no ray, as Ee parallel to and remote from the axis Drq, can be refracted to the focus q; [f] but it will cut the axis rq, in ſome point y, between q and r; and therefore a ray parallel to pb, that ſhall be refracted perpendicularly to the ſecond ſurface AB, muſt be ſome ray hk, between Ee and the axis pb, if the centers R, r are on different ſides, *fig.* 81, 82; otherwiſe hk Fig. 79 *to* 84.

[f] 134. I.

will be farther from the axis than E*e*, *fig.* 83, 84; and conſequently this ray, will cut the axis *sf* in ſome point *x*, between *f* and *s*. But it has been ſhewed above, that if the lens had no aberration, *f* would be the focus of all the rays parallel to the axis *pb*B*f*; and as the aberrations at the different ſurfaces, are contrarily, it is ſufficiently manifeſt that the focus of thoſe rays that are neareſt to the axis *pb*B*f*, will not be remote from *f*. *Q. E. D.*

222. *Schol.* In every lens whatſoever, the true focus *g* of an oblique pencil of parallel rays will be between *f* and *s*; that is, *sg* will be leſs than *sf* or SF.

Becauſe if there was no aberration at either ſurface of the lens, *sg* would be [g]equal to *s*F or *sf*; it follows that if the aberrations at each ſurface were equal, they would deſtroy each other. Let rays of light be within the lens, on each ſide of the axis *b*B, and let their inclination to that axis be ſuch, as that they would emerge at the ſide *b* parallel to *bp*. Now it is manifeſt, that a ray parallel to the oblique axis *pb*BP, and between the ſaid axis and the perpendicular *dr*, will be more oblique to the emergent ſide A, than to the incident ſide *a*; and therefore in this caſe, the aberration at the ſide A is the greateſt, and this exceſs of the aberration, contracts the focus nearer to *s* than *f*, as to *g*. In like manner, if the parallel ray be on the other ſide of the oblique axis, the greateſt aberration will be on the incident ſide *a*; and in the preſent caſe likewiſe, this exceſs of aberration, contracts the focus nearer to the lens, as to *g*; and it is manifeſt, that the more oblique is the pencil, the greater will be *fg*. Therefore, *&c.* *Q. E. D.*

[g] 221. I.

Fig. 81, 82.

§. 3. *Of diverging or converging rays, and of images.*

PROP. XXXIV.

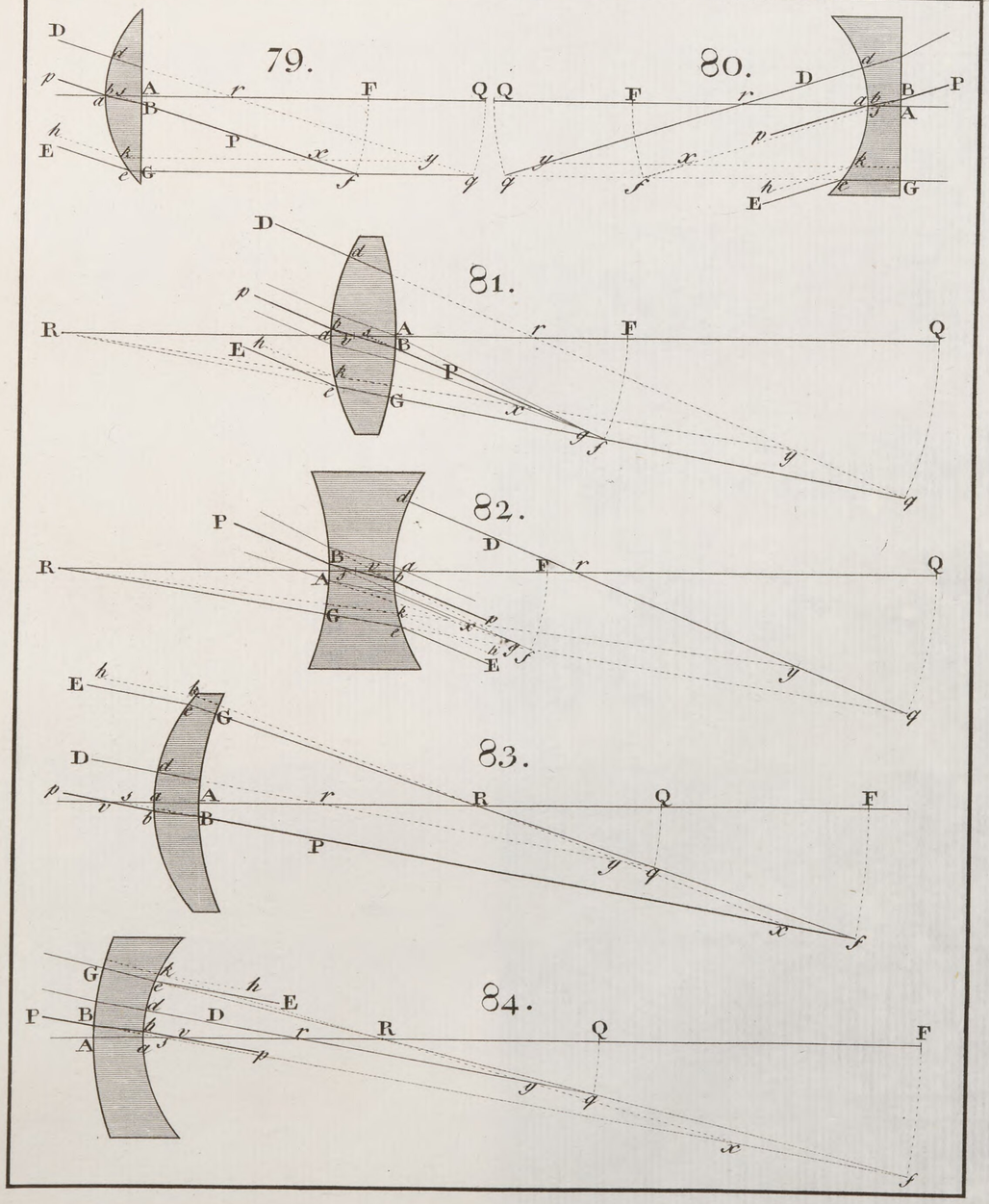
223. *The focus of incident rays, either diverging or converging upon any lens, being given; to find the focus of the emergent rays.*

Let F*f* be the axis of any lens B*b*, whoſe focal centers are *s*, *v*; and principal foci are F, *f*. Let any point Q in the axis of the lens be the focus of incident rays, either diverging as QB, or converging as MB; and let *f* be the geometrical focus of thoſe rays parallel to the axis of the lens, whoſe incidence is on the ſame ſide with the ray QB, or MB. To find the focus of the emergent rays, ſay QF : F*s* :: *vf* (F*s*) : *fq*; and placing *q* the ſame way from *f*, that F is from Q; the point *q* thus found will be the focus required.

Fig. 85 to 90.

Conſtruction. Let the point of incidence B, be not remote from the vertex of the lens: from the center *v*, deſcribe the arch *fd*, and draw *vd* parallel to the incident ray QB or MB; [a]then will *d* be the point where the emergent ray interſects the axis *vd*, and therefore the point *q*, where

[a] 221. I.


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80.
81.
82.
83.
84.



where the emergent ray bd (or Nb) produced, interſects the axis of the lens, is the focus required.

Again, draw sD parallel to qB; then will the △s QDs, vdq be [b]equiangular, and therefore QD : Ds :: [c]vd : dq. But D is a point where the incident ray qb, or Nb, after refraction by the lens, will interſect the axis sD, ſo that sD = [a]sF nearly; and when the △s QDs, vdq are vaniſhing, the point D will coincide with F, and d with f; wherefore QF : Fs :: vf : fq. *Q. E. D.*

[b] 29, & 32. I. *Euc.*
[c] 4. VI. *Eu.*

224. *Corol.* 1. Becauſe Fs = [c]vf, Fs or vf is a mean proportional between QF and fq; that is, QF : Fs :: Fs : fq.

[c] 218. I.

225. *Corol.* 2. QF : Fs :: (vf : fq ::[d]) Qs (QF ± Fs) : vq (vf ± fq).

226. *Corol.* 3. QF : Qs :: (Fs : vq ::[d]) Qs (QF ± Fs) : Qs ± vq.

[d] Vide 12. V. *Eucl.*

227. *Schol.* 1. When the lens is not very thick, in proportion to its focal length, the focal diſtances sF, vf, may without ſenſible error be reckon'd from the point O, which biſects vs; and then the preceding rules, will be convertible into the following, *viz.*

1. QF : FO :: FO (=Of) : fq.
2. QF : FO :: QO : Oq.
3. QF : QO :: QO : Qq.

228. *Corol.* 4. If q be the focus of incident rays, Q will be the focus of the emergent rays.

229. *Corol.* 5. The focus's Q, q, move both the ſame way; and always lie the contrary way from F and f. And the diſtances FQ, fq, vary reciprocally; that is, as one increaſes, the other decreaſes, but with a different velocity, according as they are differently ſituated. When Q coincides with F, q will be at an infinite diſtance; that is, the refracted rays will emerge parallel to the axis. And, in *fig.* 85, 86, when QO = 2OF, Oq will be = OQ; whence the difference of the focal diſtances of the refracted rays when QO = 2OF, and when OQ is infinite, is only equal to Of.

230. *Corol.* 6. When the focus Q, of rays diverging upon a convex lens, *fig.* 87. lies between F and the lens; the refracted rays (bN inſtead of croſſing the axis) will diverge from the focus q. And if incident rays upon a concave lens, converge towards a point that is nearer to it than F, as in *fig.* 88. the lens in this caſe will have a real focus; that is, the refracted rays will croſs the axis in q. And univerſally,

231. *Corol.* 7. When the focus's Q, q lie both on the ſame ſide of the lens; if the incident rays diverge from Q, (*fig.* 87, 90.) the emergent

Fig. 87 to 90.

rays will diverge from *q*. And if the incident rays converge towards Q, (*fig*. 88, 89.) the emergent rays will converge towards *q*. And in all theſe caſes, the nearer Q is to O, the nearer alſo will *q* be to O; and if one of theſe focus's be in O, the other will alſo coincide with it.

Fig 85,86. And the contrary happens, when Q and *q* are on different ſides of the lens; that is, rays diverging from Q (*fig*. 85.) will converge towards *q*. And rays converging towards Q (*fig*. 86.) will diverge from *q*.

232. *Schol*. 2. Convex and concave lens's, may be compared to convex and concave ſurfaces; and what has been ſhewed with reſpect to the motions of the focus's Q,*q* by the refraction of the one, is alſo applicable to the other. Thus, in *fig*. 85, 86. the focus's Q,*q* are on oppoſite ſides of the lens; and neither can approach nearer to the lens, than the focus's F,*f*. If one of the focus's is between F and the lens, *fig*. 87 to 90, the other will be alſo on that ſame ſide of the lens. In *fig*. 87, 88. the whole motion of Q is from F to the lens; but the motion of *q* is not limited to any diſtance from the lens to infinity. In *fig*. 89, 90. the place of *q* is limited from *f* to the lens; and the motion of Q, is like to that of *q* in the preceding caſe. The three caſes of convex lens's paſs from one into the other, as do alſo thoſe of concave lens's.

233. *Schol*. 3. All ſorts of convex lens's, that have equal focal lengths, will have the ſame effect if put into each other's places; becauſe the rules above, regard only their focal lengths, and not their figures. The ſame is alſo true of concave lens's. So that the ſix figures referred to, contain all the variety that can happen with reſpect to the focus's Q, *q*.

PROP. XXXV.

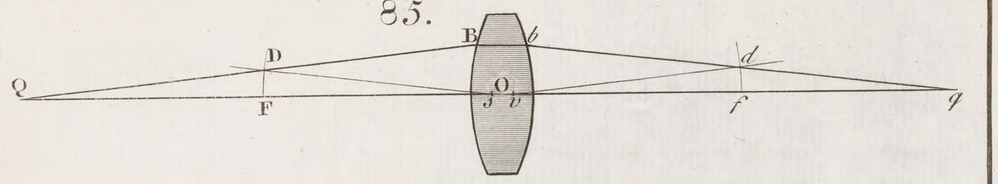
Of Images, formed by the refraction of Lens's.

Fig. 91 *to* 94. 234. *If an arch of a circle* PQR, *deſcribed from the focal center* s *of any lens, be conſidered as an object; its image* pqr *may, in moſt caſes, be alſo conſidered as a ſimilar arch, whoſe center is the other focal center* v. *The length of the object will be to the length of its image, as* sQ *to* vq: *And the image with reſpect to the object, will be erect or inverted, according as they lie on the ſame ſide, or contrary ſides of the lens.*

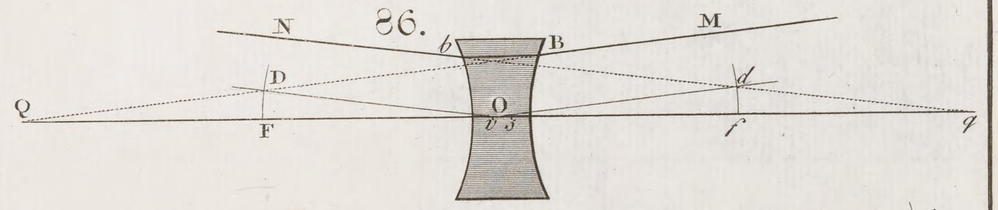
Fig. 93. This propoſition, with reſpect to a ſphere is ſelf-evident; becauſe the object being concentric thereto, (the points *s*, *v* in this caſe coinciding with the center O,) every part of the object is alike expoſed to the ſurface of the ſphere. Therefore, *&c*.

And

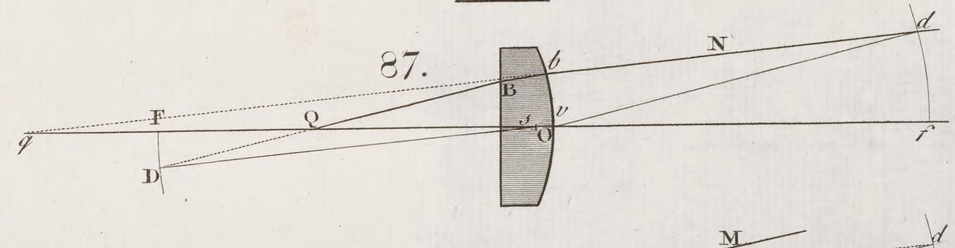
85.



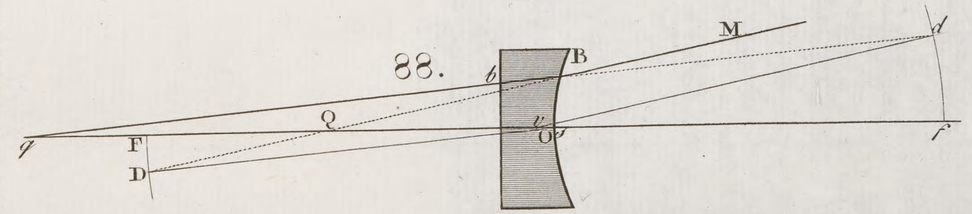
86.



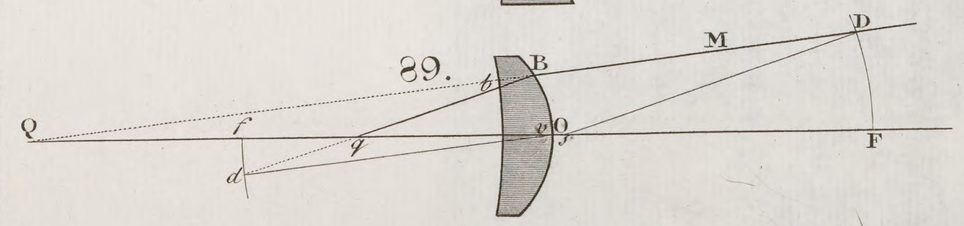
87.



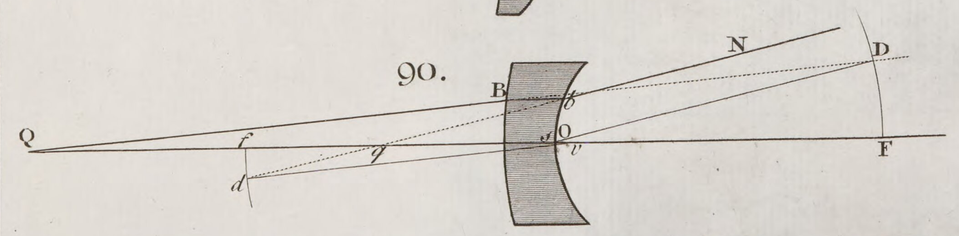
88.

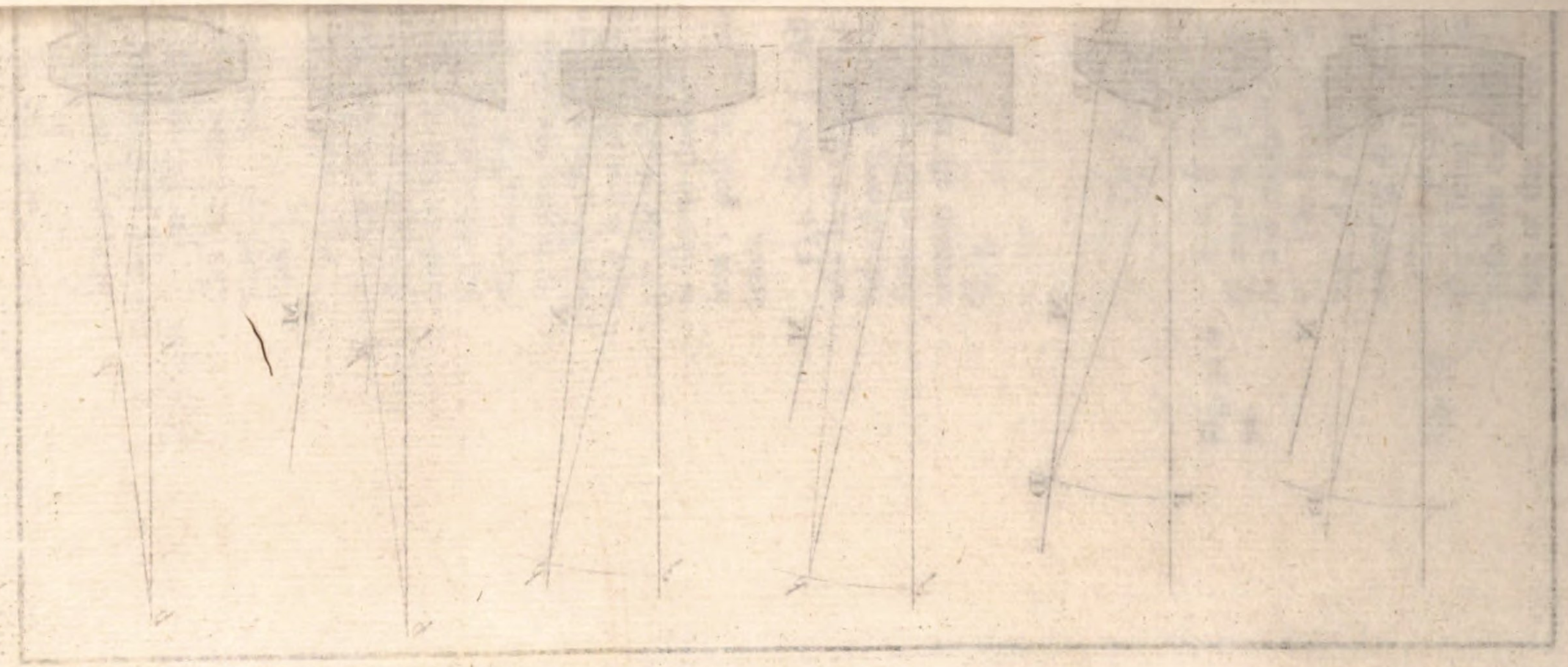


89.



90.





And in a lens, becauſe the geometrical foci of parallel rays, E,F,G; Fig.91,92, *e*,*f*,*g* [a]are nearly in arches, whoſe centers are *s*,*v* reſpectively; neglect- 94.
ing the aberrations of the lens, (or ſuppoſing the object, and the thick- [a] 221. I.
neſs of the lens, to be ſo ſmall that the refractions of the two ſides are
equal,) it will be PE : E*s* :: [b]*ve* : *ep*; QF : F*s* :: [b]*vf* : *fq*; and [b]223. I.
RG : G*s* :: [b]*vg* : *gr*. But in theſe three couplets of proportionals, the
three firſt terms [a]are nearly equal each to each reſpectively; and con-
ſequently the fourth terms are equal alſo nearly, and therefore *pqr* may
without ſenſible error, be reckoned a circular arch, whoſe center is *v*.
Q. E. D.

235. Again, becauſe the axes P*s*, *vp*; R*s*, *vr* of the oblique pen-
cils, [c]are parallel, the angles P*s*R, *pvr* are equal; and therefore the ob- [c] 205. I.
ject PQR is to its image, as their diſtances *s*Q, *vq*: And according as
their correſponding extremes P, *p*; R, *r* lie on the ſame, or on contrary
ſides of *s*, *v*; they lie alſo on the ſame or on contrary ſides of their
middle points Q,*q*; that is, the image with reſpect to the object, is
erect or inverted. *Q. E. D.*

236. *Schol.* 1. *In common caſes, or when the lens is not very thick;
[d]the axes of the oblique pencils may, without ſenſible error, be conſi- [d] 227. I.
dered as interſecting the axis of the lens in the point O, which biſects
the diſtance *s v*.

237. *Schol.* 2. If the object is a ſtrait line ſubtending a ſmall angle at
the lens, its image will be ſo too, nearly; and the ſmaller the object and
image are in proportion to their diſtances from the lens; that is, the
ſmaller are the angles which they ſubtend, the more exactly will they
correſpond. And univerſally, the images of all ſurfaces ſubtending very
ſmall angles at the focal centers of any lens, will be, accurately enough,
alike to their originals.

238. *Schol.* 3. Becauſe the focal diſtances OE, O*e*, of an oblique
pencil, [e]are leſs than the focal diſtances OF, O*f*; a circular object whoſe [e] 222. I.
center is O, will not have a ſimilar arch for its image. If the object
and its image are on different ſides of the lens, as in *fig.* 91. or if the
lens is concave; the image will have a greater curvity, than the object;
that is, the ratio of O*q* to OQ, will be greater than the ratio of O*p* to
OP: But if the object and image, are both on the ſame ſide of a con-
vex lens, *fig.* 92. the ratio of O*p* to OP, will be greater than the ratio
of O*q* to OQ. As will be manifeſt, by comparing together the rules
QF : FO :: QO : O*q*; PE : EO :: PO : O*p*.

* In the four laſt plates, the lens's are made thicker than ordinary, for the ſake of repreſenting the places of the points *s*, *v*, and the rays belonging to them diſtinctly.

Fig. 91 *to* 94.

239. *Schol.* 4. Neglecting the aberrations of the lens's, a ſtrait object (PQR) at right angles to the axis of any lens, will have a curvilinear image: And if the object and image are on different ſides of the lens, as in *fig.* 91. the image will be concave towards the object; and the image will be convex towards the object, if they are both on the ſame ſide, as in *fig.* 92, 93, 94. For it being QF : OF :: $^{f}Of : fq$; and PE : OE :: $^{f}Oe : ep$; it will alſo be,

[f] 234, & 236. I.

QO (QF ± OF) : OF :: Oq (Of ± fq) : fq; and
OF (=OE) : PO :: ep : Op; and by multiplying theſe two rows of proportionals together, we ſhall have,
QO : Oq × ep :: PO : Op × fq. But in *fig.* 91, 94. (becauſe QF is leſs than PE, and the middle terms OF, Of, OE, Oe are equal,) it is manifeſt that ep is leſs than fq; and therefore the ratio of Oq to OQ, is greater than the ratio of Op to OP. And in *fig.* 92, 93. ep is greater than fq; whence there the ratio of Oq to OQ, is leſs than the ratio of Op to OP. Whence in all caſes, the image is curvilineal; and in *fig.* 91. the image is concave, and in *fig.* 92, 93, 94. it is convex, towards the object. *Q. E. D.*

If the aberrations of the oblique pencils be conſidered, becauſe then the focal diſtances OE, Oe are leſs than OF, Of; it is manifeſt that the curvity of the image will be ſtill greater, and the ſame way as before. And the greater is the angle QOP, the leſs in *fig.* 91, 94. and the greater in *fig.* 92, 93. will be the focal diſtance Op. So that qp is not a circular arch, but approaches towards one or other of the conic ſections, having q in its vertex.

And it may be farther obſerved, that the images of ſtrait objects, by convex or concave lens's, correſpond to thoſe of like objects, by convex or concave ſurfaces. *Vide Art.* [153.] I.

240. *Schol.* 5. When an object, as PQR, is farther from a convex lens (*fig.* 91.) than its principal focus F, there will be formed a real image pqr, whoſe poſition with reſpect to the object, is inverted.

But an object placed nearer a convex lens, (*fig.* 92, 93.) than its principal focus F, or any where before a concave lens, (*fig.* 94.) will have no real image; the rays b,b; a,a; c,c in each pencil, after their emerſion out of the lens, diverging from the foci p, q, r, reſpectively. In both theſe laſt caſes, the image pqr is erect; and in the firſt caſe (*fig.* 92, 93.) the image is always greater, and in the ſecond caſe, (*fig.* 94.) the image is always leſs than the object.

Fig. 95.

241. *Schol.* 6. The theory of real images, may be thus illuſtrated by an experiment. Upon a long table draw the line BD, and over ſome point

point there in A, place the convex lens O, whoſe principal focus's are F, *f*, ſo that the axis F*f* of the lens, be parallel to BD. In the line BD, take AF, A*f* each equal to OF or O*f* in the axis Q*q*. On one ſide of A divide the line AB into the parts 1, 2, 3, 4, *&c.* each equal to AF; and on the other ſide take *f* 1 equal to A*f* or AF, and divide it into $f\frac{1}{2}$, $f\frac{1}{3}$, $f\frac{1}{4}$, *&c.* ſo that theſe parts be reſpectively equal to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, *&c.* of *f* 1 or A*f*. Now if the room be darkened, and a lighted candle be placed over any one of the diviſions in the line AB, as at 2; the image of the candle, will be ſeen diſtinct but inverted, upon a paper held over the correſponding fraction on the other ſide, as at $\frac{1}{2}$. If the candle be placed at the point 3 or 4, *&c.* the paper for receiving the image muſt be held over $\frac{1}{3}$ or $\frac{1}{4}$, *&c.* So that if the candle be moved from 2 to an infinite diſtance, the whole motion of the image will be from $\frac{1}{2}$ to *f*. If the candle be placed at 1, the image will be at 1, at the ſame diſtance from the lens on the other ſide. If the candle be brought nearer to F, the motions of the image and candle will now be reciprocal, to what they were before. But if the candle be placed any where, between F and the lens, there will be no image formed.

242. *Schol.* 7. Since an object placed at any diſtance, and its image, ſubtend [s] equal angles at the focal centers *s*, *v* of any lens; the angle under which any remote object appears, being given, it will be eaſy to find the diameter of its image. [s] 235. I.

Let *q* be the principal focus of the convex lens O, and let the angle P*s*R = *pvr* be that under which the ſun (or any other very remote object) appears to the naked eye: Biſect the ∠ *pvr* with the line *vq*; then in the right-angled △ *vqp* or *vqr*, we ſhall have the ſide *vq* and all the ∠s, to find *qp* or *qr*, the ſemi-diameter of the image. Fig. 96.

PROP. XXXVI.

Of the Camera Obſcura.

243. *By a* Camera Obſcura *in optics, is meant any darkned room or box, out of which all light is excluded, but what comes through a lens upon a white ſheet properly placed, ſo that the pictures of objects that are without, are ſeen thereon. This phænomenon is very curious, and being altogether explicable upon the principles already delivered; its * theory may not improperly be explained here.*

Let OB be a darkened room or box, into which no light is admitted, but what comes through the lens O, whoſe principal focus is F, and axis is Q*q*: And let *q* be the focus, after refraction, of rays which come from Q. Fig. 97.

244.

* The mechaniſm of different ſorts of *Camera Obſcura's*, will be exhibited in Book III.

244. *Phænomenon.* Now if a ſheet, of * white paper or canvas, be placed at *q* perpendicular to the axis O*q*; the pictures of objects that are before the lens, at the diſtance OQ, will be fairly and orderly repreſented upon the paper, in their natural colours and proportions, but inverted.

In the figure, to prevent confuſion from too many lines, there are drawn only the three pencils that come from the extremes P, R, and middle Q, of the object PQR; and in theſe pencils, there are alſo drawn only the two extreme rays, beſides the axes. But the rays flowing from any point P, for inſtance, upon the lens, are innumerable, the whole conical ſpace *b*P*d*, being filled with them; and theſe being all collected and united at the focus *p*, and there received upon the white paper, whoſe ſurface is uneven, are reflected by it all manner of ways; after the ſame manner, as objects themſelves reflect the light falling upon them. So that *p* to a ſpectator in the room, is now, as it were, a real object, exactly ſimilar to the phyſical point P, and in proportion to it, as O*p* to OP. And *p* is of the ſame colour with P, becauſe the rays flowing upon the lens from P, are united at *p*, diſtinct and ſeparate from the rays coming from other parts of the object.

In like manner, every other phyſical point of the object; ſends forth its cone of rays, which by the lens are brought to unite orderly at *pqr*; and being there reflected by the paper, the image of the whole object is diſtinct and viſible, like a picture drawn upon canvas; but much more lively and perfect, than the beſt finiſhed drawings of the greateſt artiſt.

If the objects are very remote, in proportion to the focal length of the lens, we ſhall have the pictures of thoſe that are in the ſame neighbourhood, pretty diſtinct at the ſame time, though they are at different diſtances from the lens; becauſe in that caſe the focal diſtances of the refracted rays, differ inſenſibly.

245. *Schol.* 1. If the paper be moved nearer towards the lens, as to *x*, or farther from it, as to *y*; the picture will begin to be confuſed; becauſe the rays proceeding from the next adjacent points of the objects begin to interfere and mix together, as the rays from the point *a*, will be mixed with thoſe from P, *&c.* The diſtinctneſs of the picture, is entirely owing to the ſeparation of the rays belonging to every point of the object, upon their reception by the paper. And if the rays belonging to any phyſical point of an object could be ſeparated from all others, the image of that point would be diſtinct upon the paper placed at any diſtance either before or behind the focus; only it would be leſs bright, becauſe

* The picture appears beſt upon white, becauſe white bodies reflect light more uniformly and copiouſly, than bodies of other colours; and paper reflects light more uniformly than canvas.

becauſe it would contain fewer rays, according to the greater diſtance of the paper from the focus.

If the paper be moved farther and farther from the focus, the picture will become more and more indiſtinct, and at length totally vaniſh, no one part being diſtinguiſhable from the reſt; becauſe now the rays, even from remote parts, are mixed together. It is for a like cauſe, that the room muſt be dark; otherwiſe the foreign light mixing with the pencils that form the picture upon the paper, weaken that picture, till at laſt as the light grows ſtronger it becomes inſenſible. A picture will be perfectly formed, upon a paper placed in the focus of a lens, in the open field; but the paper being at leaſt equally enlightened from all other parts, and its uneven ſurface reflecting the light it receives all manner of ways; the picture formed by the lens is quite obliterated, and nothing is ſeen but the paper itſelf.

246. *Schol.* 2. Why the image is inverted, is evident by a bare inſpection of the figure; and it is manifeſt alſo, that this inverſion is not owing to the lens: For if the lens be removed, and the light be admitted into the chamber, through a moderate hole of about an inch or leſs in diameter, we ſhall have on the oppoſite wainſcot or wall, an inverted (tho' a very imperfect) picture of the objects without, after the ſame manner as when the light came through the lens upon the paper ſcreen; the ſeveral pencils in both caſes neceſſarily croſſing each other, as well in the open hole, as at the lens. But without the lens, the picture will be very faint, and alſo confuſed: It is faint for want of ſufficient light; there being now but very few rays for forming each point of the image, and theſe being reflected by the wall or paper all manner of ways, the ſeveral pencils that enter the eye are ſo rare, or contain ſo few rays, as to be ſcarce ſenſible: And the picture is confuſed, becauſe the rays coming from the adjacent objects, interfere and mingle together; and the larger the hole, the greater will be this confuſion.

§. 4. *Of the aberrations of rays, from the geometrical focus's of lens's.*

PROP. XXXVII.

Having the angle of incidence upon any given lens, of a ray parallel to the axis; to find the proper focus of the emergent ray.

It will be ſufficient to trace the progreſs of the ray, through the ſphere, and the three convex lens's; the method of tracing the ray through the concave lens's, being much the ſame. This admits of four

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§. 4. *Of the aberrations of rays, from the geometrical focus's of lens's.*

PROP. XXXVII.

Having the angle of incidence upon any given lens, of a ray parallel to the axis; to find the proper focus of the emergent ray.

It will be ſufficient to trace the progreſs of the ray, through the ſphere, and the three convex lens's; the method of tracing the ray through the concave lens's, being much the ſame. This admits of four

Fig. 98 *to* 102. caſes: in each of which A*a* is the axis of the lens; DB is the incident ray parallel to the axis; RA = RB the radius of the ſide AB; and in *fig.* 101, 102. *ra* = *rb* is the radius of the ſide *ab*, and *rbk* is a ſtrait line. Q is the geometrical focus of the firſt ſurface AB, *q* the proper focus of the ray after its firſt refraction by that ſurface; F the geometrical focus of the lens, and *f* the focus required. In each of theſe caſes, the angle of incidence DB*d* (or its ſupplement) = ARB, is given.

Fig. 98. 247. Case 1. *Of a ſphere.*

1. Say *m* : *n* :: Sine ∠ DB*d* (= ∠ RB*b*) : Sine ∠ RB*b*; then draw B*b*[a], which is the refracted ray within the ſphere, and produce it to *q*; through the point *b*, from the center R, draw the line R*bk*. Now the ∠ *kbq* = [b]R*b*B = [c]RB*b*. 2. Say *n* : *m* :: [a]Sine ∠ *kbq* : Sine ∠ *kbf*; and draw *bf*. 3. To find the point *f*; in the △ R*bf* will be given, the radius R*b*, the ∠ *b*R*f* (= 180° — $\overline{\text{BR}b + \text{ARB}}$), and the ∠ R*bf* (= 180° — *kbf*), and conſequently the ∠ *f*; whence R*f*[d] is eaſily found. *Q. E. I.*

[a] Laws of refraction.

[b] 15. I. *Euc.*

[c] 5. I. *Eucl.*

[d] Trigonom.

Fig. 99. 248. Case 2. *A plano-convex lens, with its plane ſide towards the incident ray.*

In this caſe, the incident ray being perpendicular to the plane ſide of the lens, [a]it will paſs on directly to B unrefracted; and therefore the ∠ RBD = [b]*d*B*k* is properly the angle of incidence. Wherefore, 1. Say *n* : *m* :: [a]Sine ∠ *d*B*k* : Sine ∠ *d*B*f*, and draw B*f*. Then 2. In the △ RB*f* will be given, the ſide RB, and all the ∠s, [d]to find R*f*. *Q. E. I.*

Fig. 100. 249. Case 3. *A plano-convex lens, with its convex ſide towards the incident ray.*

1. Say *m* : *n* :: [a]Sine ∠ DB*d* (= [b]RB*b*) : Sine ∠ RB*q*. Through the point *b* (where the ray B*q* cuts the ſide *ah*) draw *bk* parallel to AR; and produce DB to *h*. Then,

2. Say *n* : *m* :: [a]Sine ∠ *kbq* (= RB*h* — RB*q*) : Sine ∠ *kbf*; and draw *bf*. To find the point *f*. 3. In the right-angled △ B*hb* are given the ſide B*h* (= A*a* — A*v*) and the ∠ *b*B*h* (= RB*h* — RB*q*); [d]to find *hb*. 4. In the right-angled △ *baf*, will be given the ſide *ba* (= B*v* — *bh*), and the ∠ *abf* (= 90° — *kbf*); [d]to find *af*. *Q. E. I.*

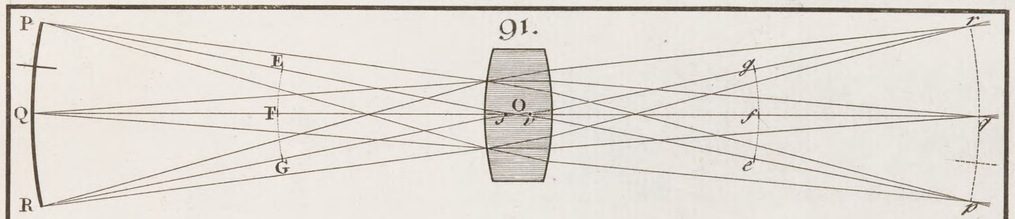
Fig. 101, 102. 250. Case 4. *When both ſides of the lens are ſpherical.*

1. Make as before, *m* : *n* :: [a]Sine ∠ DB*d* (= RB*h*) : Sine ∠ RB*b*, or RB*q*.

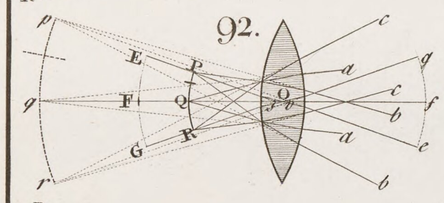
[c]29. I. *Euc.* 2. In the △ *r*RB, are given the ſides *r*R, RB, and the ∠ R (= [c]DB*d* or its ſupplement); to find the ∠s R*r*B, RB*r*, and the ſide *r*B.

3.

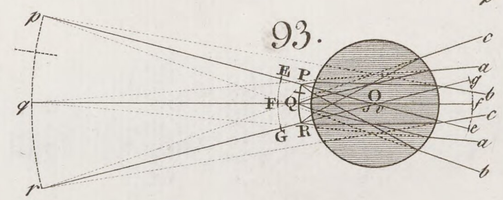
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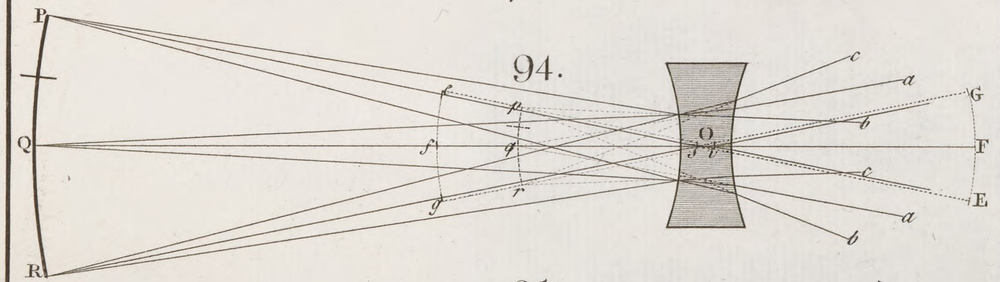
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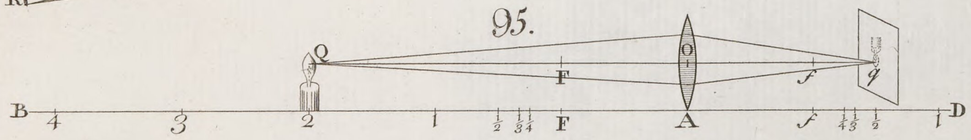
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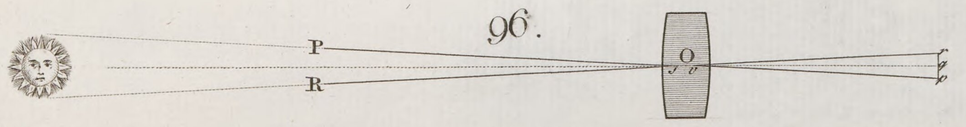
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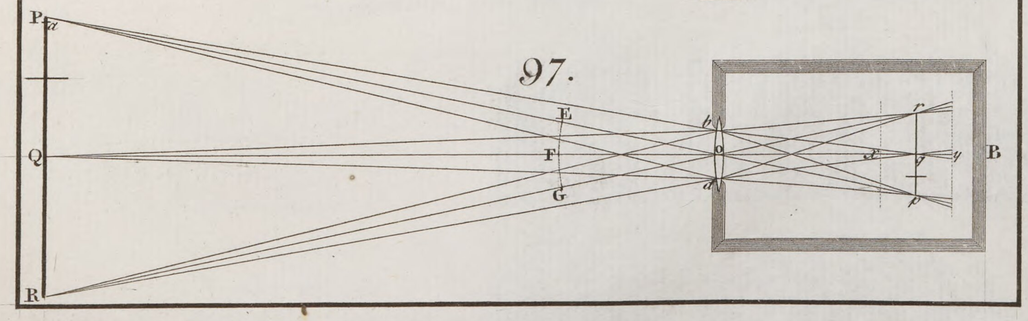
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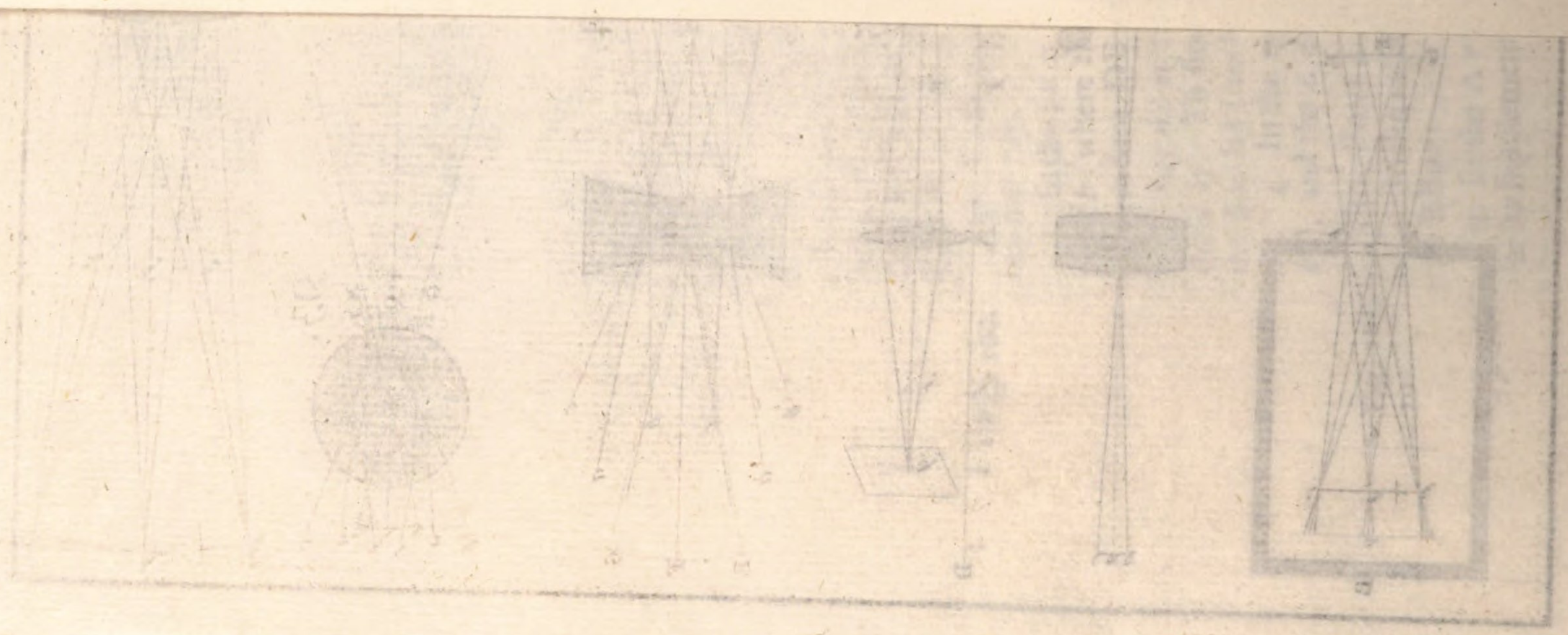


96.



97.





3. In the $\triangle rBb$, will be given the ſides rB, rb, and the $\angle rBb$ ($=$ $RBq \pm RBr$); [d] to find the $\angle Brb$, and alſo the $\angle rbB = [b] kbq$.

4. Say $n : m$ :: [a] Sine $\angle \frac{kbq}{rbq}$: Sine $\angle \frac{kbf}{rbf}$; and draw bf.

5. In the $\triangle rbf$, will be given the ſide rb, the $\angle brf$ ($= BrR \mp BrB$, or their ſupplement) and the $\angle rbf$; [d] to find rf. *Q. E. I.*

If a ray HG parallel to the axis, enters the concave ſide of a meniſcus lens; the point L where the emergent ray cuts the axis, and conſequently the aberration LM, from the geometrical focus M, may, as above, be found thus: Fig. 102.

1. Let it be $m : n$ :: Sine $\angle rGH$: Sine $\angle rGK$; and produce KG through Z to g; then will GZ be the refracted ray, within the lens.

2. In the $\triangle RrG$, are given the ſides Rr, rG, and the $\angle r$; to find the $\angle$s rRG, rGR, and the ſide RG.

3. In the $\triangle RGZ$, will be given the ſides RG, RZ, and the $\angle RGZ$ ($= 180° - \overline{KGr + rGR}$); to find the $\angle ZRG$, and alſo the $\angle RZG = eZg$.

4. Say $n : m$ :: Sine $\angle eZg$: Sine $\angle eZL$; and draw ZL.

5. In the $\triangle RZL$, are given the ſide RZ, and all the $\angle$s (*viz.* the $\angle R = 180° - \overline{ZRG + GRr}$, and the $\angle RZL = 180° - eZL$); to find the ſide RL. *Q. E. I.*

251. *Schol.* 1. Having the foci F, f; the longitudinal aberration Ff, of a given ray DB, is therefore given: And to find the lateral aberration FG; in the right-angled $\triangle fFG$, are given the ſide fF, and the $\angle FfG = Rfb$. Fig. 98 *to* 101.

252. *Schol.* 2. The point of incidence B, was taken at ſome diſtance from the edge of the lens; becauſe a lens let in a frame or cell for uſe, has a part, towards the edge, always covered. This makes no alteration as to the two firſt caſes; becauſe a whole ſphere cannot be uſed, as a lens: And in the ſecond caſe, the incident ray undergoes no refraction by the firſt ſide. But in the two laſt caſes (*fig.* 100 to 102.) if the point B be taken at the edge, that is, if $ABvZ$ be ſuppoſed the lens, the calculation will be ſhorter. Thus,

CASE 3. 1. Say $m : n$ :: Sine $\angle DBd$: Sine RBq. Fig. 100.

2. Make $n : m$ :: Sine $\angle hBq$: Sine $\angle hBf$; and draw Bf.

3. In the right-angled $\triangle Bvf$, are given the ſide vB, and the $\angle f$ ($= hBf$); to find vf.

Fig. 101, 102.

Case 4. 1. Say $m : n ::$ Sine $\angle$ DBd (=RBb) : Sine $\angle$ RBq.

2. In the $\Delta\, r$BR, are given either the three ſides, or two ſides and the $\angle$ R; to find the $\angle$ RBr (whoſe ſupplement, *fig.* 101. is lBR) and the $\angle$ RrB.

3. Say $n : m ::$ Sine $\angle \begin{matrix} l\mathrm{B}q = l\mathrm{BR} - \mathrm{RB}q \\ r\mathrm{B}q = \mathrm{RB}q - \mathrm{RB}r \end{matrix}$: Sine $\angle \begin{matrix} l\mathrm{B}f \\ r\mathrm{B}f \end{matrix}$.

4. In the $\Delta\, r$Bf, will be given the ſide rB, and all the $\angle$s (*viz.* the $\angle r$, and the $\angle\, r\mathrm{B}f$ (=180—lBf, *fig.* 101.) to find rf. *Q. E. I.*

253. *Schol.* 3. Suppoſing the ſines of incidence and refraction from glaſs to air, to be as 3 to 2; the following table (if no miſtake crept into the calculation, which 'twas difficult to avoid) gives the aberrations of the lens's therein ſpecified, in ſuch parts that the focal length of each lens is 100,000 nearly; and the ſeveral apertures that correſpond, are alſo the ſame nearly; that is, making the focal length of each lens Radius, the correſponding angles of incidence will be nearly the ſame; or AB in each lens, and *ab* in the ſphere, will be nearly of the ſame breadth.

Fig. 98.		*Fig.* 99.			*Fig.* 100.		*Fig.* 101.	
RF = 100,000		AF = 100,000			RA = 50,000		RA = rv = 100,000	
$\angle a$Rb	Ff	$\angle$ ARB	Ff	FG	$\angle$ DBd	Ff	$\angle$ DBd	Ff
° ′		° ′			° ′		° ′	
3 : 0 +	697.6	4 : 0	548.63	19.217	4 : 0	142.61	2 : 0	203.41
1 : 30 +	173.93	2 : 0	135.2	2.36	2 : 0	35.867	1 : 0	50.33
0 : 45 +	44.33	1 : 0	34.1	0.297	1 : 0	8.888	0 : 30	11.9
0 : 22½	11.31	0 : 30	8.1	0.035	0 : 30	2.295	0 : 15	3.07

Note, In computing the aberrations of the two laſt lens's, I neglected their thickneſſes; that is, I ſuppoſed each ray to enter at the edge of the lens.

254. *Schol.* 4. By the method ſhewed in the preceding propoſition, may be found the exact quantity of the aberration of parallel rays, from the geometrical focus of any given lens, having a given aperture. Equations have been alſo formed for this purpoſe; but as the method of raiſing them is unavoidably tedious, and their application very laborious, on account of their involving a great many terms; and aberrations from the figure, are but of ſmall moment, in compariſon of the aberrations on account of the different refrangibility of light; as will be ſeen in the next ſection. I ſhall therefore omit inſerting theſe equations

tions here; but ſhall take the liberty of borrowing from * M. *Huygens*, (who, I think, was the firſt that took the trouble of making the computation,) the proportions which the longitudinal aberrations of the extreme rays parallel to the axis, nearly bear to the thickneſſes of the following glaſs lens's; the point where they enter, being ſuppoſed not very remote from the vertex, and alſo in the edge of the lens; as the point B in the lens ABvZ, *fig.* 99, to 102.

255. When the incident ray falls upon the plane ſide of a plano-convex glaſs, $Ff = \frac{2}{3}Av$, nearly. This has been already demonſtrated in *Art.* 138.

256. When the incident ray falls upon the convex-ſide of a plano-convex glaſs, $Ff = +\frac{1}{6}Av$, nearly. And therefore the longitudinal aberration of a plano-convex glaſs, is near four times leſs in this caſe than in the former; being as $\frac{1}{1}\frac{4}{2}$ to $\frac{5}{1}\frac{4}{2}$, or as 7 to 27.

257. When the glaſs is a double convex, of equal convexities, $Ff = \frac{5}{3}Av$ nearly ‖. And therefore the aberration of this glaſs to that of a plano-convex in its beſt poſition, is as $\frac{5}{3}$ to $\frac{7}{6}$, or as 10 to 7.

258. A double convex glaſs having the radius of the firſt ſurface upon which the ray falls, to the radius of the ſecond ſurface, as 2 to 5; is juſt as good as the plano-convex in its beſt poſition, it being in both $Ff = \frac{7}{6}Av$.

259. A double convex, having the radius of its firſt ſurface, to that of the ſecond, as 1 to 6, is the beſt glaſs of all; and its aberration $Ff = \frac{1}{1}\frac{5}{4}Av$. But if the other ſide of this glaſs be expoſed to the parallel rays, Ff will be $= \frac{1}{4}\frac{4}{2}\frac{5}{}Av$.

260. A plano-convex glaſs, with its $\left\{\begin{matrix}\text{convex}\\\text{plane}\end{matrix}\right\}$ ſide, towards the incident parallel rays, has leſs aberration than any meniſcus, with its $\left\{\begin{matrix}\text{convex}\\\text{concave}\end{matrix}\right\}$ ſide expoſed to parallel rays. Whence it neceſſarily follows, that that meniſcus is beſt, which approaches neareſt in ſhape to a plano-convex lens.

261. The above proportions of Ff to Av, will be alſo the ſame in concave glaſſes under like circumſtances; Av being now taken at the circumference of the lens. See *Art.* 137 & 199. I.

* *Huygens*'s Dioptrics, Prop. XXVII. See alſo Dr. *Smith*'s Optics, Book II. Chap. 13.

† This may be proved from *Art.* 138, 61. For by *Art.* 138. the aberration Qq of the firſt ſurface is $= \frac{4}{3}Av$; this then will cauſe an aberration from $F = \frac{2}{3}$ of $\frac{4}{3}Av$ nearly $= \frac{8}{9}Av$. And by *Art.* 61. the aberration Ff of the plane ſurface, if there had been no aberration cauſed by the firſt ſurface, would have been (by *Art.* 58.) $= \frac{5}{1}\frac{}{8}Av$ nearly (becauſe QA is thrice RA); by adding theſe together we have $(\frac{2}{1}\frac{1}{8}\ Av =)\ \frac{7}{6}Av$ for the aberration Ff.

‖ So far reaches the Table, *Art.* 253, with which the above numbers agree nearly.

262. *Schol.* 5. Which ever ſide of a lens is beſt when expoſed to parallel rays, that ſide will be the worſt when expoſed to rays diverging from a focus, near the principal focus of the lens: And *vice verſa*. This is ſufficiently manifeſt of it ſelf.

PROP. XXXVIII.

263. *Having the focus of a ray, either diverging or converging upon a given point, of any given lens; to find the focus of the emergent ray.*

It is needleſs to have figures on purpoſe to illuſtrate this propoſition; but in thoſe laſt referred to, in the axis produced, we may ſuppoſe ſome point Y, to be the focus of the incident ray. The propoſition may be reduced into two caſes.

Fig. 98, 100, 101, 102.

Case 1. *When the incidence is upon a ſpherical ſide.*

Let B be the point of incidence, and in the $\triangle$ YRB will be given the ſides YR, RB, and the $\angle$R, to find the angle of incidence Y; then the reſt of the proceſs will be as in the preceding propoſition.

Fig. 99.

Case 2. *When the incidence is upon the plane ſide of a plano-convex lens.*

Let the point of incidence be *h*; and in the right-angled $\triangle$ Y*ah*, will be given the two ſides Y*a*, *ah*; to find the $\angle$ of incidence Y. 2. Find the refracted $\angle$ at *h*, and ſuppoſe for inſtance the point O to be the focus of the ray refracted at *h*; to find the point O, in the right-angled $\triangle$ *ha*O will be given the ſide *ah*, and the $\angle$ *h*; to find *a*O. Let B be the point where the ray emerges out of the lens: In the $\triangle$ RBO will be given the ſides RB, RO, and the $\angle$ O; to find the $\angle$ RBO, whoſe ſupplement is the $\angle$ *d*BO; then ſaying *n* : *m* :: Sine $\angle$ *d*BO : Sine $\angle$ *d*B*f*, we may draw the ray B*f*. Laſtly, in the $\triangle$ RB*f* will be given, the ſide RB, and all the $\angle$s, to find R*f*. *Q. E. I.*

PROP. XXXIX.

Fig. 99 *to* 102.

264. *In every lens having different apertures, the longitudinal aberrations* (Ff) *of the extreme rays parallel to the axis, are as the ſquares, and their lateral aberrations* (FG) *as the cubes, of the diameters of thoſe apertures, nearly. Or, the longitudinal and lateral aberrations of parallel rays, incident at different diſtances from the axis of any lens, are reſpectively as the ſquares and cubes of theſe diſtances, nearly.*

[a] 140. I. This has been already demonſtrated of [a] ſingle ſurfaces, and conſequently alſo of plano-convex and concave lens's, with their plane ſides expoſed to the incident rays. And from what has been ſhewed in *Art.* 60, 136, it may be gathered, that in all lens's, the longitudinal aberrations *

tions * are in ſome conſtant ratio to Av the thickneſs of the lens, at leaſt nearly; and Av [b] is as the ſquare of the breadth BZ nearly. [b] 36. I. Therefore, *&c.*

Again, whilſt the aperture BZ remains the ſame, FG will be in the ſimple ratio of Ff, that is as BZ^2; but as the aperture BZ is increaſed or diminiſhed, FG at the ſame time increaſes or diminiſhes, not only as Ff, but moreover in the ſimple ratio of the ſine of AB; that is in the whole, as the cube of the ſine of AB, or as the cube of the chord of BZ, nearly. *Q. E. D.*

265. *Schol.* 1. In different lens's, that have the ſame apertures and focal lengths; the lateral aberrations of parallel rays in each, are nearly as the longitudinal ones: But in that lens which has the greateſt aberration, the ratio of the lateral aberration to the longitudinal one, is the greateſt. Let OF, *fig.* 99. be $=$ fF, *fig.* 100. then will Fp in the former be $=$ FG in the latter; vB and vF in each being ſuppoſed reſpectively equal. Now, *fig.* 99. the ratio of GF to Ff, is greater than the ratio of pF to FO, becauſe the $\angle$ O is leſs than the $\angle$ f, and the $\angle$ F in both $\triangle$s is common. Therefore, *&c. Q. E. D.*

266. *Schol.* 2. Now it is manifeſt that no lens can gather all the rays into one point; and that the focus's of all the rays, as they are more and more remote from the axis, will lie orderly nearer and nearer to the lens, than F. Nevertheleſs it is manifeſt alſo, that F or a point very near to F, is the burning focus of the lens, or that place where the rays are denſeſt of all.

Let the arches Aa, ab, bc, cB, cC, on each ſide of the axis AF, intercepted between the parallel rays 0, 1, 2, 3, 4 at their emerſion out of the plano-convex lens BAC, be all equal; and let F be the geometrical focus of the lens, and f the focus of the extreme rays 4, 4. At the point F, upon the axis AF, raiſe on each ſide the perpendiculars FG, FH. Divide Ff into as many equal parts, as the ſquare of the number of equidiſtant rays on each ſide the axis, contains units; that is, in this example divide Ff into 16 equal parts; and from the point F towards f, lay off the ſquares of the numbers 1, 2, 3, *&c.* reſpectively; that is, let F1 be $=$ 1, F2 $=$ 4, F3 $=$ 9, and F4 or Ff $=$ 16; and thro' the points 1, 2, 3, f draw the rays a1, a1; b2, b2, c3, c3; Bf, Cf, and produce them till they meet the line GH; then will the lines FG, FH be divided at the points 1, 2, 3, 4, as the cubes of theſe numbers, nearly; that is F1, F2, F3, F4, will be as the numbers 1, 8, 27, 64, nearly. Or if GH be firſt divided after this manner, rays drawn to the reſpective points therein will divide Ff as above, nearly. Fig. 103.

267.

* See the Table, *Art.* 253.

267. *Schol.* 3. The rays refracted by the lens, will be ſome of them ſpread all over a circle, whoſe diameter is GH; and as they increaſe in denſity from G and H towards F, *ad infinitum*; and from F decreaſe in the ſame manner, ſo as at G and H the denſity becomes nothing; at the center F, they will be infinitely denſe; and at the extremities G, H, they will be infinitely rare, and therefore cannot there be ſenſible. Let
Fig. 104. the line GH, *fig.* 104. be divided in the ſame manner with GH, *fig.* 103. And from the center F, deſcribe the circles 1F1, 2F2, 3F3, 4F4. Now the light contained in the ſmalleſt circle, is equal to that which is contained in the ſpace between the peripheries of any two adjacent circles; and the light contained in the circle whoſe diameter is 2F2, is equal to all the reſt: Whence it ſeems probable, that in a lens having a large aperture, the diameter of the greateſt circle of aberration that is ſenſible cannot much exceed 2F2; at leaſt I think we ſhall be much within compaſs if we aſſume this diameter not leſs than $\frac{1}{4}$ GH.

Fig. 103. 268. *Schol.* 4. The leaſt circle, through which all the rays refracted by any lens can paſs, has for its diameter a certain line *gh*, drawn between the remoteſt points *g*, *h* from *f*, where the extreme rays cut any of the other rays on contrary ſides of the axis. If the refraction be by a ſingle ſurface, or by a plano-convex lens with its plane ſide expoſed to parallel rays, the line *gh* will be $=$ *$\frac{1}{4}$GH. Alſo the diameter of the leaſt circle, through which all the parallel rays, reflected by a concave ſpherical ſpeculum, can paſs, will be $=$ †$\frac{1}{2}$FG.
Fig. 24.

PROP. XL.

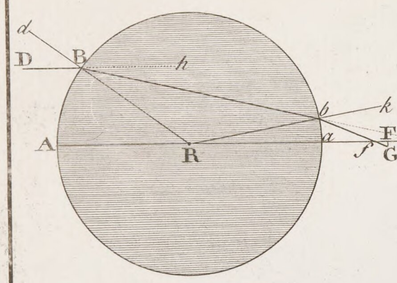
269. *If an incident ray that is not very oblique to the axis of any lens, be conſidered as having its focus in the point where it interſects that axis; or as belonging to the oblique pencil, whoſe axis is parallel to the ſaid ray; the lateral aberrations of the emergent ray in both caſes, will be as the diſtances of its focus's from the lens, nearly.*

Fig. 105, 106. Let F*f* produced be the axis of the lens, Q the focus of an incident ray, either diverging as QB, or converging as MB, to the point B. Parallel to QB draw the axis O*b*. Let *q* be the geometrical focus of emergent rays, whoſe focus of incidence is Q; *b* the geometrical focus of rays parallel to O*b*; and let B*y* produced be the emergent ray. Upon the axis's O*q*, O*b* raiſe the perpendiculars *qt*, *bs*, till they meet the emergent ray. I ſay the lateral aberrations *qt*, *bs* are as the diſtances from the lens B*q*, B*b*, or O*q*, *of*, nearly; that is, *qt* : *bs* :: O*q* : *of*; nearly.

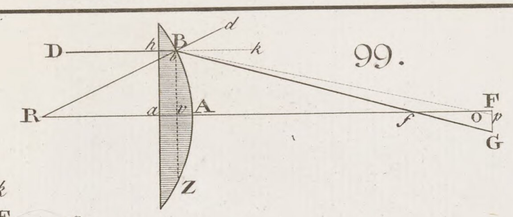
In the △s B*bs*, B*qt*; had the ∠s *b* and *q*, or *s* and *t* been equal, it would have been exactly *qt* : *bs* :: B*q* : B*b*. But in theſe △s, the ∠ B being very

* *Newton*'s Optical Lectures, page 184, 185. † Dr. *Smith*, ibid.
Alſo Dr. *Smith*'s Optics, *Art.* 339.

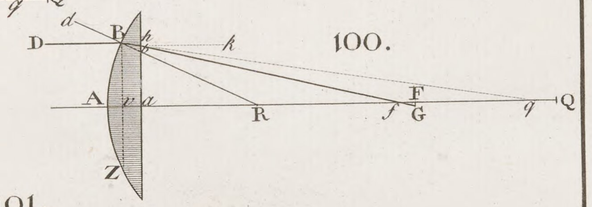
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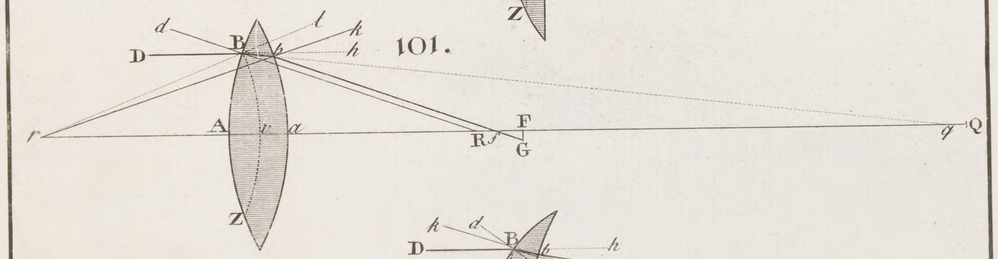
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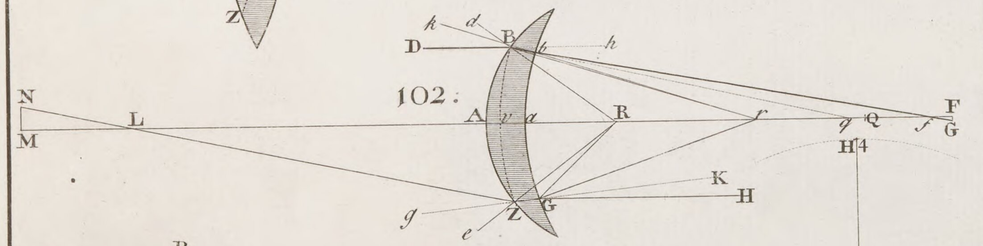
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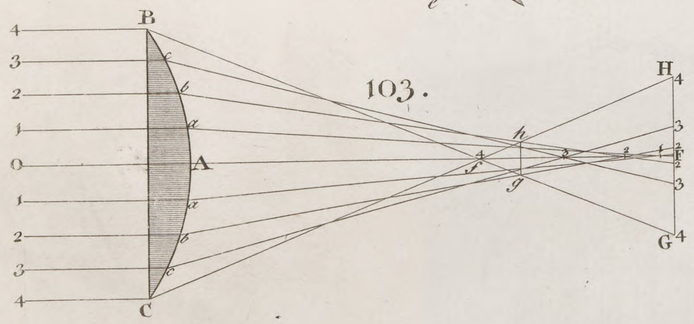
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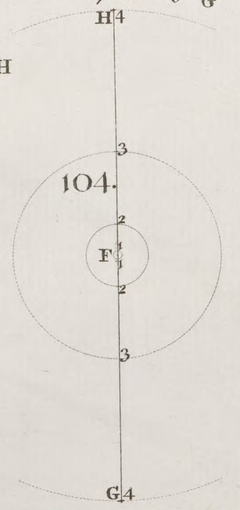
102.



103.



104.



very acute, and the ∠s q,b being not very oblique; the inequality of theſe ∠s can create but a very ſmall and imperceptible error in the above proportion of Bq : Bb :: qt : bs. And the nearer B is to O, the leſs will that error be; alſo in that caſe, the nearer will Bq, Oq, and Bb, Of be to a ratio of equality. Therefore, *&c.* *Q. E. D.*

270. *Schol.* 1. From what has been before ſhewed about the aberrations of ſingle ſurfaces, it is manifeſt that this propoſition is univerſal for them alſo; and likewiſe for concave, as well as convex lens's. See *fig.* 43 to 48.

271. *Corol.* When Oq is greater than Ob or Of; the ratio of qt to bs, is leſs than the ratio of Bq to Bb. And when Oq is leſs than Of, the ratio of qt to bs, is greater than the ratio of Bq to Bb.

272. *Schol.* 2. In a lens having a moderate aperture, the aberration bs, is nearly equal to the aberration fv, of a ray NB parallel to the axis. For in the △s Bbs, Bfv, the ∠s bBs, fBv are nearly equal; and ſo are alſo the other correſpondent ∠s, and the ſides Bb, Bf are alſo nearly equal. Therefore, *&c.*

273. In order to avoid the aberrations, ariſing from the ſpherical figures of lens's, ſeveral of the * writers of dioptrics, before Sir *Iſaac Newton* publiſhed his optical diſcoveries, had beſtowed much pains in inveſtigating the figures of lens's, that would refract a direct pencil of (homogeneal) rays, having a given focus, into a point, without any aberration. But theſe authors, not regarding the oblique pencils, their † *theoretical lens's*, if they really could have been made, would not have anſwered all purpoſes better than thoſe we now have; and upon the whole they would not have been ſo good. For if an object of ſome breadth be placed in the given focus, the extremes of the object ſeen through ſuch a lens, would be far leſs diſtinct than the middle part; and this indiſtinctneſs would increaſe very faſt, as the object ſubtended a larger angle; and in common lens's, we do not ſo much want to correct the aberrations of the direct pencils, as we do to correct thoſe of the oblique pencils. Again, if the focus of incident rays be changed, that

* Monſ. *Huygens* in particular, ſeems to have beſtowed more pains upon Dioptrics, and to have treated that ſcience in a more elegant manner, than any author before him. But this truly great man no ſooner ſaw Sir *Iſaac Newton*'s diſſertations about the different refrangibility of light, than he ingenuouſly embraced this new theory, and laid aſide all farther thoughts of improving refracting teleſcopes, which was his principal aim, by correcting the figures of their glaſſes.

† Sir *Iſaac Newton* hath ſhewed (*Newton*'s Optics, 8vo *Engliſh* edit. page 90.) how to make a better lens for all purpoſes, than can be made of one piece of glaſs, by cementing two like concavo-convex glaſſes, with water between them, together; the radii of the ſides of each glaſs having a certain ratio to each other. A contrivance ſomewhat like this, had been alſo thought of by Dr. *Hook*, before Sir *Iſaac Newton*. But as the aberrations cauſed by the figures of common lens's, are altogether inconſiderable when compared to the aberrations from the different refrangibility of light; this contrivance will not anſwer the pains, as Sir *Iſaac* himſelf hath ſhewed.

that is, if an object be placed nigher to, or farther from the lens, than the focal diſtance for which it was formed, there would be then none of the pencils of light, that came from the object, free from aberrations after refraction by the lens; and a ſpherical lens in this caſe alſo, would do better than the other. So that upon the whole, ſpherical ſurfaces, or common lens's, as well for the reaſons above aſſigned, and alſo becauſe they are the eaſieſt formed, may be reckoned preferable to any other.

SECTION V.

Of the aberrations cauſed by the different refrangibility of the rays of light.

Rays of light differently refrangible; diſcovered by Sir *Iſaac Newton*.

274. FROM the common *phænomenon of a diſtorted coloured image produced by a glaſs triangular priſm, and from ſeveral experiments deviſed by himſelf; Sir *Iſaac Newton* hath clearly demonſtrated, that the rays of light are not all equally refrangible: that is, rays of light impinging any refracting ſurface, at the ſame degree of obliquity from the perpendiculars to that ſurface at the points of incidence, will not be all equally bent or refracted towards or from the ſaid perpendiculars; but ſome will be more, and others leſs refracted, and that in all poſſible degrees between the limits of the leaſt and moſt refrangible rays.

Cauſe of colours, diſcovered alſo by the ſame author.

275. The ſame great author alſo diſcovered, that the cauſe why bodies appear of different colours, is their reflecting ſome of the differently refrangible rays, more copiouſly than others; the moſt refrangible rays producing *violet*, and the leaſt refrangible rays *red* colours. And ſo others through all the intermediate degrees of refrangibility, producing all the varieties of different colours. And hence, the peculiar colours of particular bodies, are owing to ſome peculiar textures or properties of theſe bodies, that diſpoſe them to reflect ſuch rays of light, as are ſuited to produce the idea of ſuch and ſuch a colour.

276.

* The priſm was in many hands, and the phænomenon above well known, but not the cauſe of it, before Sir *Iſaac Newton*'s time. There is no meddling with the ſubject of this ſection, without borrowing from Sir *Iſaac Newton*'s Optics; but I ſhall be very brief in my extracts from it, and refer the reader to the book it ſelf, in which are many curious and ſurprizing diſcoveries about light, which do not immediately concern us here. And of all Sir *Iſaac*'s great performances, his Optics is juſtly eſteemed as one of the chief. That Edition I refer to, is the *Engliſh* Octavo.

276. The different refrangibility of light, is a great impediment to the perfection of optical inſtruments. But this diſadvantage is infinitely outweighed, by the ineſtimable benefits accruing from the diverſities of colours; theſe, by their variety, not only contribute greatly towards the beauty of the material world, but alſo help us to diſtinguiſh bodies and things one from another. And it is almoſt demonſtrable, that if all the rays of light had been perfectly alike in their properties, all the bodies in the univerſe would have appeared of one colour.

277. After what manner the ideas of different colours are produced in the mind, is a propoſition as far above our reach, as how we have the ideas of different ſmells, taſtes, *&c.* But it cannot be doubted, that the impulſes of light upon the optic nerve, which excite the ideas of different colours, muſt be themſelves different; and this difference muſt ariſe from a difference in the conſtitution of light itſelf. And as thoſe rays that are the leaſt refrangible, undergo the leaſt change by the refraction of bodies; it ſeems probable that their impetus ſhould be greater than that of rays which are more refrangible: And this greater impetus of the leaſt refrangible rays may proceed, 1. from the particles of theſe rays being larger than thoſe of the other rays; or, 2. theſe particles having a greater denſity, their magnitudes being the ſame with thoſe of the reſt; or, 3. from their having a greater velocity; or laſtly, from theſe ſeveral cauſes combined together.

278. *Defin.* Rays of light that are alike refrangible, are called *homogeneal rays*; alſo *ſimple*, and *ſimilar rays*: And thoſe rays which differ in their refrangibility, are called *heterogeneal*, *compound*, or *diſſimilar rays*.

279. *Phenom.* If the ſun's rays admitted into a dark chamber, through Fig. 107.
a ſmall round hole O in the window-ſhutter, be intercepted by a triangular glaſs priſm BAC, and the priſm be turned * ſlowly round its axis until the ſun's image be thrown on the oppoſite ſide of the room; inſtead of a round image, there will be formed an oblong one PT, terminated with two rectilinear and parallel ſides, and two ſemi-circular ends. The ſides will be pretty well defined, but not the ends, the light there decaying and vaniſhing by degrees. The breadth of the oblong image, anſwers to that of the ſun's image at a like diſtance from the hole, when its rays are not refracted; and its length being much greater, ſhews that the rays which go to T, are not ſo much refracted as thoſe which go to P; and ſo all the rays that fall from T towards P, have greater degrees of refrangibility, as they are more remote from T. So the oblong

* See this experiment, with all the minute circumſtances that are neceſſary in order to make it ſucceed beſt, deſcribed at length in Sir *Iſaac Newton*'s Optics, p. 21, *&c.* The ſhort account above is ſufficient here, being only intended as an introduction to what follows.

Fig. 108. long image PT is compoſed, as it were, of innumerable circles (*p*, *q*, *r*, *s*, *t*, &c.) each anſwering to the ſun's image, and whoſe centers lie all in a ſtrait line betwixt T and P. And hence it is, that the ſides of PT are rectilinear, and well defined, and the ends ſemi-circular and faint.

Fig. 107. 280. The ſpectrum PT conſiſts, as the phraſe is, of all the colours of the rainbow, all orderly ranged, as expreſſed in the figure. The dotted lines ſhew the confines of the colours, all the ſpace between them, oppoſite to the name, being allotted to ſuch a colour. But the colours are not totally ſeparated in any one part, but contiguous, and running into one another. For example, the ſpace allotted to red, contains innumerable degrees of red; the laſt of the red being not diſtinguiſhable from the firſt of the orange, *&c.*

281. The number of primary colours ſeems to be altogether arbitrary: Sir *Iſaac Newton* was pleaſed to diſtinguiſh them into ſeven, *viz.* *violet*, *indigo*, *blue*, *green*, *yellow*, *orange*, *red*, as expreſſed in the figure; out of theſe *white* and *black*, vulgarly called colours, are excluded: Whiteneſs ariſing from a mixture of light of all colours in equal proportion; and blackneſs from a reflection of a very ſmall quantity of a mixture of rays of all ſorts. The ſpace PT is divided (by the dotted lines ſeparating the colours) as a muſical chord, agreeable to Sir *Iſaac Newton*'s method: See his Optics, page 110. The circular ſpaces at the ends repreſent a little faint penumbra.

282. † Rays of the ſame colour, have always the ſame degree of refrangibility. And the colour exhibited by any ray ſeparated from the reſt, always continues invariably the ſame; however that ray may be refracted or reflected, ſo long as it is not blended with other coloured rays: For inſtance, a red ray always exhibits redneſs, and never any other colour; and a green ray always retains its greenneſs, *&c.* In like manner, all ſorts of rays mingled together, always exhibit whiteneſs, whether theſe rays are parallel, condenſed in a focus, diverging or converging, ſo long as they are not ſeparated aſunder. But if any of the coloured rays are wanting in the mixture, whiteneſs will not be produced; and the colour will be different, according to which rays are moſt predominant. And hence, all the varieties of colours of natural bodies, proceed from the different proportions in which they reflect ſome, and abſorb others of the different coloured rays of light. And when moſt of the rays are abſorbed, and but few reflected, ‖ blackneſs is

† See *Newton*'s Optics, Book I. Part II. Alſo Book II. and III.

‖ There ſeems to be ſomething in the conſtitution of black, beſide a mere deficiency of light; for bodies will retain their colours, in very different degrees of light. However black is not one of the priſmatic colours; and perhaps it may proceed from ſome kind of mixture of ſome peculiarly coloured rays, when others are totally abſorbed, or reflected a different way.

is exhibited. So that colours are not in bodies, for they will appear of any colour, if they be enlightened only with rays of ſuch a peculiar colour; but they are ideas excited by ſome difference in the impulſes upon our optics, of the differently refrangible rays *.

283. If the round hole in the window-ſhutter enlightened only by the clouds, be ſeen by rays refracted by a priſm, its image inſtead of appearing round will be oblong, and coloured like the ſun's image, as before deſcribed. And univerſally, the light reflected from all natural bodies, when refracted in the ſame manner, will have like effects. Alſo priſms made of water (by confining it between three glaſs plains cemented together,) or any other tranſparent medium hitherto known, will have like effects; only the length of the coloured image will differ, according to the different degrees of the refrangibility of the priſm.

284. Let RS be any refracting ſurface, ſuppoſe of glaſs, PO a perpendicular thereto at the point B, and let the line DB make an oblique angle with PB. If the ſolar rays be conceived to flow along DB, continually ſucceſſive to one another, ſo that they may one after another impinge on the point B, and there be refracted into † a denſe medium; or, if DB be ſuppoſed an indefinitely ſlender pencil of parallel rays, falling on the refracting ſurface at the points neareſt to B; theſe rays after being refracted, inſtead of keeping cloſe, as it were, in one line, as before, will diverge from one another into different lines B*p*, B*q*, B*r*, B*s*, B*t*, &c. and into innumerable others through the intermediate ſpaces, according as each ray is ſuited to ſuffer a greater or leſs refraction. The rays B*p* that are refracted the moſt produce purple colours, and thoſe B*t* refracted the leaſt produce red colours, and all the intermediate rays produce different colours ‖. Fig. 109.

285. *Schol.* If RS be a plane ſurface, and the medium be terminated by another plane ſurface TV parallel to the former; the rays after both refractions will emerge in lines [a] parallel to the incident ray DB.

[a] 67. I.

286. In like manner, if DB be ſuppoſed a very ſlender pencil of parallel rays, incident upon any refracting ſurface or lens BA*b*, at points infinitely near to B; theſe rays after refraction will interſect the axis AF of the lens, at different points F, *f*; that is, the moſt refrangible rays will have their focus at *f*, nearer to the lens; and the leaſt refrangible rays, will have their Fig. 110, *to* 114.

* See 277. I.

† The caſe is the ſame when the incidence is from a denſe into a rare medium, only the moſt refrangible rays will be fartheſt from the perpendicular.

‖ If the different ſorts of coloured rays follow one another in the ſame line, their velocity is ſo exceeding great, that no interruption between the different ſorts can be perceived. And if they come together like threads, they are ſo cloſely united and prodigious fine, that the ſmalleſt perceivable thread or pencil, or what we would call a ray of light, conſiſts of an indefinite number of different ſorts of coloured rays.

focus at a point F, more remote from the lens: and innumerable other rays will interſect the axis in different points, all lying orderly between F and *f*, according as thoſe rays differ in degrees of refrangibility.

287. The ſines of incidence and refraction of all ſorts of rays *, are in a conſtant ratio; that is, whatever is the ratio betwixt theſe ſines, at any one degree of incidence of any particular ray, there will be the ſame ratio between them, at all other degrees of incidence of the ſame ray; at leaſt, as exactly as can be diſtinguiſhed by experiments.

288. If the common ſine of incidence from glaſs to air is 50 †, the ſines of refraction of the leaſt and moſt refrangible rays will be 77 and 78 reſpectively. Whence in ſmall refractions (that is, ſuch that the angles and their ſines are nearly proportional) the refraction of the leaſt refrangible rays, is to the refraction of the moſt refrangible rays, as 27 to 28 nearly; or [a] $\angle EBF : \angle EBf :: 27 : 28$, nearly. And the difference of the refractions of the leaſt refrangible and moſt refrangible rays, is about $\frac{1}{2}\frac{1}{7}\frac{1}{2}$ or $\frac{1}{5}\frac{1}{5}$ of the whole refraction of the mean refrangible rays.

[a] Fig. 110.

289. By dividing the difference between 77, 78, the ſines of the leaſt and moſt refrangible rays, in the ſame manner as the *ſpectrum* PT, *fig.* 107. is divided by the dotted lines; Sir *Iſaac Newton* ‖ computes the ſines of refraction from glaſs to Air, of the ſeveral ſorts of rays contiguous to theſe dotted lines, to be $77, 77\frac{1}{8}, 77\frac{1}{5}, 77\frac{1}{3}, 77\frac{1}{2}, 77\frac{2}{3}, 77\frac{7}{9}$, 78; their common ſine of incidence being 50; that is, the ſines of incidences of all *red-making rays* out of glaſs into air, are to the ſines of their refractions from 50 to 77, and 50 to $77\frac{1}{8}$. The common ſine of incidence being 50, the ſines of refractions of the *orange-making rays* are from $77\frac{1}{8}$ to $77\frac{1}{5}$; thoſe of the *yellow-making* from $77\frac{1}{5}$ to $77\frac{1}{3}$; thoſe of the *green-making* from $77\frac{1}{3}$ to $77\frac{1}{2}$; of the *blue* from $77\frac{1}{2}$ to $77\frac{2}{3}$; of the *indigo* from $77\frac{2}{3}$ to $77\frac{7}{9}$; and thoſe of the *violet* from $77\frac{7}{9}$ to 78.

290. If the above numbers be made the ſines of incidences from air to glaſs, of the ſeveral ſorts of rays reſpectively; [a] the ſine of refraction of them all will be 50.

[a] 13. I.

PROP.

* *Newton*'s Optics, p. 64 to 70. & p. 82.
† *Newton*'s Optics, p. 73.
‖ Ibid. p. 111.

PROP. XLI.

291. *If heterogeneal rays or rays of all ſorts are incident upon the ſpherical ſurface of a rare medium, whether it be concave or convex*, or upon the plane ſide of a plano-convex lens, parallel to the axis and not † very remote from it;*

The focal diſtance from the refracting ſurface or lens of the leaſt refrangible rays, will be

To the focal diſtance of the moſt refrangible rays;

As the difference between the ſines of incidence and refraction of the moſt refrangible rays,

To the difference between the ſaid ſines of the leaſt refrangible rays.

Let O be the center of the ſpherical refracting ſurface BA, OF a line perpendicular to that ſurface, and alſo to the plane ſurface B*vb*; DB a line parallel to the axis OF, along which let the incident rays be ſuppoſed to flow; F the focus of the leaſt refrangible rays, and *f* the focus of the rays that are moſt refrangible. Let the greateſt of the ſines of incidence and refraction of the moſt refrangible rays, be called M; and the greateſt of the ſaid ſines of the leaſt refrangible rays, be called L; and the leaſt of the ſines of incidence and refraction of both ſorts of rays call I. I ſay AF : A*f* :: M—I : L—I. Fig. 110.

For L—I : I :: [a]OA : AF; and M—I : I :: [a]OA : A*f*. Therefore M—I : L—I :: AF : A*f*. *Q. E. D.* [a] 125, & 127. I.

292. *Corol.* 1. $M-L\ (\overline{M-I}-\overline{L-I})\frac{M}{L}-I :: fF\ (AF-Af)\frac{AF}{Af}$.

293. *Corol.* 2. If the refraction be between glaſs and air, it will be AF : A*f* :: 28 : 27.

For M—I : L—I :: [b]78—50 : 77—50 :: (28 : 27) AF : A*f*. Whence alſo the longitudinal aberration $fF = \frac{1}{27}AF = \frac{1}{27}Af$. [b] 288. I.

PROP.

* See *fig.* 41, 42.

† This limitation is neceſſary, to prevent the aberration from the figure affecting the truth of the propoſition.

PROP. XLII.

294. *If heterogeneal rays are incident upon a ſpherical ſurface, whether convex or concave*, of a denſe medium, parallel to the axis and not very remote from it; or upon the plane ſide of a plano-concave lens with like reſtrictions, it will be,*

As the diſtance of the focus of the leaſt refrangible rays from the center of the ſpherical refracting ſurface,

Is to the diſtance of the focus of the moſt refrangible rays from the ſaid center;

So is the difference between the ſines of incidence and refraction of the moſt refrangible rays,

To the difference between the ſaid ſines of the leaſt refrangible rays.

Fig. 111. That is, retaining the ſame references as in the preceding propoſition, OF : O*f* :: M—I : L—I.

[a] 125 & 126. I. For AO : OF :: [a]L—I : I; and AO : O*f* :: [a]M—I : I;
Therefore OF : O*f* :: M—I : L—I. *Q. E. D.*

295. *Corol.* 1. The refracting mediums and the radii AO, *fig.* 110, 111. being the ſame, the longitudinal aberrations *f*F of heterogeneal rays in both caſes will be the ſame.

[b] 126 & 127. I. For AF and A*f*, *fig.* 110. are [b] equal reſpectively to OF and O*f*, *fig.* 111.

Fig. 111. 296. *Corol.* 2. $OF : Of :: \frac{M}{L} AF : Af$. For,

[c] 126. I. M—I : M :: [c]AO : A*f*;
And AO : AF :: [c]L—I : L. Wherefore
M—I : L—I :: M × AF : L × A*f*. Therefore, *&c.*

297. *Corol.* 3. Becauſe the ratio of AF to A*f*, is leſs than the ratio of OF to O*f*; the ratio of *f*F, to AF or A*f*, *fig.* 110. is greater than the ratio of *f*F to AF or A*f*, *fig.* 111. And if the medium, *fig.* 111. is glaſs, it will be 28 : 27 :: 78 AF : 77 A*f* :: OF : O*f*; or AF : A*f* :: (28 × 77 : 27 × 78 ::) 2156 : 2106; whence *f*F is about $= \frac{1}{44}$ AF.

* See *fig.* 39, 40.

PROP. XLIII.

298. *If heterogeneal rays, are incident upon any lens whatſoever, whoſe thickneſs is very ſmall, parallel to the axis, and not very remote from it;*

The focal diſtance from the lens of the leaſt refrangible rays,

Will be to the focal diſtance of the moſt refrangible rays;

As the difference between the ſines of incidence and refraction of the moſt refrangible rays,

To the difference between the ſaid ſines of the leaſt refrangible rays.

Let the focal diſtance after refraction, of the parallel rays that are the leaſt refrangible, be called F; and of thoſe that are moſt refrangible, be called f; and let L, M, I ſignify the ſame as in the two preceding poſitions.

I ſay F : f :: M—I : L—I, very nearly.

CASE 1. Let P and p be the foci after refraction, of the leaſt and moſt refrangible rays, whoſe incidences were on the plane ſide of a plano-convex lens; then it will be accurately, Fig. 112.

M—I : L—I :: [a]AP : Ap. *Q. E. D.* [a] 291. I.

CASE 2. When the incidences of the parallel rays are upon the convex ſide of a plano-convex lens. Let F and f be the foci after refraction, of the leaſt and moſt refrangible rays reſpectively; and let s, be one of the focal centers of the lens; then will $sF = {}^{b}AP$, and $sf = {}^{b}$ Ap. Wherefore, as above, [b] 218. I.

M—I : L—I :: sF : sf. *Q. E. D.*

CASE 3. When both ſides of the lens are ſpherical. Retaining the ſame references as above; let moreover the radius of the flatteſt ſide (A) be called R; and the radius of the other ſide (a) call r; alſo let the thickneſs Aa of the lens be called t; and let s be that focal center of the lens which belongs to the refracted rays. Fig. 113, 114.

$$\text{Now } \frac{\frac{LIRr}{L-I} - IRt}{LR \pm Lr + Lt \pm It} + \frac{IRt}{LR \pm Lr} : sF ::$$

[c] 220. I.

$$\frac{\frac{MIRr}{M-I} - IRt}{MR \pm Mr \mp Mt \pm It} + \frac{IRt}{MR \pm Mr} : sf.$$

If t be conſidered as infinitely ſmall, the quantities multiplied by it will be ſo alſo, and therefore may be neglected; then multiplying the antecedents

antecedents by $R \pm r$, and by L and M, and dividing by IR, we ſhall have $\left(\frac{MLr}{L-I} : \frac{MLr}{M-I} ::\right) M-I : L-I :: sF : sf$. *Q. E. D.*

299. *Corol.* In every lens, whoſe thickneſs is very ſmall, it will be [d] 292. I. $M-L : \frac{M}{L}-I :: {}^{d}fF : \frac{sF}{sf}$; nearly. And therefore in a glaſs lens, the longitudinal aberration $fF = \frac{1}{28} sF = \frac{1}{27} sf$, nearly.

Fig. 112. 300. *Schol.* 1. A plano-convex lens with its convex ſide expoſed to parallel heterogeneal rays; the thicker is the lens, the leſs will be the aberration fF: but the difference on account of the thickneſs is very inconſiderable. Let s be the focal center of the lens for the leaſt refrangible rays, and t the focal center for the moſt refrangible rays. Now as the thickneſs of the lens increaſes, the points s, t ſeparate; vt being always leſs than vs; for it will be [e] 209. I. $\frac{L}{M} : I :: {}^{e}v A : \frac{vs}{vt}$. But sF, tf are [f] 175, & 218. I. invariable quantities; and therefore as the point t recedes from s, the focus f approaches nearer to F; that is, fF will be ſo much diminiſhed, as st is increaſed.

The variation of fF on account of the different thickneſſes of lens's having both their ſides ſpherical, is alſo very inconſiderable.

301. *Schol.* 2. In computing the aberration above mentioned, the incident rays were ſuppoſed to be the neareſt of all to the axis; and in that caſe, all lens's whatſoever whoſe thickneſſes are very ſmall, are equally good. But when the incident rays are remote from the axis, there will be an aberration by the figure; and that aberration will increaſe the aberration of heterogeneal rays. Therefore that lens is beſt which has leaſt aberration of homogeneal rays.

302. *Schol.* 3. What has been above demonſtrated of convex lenſes, is alſo true of concave ones.

PROP. XLIV.

303. *In the ſame lens, the longitudinal aberration* F f *of parallel hete-rogeneal rays is invariable: And the lateral aberration* F G *increaſes or decreaſes in the ſimple ratio of the aperture of the lens; and is to the ſemi-aperture of the lens, as* F f *to* f v. Fig. 110, to 114.

Let fall the perpendicular B *v*.

1. F *f* is invariable, becauſe it has always the [a] ſame ratio to the invariable quantities AF or A *f*. [a] 291, 294, & 298. I.

2. The right-angled △s *f v* B, *f* F G are ſimilar, and therefore, FG : *v*B :: *f*F : *f v*. Therefore, *&c. Q. E. D.*

304. *Corol.* 1. In any glaſs lens whoſe thickneſs is very ſmall, FG = $\frac{1}{27}$ *v* B, nearly.

For M — L : L — I :: [b] *f*F : (*f s* or) *f v* :: FG : *v*B :: 1 : 27. [b] 299. I.

305. *Corol.* 2. In the ſame lens, the angle of aberration of parallel heterogeneal rays, is as the aperture of the lens nearly. For that angle is as FG nearly; and FG is to *v*B in the conſtant ratio of F *f* to *f v*.

PROP. XLV.

306. *In different lenſes that are alike figured, their greateſt angles of aberrations will be equal, if their linear apertures be as their focal lengths, and the focus of incident rays be alſo at like diſtances from the lenſes.*

1. Let DB, *db* be ſlender pencils of heterogeneal rays parallel to the axes AF, *af* of the lenſes A, *a*; and let AB, *ab* be the ſemi-apertures of the ſaid lenſes. And neglecting the aberrations from the figures of the lenſes, let F, *f* and K, *k* be the focus's of the leaſt and moſt refrangible rays reſpectively. Now if it be, AB : *ab* :: AF : *af* :: AK : *ak*; the △s FAB, *fab*, and KAB, *kab* will be [a] ſimilar; and therefore the ∠s FBK, *fbk* will be equal. *Q. E. D.* Fig. 116, 117. [a] 6. VI. *Euc.*

2. Let *r*, *t*, and *s*, *v* be the proper focus's of the leaſt and moſt refrangible rays reſpectively. Now if the lenſes are alike figured, the aberrations F *r*, *ft*, and K *s*, *kv* will be in the [b] ſame given ratio to their thickneſſes. And when the apertures of the lenſes are as their focal lengths, their thickneſſes are alſo in the ſame proportion; therefore AB : *ab* :: F *r* : *ft* :: K *s* : *kv*; and conſequently the △s A *r*B, *atb*, and A *s*B, *avb* being ſimilar, the ∠s *r*B *s*, *tbv* are equal. *Q. E. D.* [b] 60, 136, & 264. I.

307. *Schol.* 1. If rays of all ſorts are ſuppoſed to flow along the lines FB, *fb* or *r*B, *tb*; the leaſt refrangible rays will emerge along the lines BD, *bd* parallel to the axis; and the moſt refrangible rays

will emerge along the lines BE, *be*, ſo as to make the angles of aberrations DBE, *dbe*, equal to the angles FBK, *fbk*, or *r*BS, *tbv*, reſpectively; and conſequently the ſaid angles DBE, *dbe*, will be equal one to the other.

308. *Schol.* 2. In different lenſes having the ſame apertures, if theſe apertures are but moderate, ſo that the aberrations from the figures of the lenſes may be neglected; the greateſt angles of aberrations of heterogeneal rays by theſe different lenſes, will be nearly in the inverſe ratio of their focal lengths.

Fig. 114, 116. For AB : *radius* :: A*f* : *co-tang.* ∠A*f*B :: AK : *co-t.* ∠AKB. And in the different figures, the co-tangents of the ∠ˢ AFB will be as AF in each; and theſe are to each other, as A*f*, AK. But the tangents of ſmall angles are nearly as thoſe angles; and therefore

∠FB*f* : ∠FBK :: AK : A*f* nearly. *Q. E. D.*

309. *Schol.* 3. In different lenſes having moderate apertures; the angles of aberration FBG are nearly, as the ſemi-apertures AB directly, and focal lengths AF inverſely. For theſe angles may be conſidered as being as their chords directly, and as FB inverſely; and theſe chords may be reckoned as FG, and FB as FA. But in the ſame lens, FG is always as AB. Therefore, *&c. Q. E. D.*

310. *Corol.* The angles of aberrations by all lenſes that are alike figured, and having the focus's of the incident and emergent rays alike ſituated; will be the ſame, if the apertures of the lenſes be as their focal lengths.

PROP. XLVI.

Fig. 110, to 114. 311. *The diameter of the leaſt circle, through which all the heterogeneal rays will paſs, is a line* gh, *drawn between the points of interſection of the moſt and leaſt refrangible rays, proceeding from the oppoſite ſides of the lens: And* gh : FG :: Bh : BF.

The firſt part of the propoſition is evident by bare inſpection of the figures: and it is alſo manifeſt, that *gh* is parallel to GH; whence it follows, *gh* : FG :: [a] B*h* : BF. *Q. E. D.*

[a] 4. VI. *Euc.*

312. *Corol.* 1. $gh : FG :: \frac{vf+vF}{2} : vF :: vf : \frac{vf+vF}{2}$, nearly.

Alſo $gh : Bv :: fF : \frac{vf+vF}{2}$, nearly. Hence, when the verſed ſine A*v* is very ſmall, in compariſon of the focal diſtance from a ſpherical refracting ſurface or lens, it will be nearly;

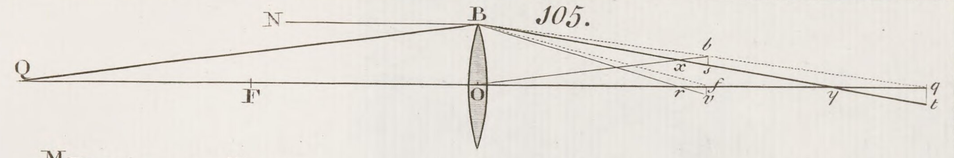
[b] 292, & 299. I. 313. *Corol.* 2. $gh : Bv :: (fF : \frac{vF+vf}{2} ::$ [b]$)\ M-L : \frac{M+L-2I}{2}$.

And therefore if the refracting ſurface or lens, is glaſs; it will be,

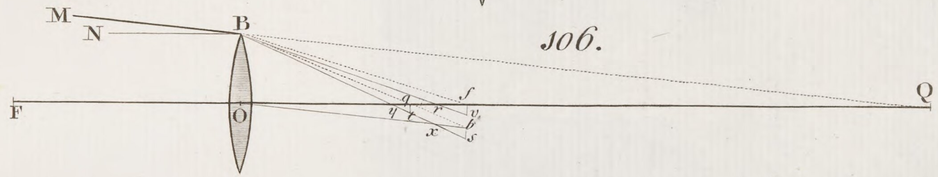
gh : B*b* :: 1 : 55 (:: 78 — 77 : 78 + 77 — 100).

314.

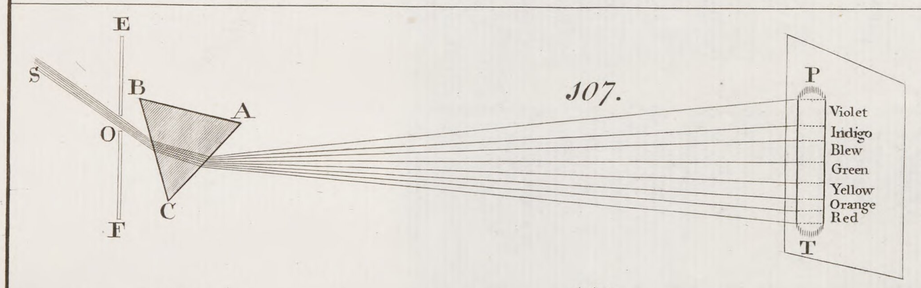
105.



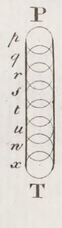
106.



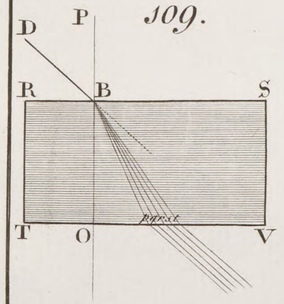
107.



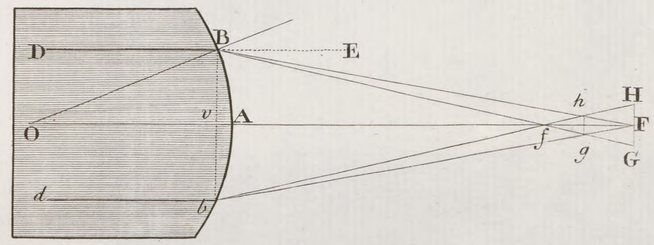
108.



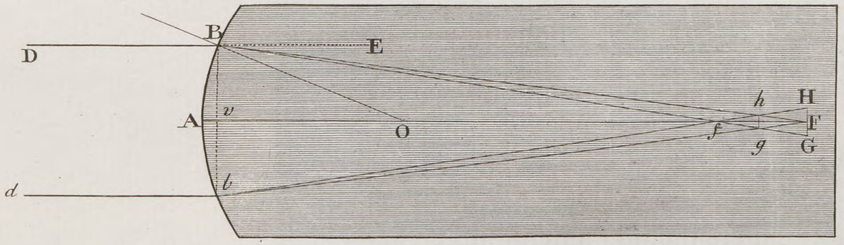
109.

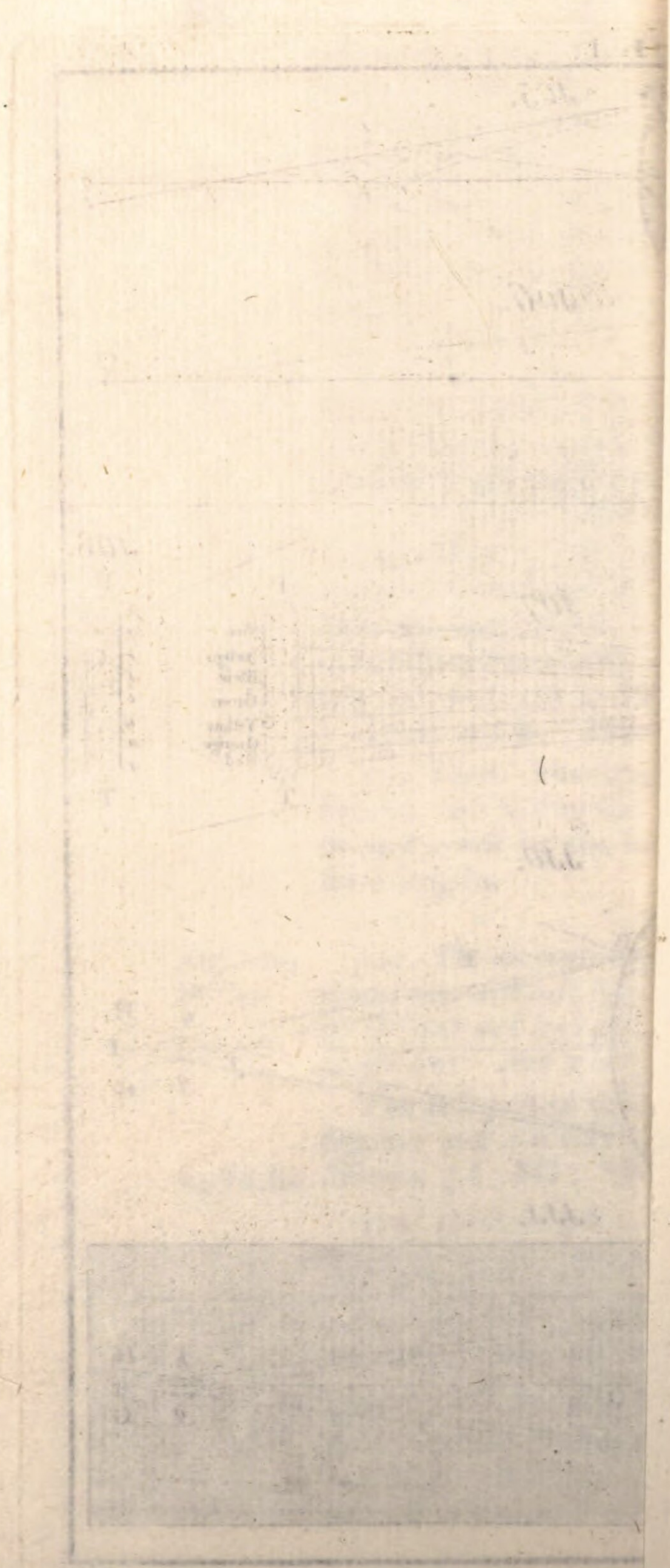


110.



111.





314. *Schol.* The diameter of the leaſt circle that can receive all the rays of any two colours, is after the ſame manner determinable from the proportion of their ſines. Thus all the *orange* and *yellow* will be contained in a circle whoſe diameter is about the $\frac{1}{200}$ part of the aperture of a glaſs lens, when that aperture is but ſmall in proportion to the focal length; the ſines of the outermoſt orange and yellow, being to the common ſine of incidence as [c] $77\frac{1}{8}$ and $77\frac{1}{3}$ to 50; and putting L, M and I for theſe ſines, it will be, [c] 289. I.

M—L : M+L—21 :: 1856—1851 : 3707—2400 :: 1 : 261.

PROP. XLVII.

315. *The rays contained in the circle of aberration whoſe diameter is* gh, *are not uniformly ſpread over that circle; but are denſer at the center than in any other part, and from thence towards the circumference they grow continually rarer. And the denſity of the light in any point of this circle, is directly as the diſtance of that point from the circumference, and reciprocally as its diſtance from the center, nearly.*

For, ſuppoſing the heterogeneal rays that are refracted at the points B, *b* in the periphery of the lens, to be ſpread uniformly over the phyſical line *g h*; the rays tranſmitted through every other point in that periphery, will be alſo ſpread uniformly over lines all equal to *g h*, and interſecting one another at the center *c*; ſo that the whole light tranſmitted as above, will be ſpread over a circle whoſe diameter is *g h*. And upon the above ſuppoſition, the denſity of the light in any point of this circle, will be reciprocally as the diſtance from the center; that is, the denſity in any point *a*, will be to the denſity in any point *b*, as *c b* to *c a*. Fig. 110, to 114. Fig. 115.

If the lens or medium be ſuppoſed to be divided into an indefinite number of ſmall concentric rings, all equal in breadth; the quantity of light tranſmitted through each ring, will be as the rings themſelves, that is, as their diameters; and this light will be ſpread over circles of aberrations, whoſe diameters [a] are alſo as thoſe of the reſpective rings. Now in each circle of aberration conſidered apart from the reſt, the denſity of the light will be reciprocally as the diſtances from the center; and the whole quantity of light in each circle being as its diameter; the denſities in each circle at equal diſtances from the common center are equal. For, in the circle *h g c*; [a] 303. I.

denſity in *a* : *denſity* in *b* :: *c b* : *c a*; and in the circle *b d c*,
denſity in *b* : *denſity* in *a* :: *c a* : *c b*. Wherefore,
denſity in *b* : *denſity* in *b* :: *c b* : *c b*. Inſtead of the denſities in *b* and *b*, let us ſubſtitute their ratio's, and it will be in the circle $\frac{hgc}{bdc}$;

denſity in $a : \frac{cb}{cb} :: \frac{cb}{cb} : \frac{ca}{ca}$. Whence it is manifeſt that the denſity at a in each circle is the ſame. And therefore the whole denſity in any point of the circle of aberration, is directly as the diſtance of that point from the periphery, and reciprocally as its diſtance from the center, nearly; that is, the denſity of the light in any point a or b, is as $\frac{ba}{ca}$ or $\frac{bb}{cb}$, nearly. *Q. E. D.*

316. *Corol.* 1. Let the ſemi-diameter cb be biſected in n; and the denſity of the light in any point b: will be to its denſity in $n :: bb : bc$.

For by the preceding,

denſity in b : *denſity* in $n :: \frac{bb}{cb} : \left(\frac{bn}{cn}\right) 1 :: bb : cb$.

317. *Corol.* 2. Hence the light at the center is infinitely denſe, and at the circumference infinitely rare, in compariſon of the light at the point n.

318. *Schol.* 1. The quantity of light in the greateſt circle bgc: is to the quantity of light in any leſſer circle $bdc :: bc^2 : bc^2 - bb^2$.

Let the ring $bbdg$ be divided into any number of indefinitely ſmall concentric rings, all equal in breadth; and let the radii of theſe rings
[b] 315. I. be co, cp, cq, cr, &c. Now the denſity of the light in each ring [b] will be as $\frac{bo}{co}$, $\frac{bp}{cp}$, $\frac{bq}{cq}$, $\frac{br}{cr}$, &c. and the rings are as co, cp, cq, cr, &c. Wherefore multiplying each ring by its reſpective denſity, we ſhall have the quantity of light in each, as bo, bp, bq, br, &c. and the ſum of all theſe is $= bb \times \frac{1}{2} bb = \frac{1}{2} bb^2$. If the whole circle be divided in like manner into theſe ſmall rings, till we come to the center; the ſum of the light in them all, will be as $bc \times \frac{1}{2} bc = \frac{1}{2} bc^2$. And therefore, the quantity of light in the circle bgc: is to the quantity in the circle $bdc :: \frac{1}{2} bc^2 : \frac{1}{2} bc^2 - \frac{1}{2} bb^2 :: bc^2 : bc^2 - bb^2$. *Q. E. D.*

319. *Schol.* 2. Sir *Iſaac Newton* * obſerves, that the *yellow* and *orange* being the moſt luminous of the priſmatic colours, affect the ſenſe more ſtrongly than all the reſt together; and next to theſe in ſtrength are the *red* and *green*. The *blue* compared to theſe is a faint and weak colour; and the *indico* and *violet* are ſtill darker and fainter; ſo that theſe, compared to the ſtronger colours, are little to be regarded. Whence, and from the greater denſity of the rays of light near the center of a circle of aberrations, Sir *Iſaac* infers, that the ſenſible image of a lucid point in the focus of a glaſs lens having but a ſmall aperture, is not much broader than $\frac{1}{250}$ of the diameter of that aperture.

320. *Schol.* 3. The images of objects are therefore to be placed or reckoned, not in the focus of the mean refrangible rays, which are in the

* *Newton*'s Optics, p. 85, 86.

the confine of green and blue; but in the focus of thoſe rays which are in the middle of the orange and yellow. The ſines of incidence and refraction of theſe rays between glaſs and air, are as 17 and 11: And from theſe proportions therefore, the focal lengths of glaſſes for optical uſes, are to be computed †.

PROP. XLVIII.

321. *In any lens retaining the ſame aperture, and neglecting the aberration from its figure:*

The longitudinal aberration of heterogeneal rays, diverging from, or converging to, a given focus in the axis of the lens,

Is to the longitudinal aberration of parallel heterogeneal rays by the ſaid lens;

As the diſtance between the focus of incidence, and the focus of the moſt or leaſt refrangible rays after refraction by the lens,

Is to the diſtance between the focus of incidence, and the principal focus of the leaſt or moſt refrangible rays.

Let Y be the focus of the incident heterogeneal rays; Q, q the focus's of the leaſt and moſt refrangible rays, after refraction by the lens O; and F, f the principal focus's of theſe rays reſpectively. Fig. 118, to 123.

I ſay $Qq : Ff :: Yq : YF :: YQ : Yf$.

For $YF : YO ::$ [b] $YO : YQ$; and $Yf : YO ::$ [b] $YO : Yq$; wherefore [b] 227. I.

$YF : Yf :: Yq : YQ$; and therefore,

$$Qq\,(YQ \backsim Yq) : \frac{Yq}{YQ} :: Ff\,(Yf \backsim YF)\,\frac{YF}{Yf}.$$ Q. E. D.

322. *Corol.* 1. $YF : Yf :: Fq : fQ :: Yq : YQ$.

323. *Corol.* 2. $OQ : Oq :: Yf \times OF : YF \times Of$.

For, $YO : OQ ::$ [c] $YF : OF$; [c] 227. I.

And $Oq : YO ::$ [c] $Of : Yf$. Therefore, &c. Q. E. D.

324. *Corol.* 3. $Qq : Ff :: \frac{Yq + YQ}{2} : \frac{YF + Yf}{2}$.

Let the point which biſects Ff be called a, and that which biſects Qq be called b; and then it will be, $Qq : Ff :: Yb : Ya$. And becauſe Ff is an invariable quantity, Qq increaſes or diminiſhes as $\frac{Yb}{Ya}$.

325. *Corol.* 4. If the lens is glaſs, it will be,

$Qq : \frac{1}{17}Of ::$ [d] $Yq : YF :: YQ : Yf$. [d] 299. I.

Alſo, $27YF : 28Yf ::$ [d] $Oq : OQ$.

PROP.

† *Newton*'s Optics, p. 86.

PROP. XLIX.

326. *In any lens retaining the ſame aperture, and neglecting the aberration from its figure:*

The lateral aberration of heterogeneal rays, diverging from, or converging to, a given focus in the axis of that lens,

Is to the lateral aberration of parallel heterogeneal rays by the ſaid lens;

As the rectangle under the longitudinal aberration of the rays belonging to that focus, and the diſtance from the lens of the principal focus of the moſt refrangible rays,

To the rectangle under the longitudinal aberration of parallel heterogeneal rays, and the diſtance from the lens of the focus of the moſt refrangible rays.

Fig. 118, 119. Let OB be the ſemi-aperture of the lens O, QS the lateral aberration for that aperture of heterogeneal rays, whoſe focus of incidence is Y; and FG the lateral aberration of parallel heterogeneal rays for the ſame aperture;

I ſay, $QS : FG :: Qq \times Of : Ff \times Oq$.

For the right-angled triangles fFG, fOB; and qQS, qOB being ſimilar; it will be,

$Ff : Of :: FG : OB$; and $qO : qQ :: OB : QS$. Wherefore

$QS : FG :: Qq \times Of : Ff \times Oq$. *Q. E. D.*

The ſame will be alſo true in the four ſucceeding caſes or figures, if the reſpective lines BF, BfG, FG, BqS, QS be drawn, changing the places of Q, q in fig. 120, 122.

327. *Schol.* 1. In any lens and with any aperture; $QS = OB \times \frac{Qq}{Oq}$.

Fig. 118, 119. 328. *Schol.* 2. In different lenſes, ſimilarly formed, having their apertures as their focal lengths, and forming equal images Qp behind them of a given object YP; the lateral aberration QS will be as the focal lengths of the lenſes. For in that caſe the angles of aberrations QBS in each are equal, and in each OQ is as OB or OF.

329. *Corol.* Hence, if a given object is to have its image upon a ſcreen magnified in a given ratio; the deeper is the lens, the more diſtinct will be the image. For in this caſe, the diſtinctneſs of the image is chiefly affected by the aberration QS. And if the diſtance OQ of the ſcreen is fixed; the diſtinctneſs of the image will not be ſenſibly different by uſing lenſes of different focal lengths, if it be but ſufficiently bright; provided always that the figures of the lenſes be ſimilar, and their apertures as their focal lengths.

PROP.

PROP. L.

330. *The circle of aberrations cauſed by the ſphericalneſs of the figure of an optic lens having a moderate aperture, compared with the circle of aberrations cauſed by the unequal refrangibility of the rays of light, is very inconſiderable.*

For a compariſon betwixt theſe two ſorts of aberrations, let us take Fig. 99. a plano-convex glaſs with its plane ſide expoſed to parallel rays; this lens being the worſt for homogeneal rays of any in the table, *Art.* 253. Let F be the principal focus of the lens, Fp the lateral aberration of homogeneal rays, for the ſemi-aperture Bv; and FG the lateral aberration of heterogeneal rays for the ſame aperture. And putting m, n for the greater and leſſer of the ſines of incidence and refraction, it will be, $Fp : FG :: \frac{Bv^3}{2RA^2} \times \frac{mm}{nn} : \frac{2}{55} Bv$.

For in any plano-convex lens having a ſmall aperture with its plane ſide to the parallel incident rays, it will be $OF =$ [a] $vA \times \frac{mm}{\overline{m-n} \times n}$ nearly. [a] 136. I.

And $vA =$ [b] $\frac{Bv^2}{2RA}$ nearly. Alſo $Fp : OF ::$ [c] $Bv : vO = AF$ nearly. [b] 13. VI. *E.*

But $AF =$ [d] $\frac{n}{m-n} RA$. Therefore $Fp = \left(\frac{Bv^2}{2RA} \times \frac{mm}{m-n \times n} \times \frac{Bv}{RA \times n} =\right) \frac{Bv^3}{2RA^2} \times \frac{mm}{nn}$. [c] 4. VI. *Eu.*

And $FG =$ [e] $\frac{2}{55} Bv$. Therefore, *&c.* [d] 175. I. [e] 313. I.

The diameter of the leaſt circle that will contain all the heterogeneal rays is = [f] FG nearly; and the diameter of that leaſt circle which will contain all the homogeneal rays is = [g] $\frac{1}{2} Fp$. If we compare theſe leſſer circles of aberrations together, their reſpective diameters will be, as $\frac{2}{55} Bv$, to $\frac{Bv^3}{4RA^2} \times \frac{mm}{nn}$. And by theſe numbers, the diameters of the reſpective circles of aberrations for any given aperture of a plano-convex glaſs, may be eaſily found. [f] 311. I. [g] 268. I.

If we take $\frac{1}{2} FG$ in the table, *Art.* 253. for the diameter of the leaſt circle of aberrations of homogeneal rays; this will be for the ſeveral apertures there ſpecified, beginning with the largeſt and ſo downwards, the $\frac{1}{13}$, $\frac{1}{54}$, $\frac{1}{213}$, $\frac{1}{906}$ of the diameters of the leaſt circles of the aberrations of heterogeneal rays, for theſe reſpective apertures, nearly. But in many caſes, the ſmalleſt even of theſe apertures is much too large; and the aberration from the figure decreaſing as [h] the cubes of the apertures, and that from the heterogenity of the rays only in the [i] ſimple ratio of the apertures; the fractions expreſſing the ratio's of theſe reſpective aberrations, will become much ſmaller as the apertures are leſſened. [h] 264. I. [i] 303. I.

If

If the diameter of the leaſt circle of aberration of heterogeneal rays, that may affect the ſenſe, be admitted to be ſo ſmall as the $\frac{1}{250}$ part of the aperture of a lens; Sir *Iſaac Newton* * computes that in a 100 foot teleſcope with a proper aperture, this aberration would be at leaſt 1200 times the aberration cauſed by the ſphericalneſs of the figure of the object-glaſs.

† It is farther obſervable, that the rules deduced from experience, for limiting the apertures and charges of teleſcopes, agree exactly with the theory of the different refrangibility of the rays of light, without at all regarding the figures of the object-glaſſes. Therefore, *&c.* *Q. E. D.*

331. *Schol.* 1. The leaſt circle of aberration from the imperfect figure of a plano-convex glaſs, with its convex ſide expoſed to parallel rays, is equal to the leaſt circle of aberration of heterogeneal rays, if the figure of the lens was perfect, when the aperture of the lens is half its focal length, nearly. That is, $\frac{1}{4}Fp = FG$ nearly, when $vB = \frac{1}{4}AF$. Fig. 99.

For $OF = {}^{*}\frac{7}{6}vA = \frac{7}{6}\frac{vB^2}{AF}$ nearly; and $Fp = \frac{Bv \times FO}{vO}$. * 256. I.

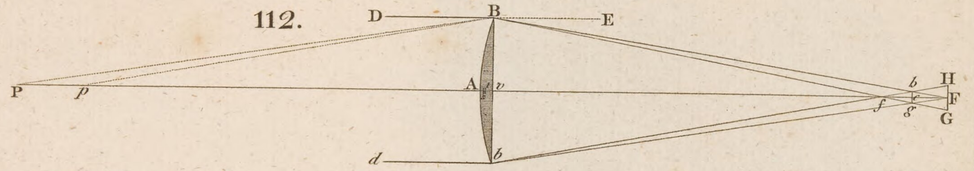
But $vO = AF$ nearly; and therefore $\frac{1}{4}Fp = \frac{7}{12} \times \frac{vB^3}{AF^2}$.

Now $FG = \frac{1}{27\frac{1}{2}}vB$, and when $vB = \frac{1}{4}AF$, $\frac{1}{4}Fp = \frac{7}{12} \times \frac{1}{16}vB = \frac{1}{27\frac{3}{7}}vB$. Therefore, *&c.* *Q. E. D.*

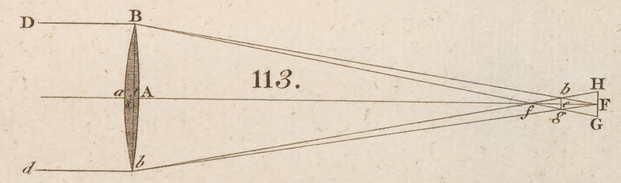
332. *Schol.* 2. The aberrations of coloured rays, being ſo very conſiderable in proportion to the aberrations ariſing from the ſpherical figures of lenſes; there is no ſenſible advantage to be expected from lenſes of other figures; nor yet from a medium of different refractive power from glaſs, unleſs that medium ſhould be more homogeneal than glaſs. For different refrangibility being a property of light itſelf, the aberrations proceeding from thence muſt alike take place, whatever be the medium it is refracted by; unleſs a medium ſhould be found that would tranſmit only the brighteſt of the priſmatic colours. And all the conſequence of having a medium of greater or leſs refractive power than glaſs, would be; that for the ſame focal lengths, lenſes made of ſuch a medium would be more or leſs flat, than thoſe we have at preſent ‖.

* *Newton*'s Optics, p. 84, 89. † Ibid. p. 89. ‖ *Note*, In the annexed plate, the aberrations Ff, Qq, are each four times too large in proportion to the focal diſtances Of, Oq.

112.



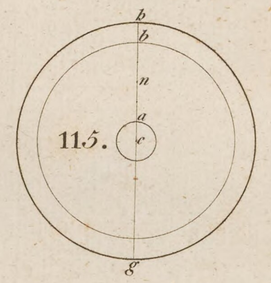
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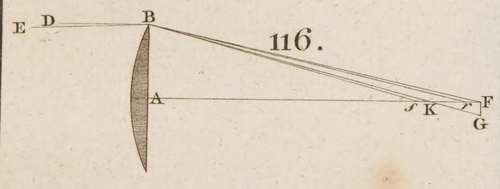
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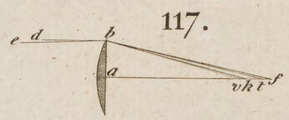
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116.



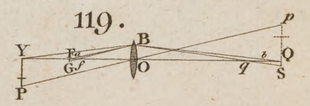
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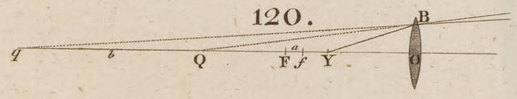
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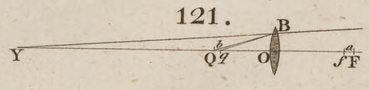
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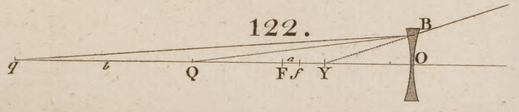
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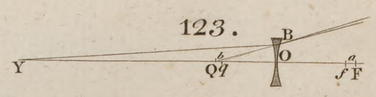
121.



122.



123.



A TREATISE OF OPTICS.

BOOK II.

Of VISION.

HAVING now gone through the general * theory of reflection and refraction of light, we are in the next place to enquire into the theory of viſion. This enquiry in itſelf is not only curious and entertaining, but without it there is no accounting for ſeveral optical phænomena; and it is alſo very neceſſary that it ſhould precede the particular theories of optical inſtruments, if we would rightly underſtand their effects in extending, as they do, ſo prodigiouſly, the natural boundaries of viſion. I have therefore beſtowed a diſtinct book upon this head, the two firſt ſections whereof are upon viſion in general; in which I have endeavoured to digeſt and ſet in the beſt light I could, what I have been able to gather upon the ſubject; the two remaining ſections are for explaining the phænomena of viſion by the reflection of poliſhed ſpeculums, and refraction of a ſingle lens.

SECTION I.

Of the eye, and manner of viſion.

§. 1. *A brief deſcription of the eye.*

1. THE eye conſiſts of ſeveral *coats*, or teguments, which form a *ball* perfectly globular, excepting on the fore part, which is a little more protuberant than the reſt. Within this ball are included three different

* This theory has no neceſſary connection with viſion, and may be perfectly underſtood by a blind man: And although a perſon who does not remember to have ſeen objects, cannot have the ſame ideas about colours that others have; yet he may perfectly comprehend the theory of the different refrangibility of light, and argue from thence as juſtly, as if he had ſeen the coloured ſpectrum formed by a priſm. But there is no underſtanding the theory of viſion, without a competent ſkill in that of light; otherwiſe it would have been perhaps more natural, to have made the deſcription of the eye, &c. the entrance into the ſcience of optics.

different ſorts of liquids, or ſoft tranſparent ſubſtances, called *humours*.
Fig. 1. The figure repreſents a ſection of the eye through the middle, by an horizontal plane, or a plane paſſing through both eyes; and for the ſake of repreſenting the parts more diſtinctly, the diameter of the figure is about twice the diameter of the human eye *.

Sclerotica and cornea. 2. The outermoſt coat, repreſented by the ſpace contained between the two outermoſt circles BFB, is called *ſclerotica*. Adjoining to the ſclerotica at the points B, B is a more globular coat, repreſented by the ſpace between the two circles BAB, called *cornea*. Theſe two coats make together one firm caſe of a proper form, for containing the reſt of the coats and humours. The cornea is tranſparent (like horn, whence it has its name) for the admiſſion of light into the eye.

Choroeides and Tunica Ruyſchiana. 3. The next coat under the ſclerotica, repreſented by the two innermoſt circles BFB, is a membrane of leſs firmneſs, called *choroeides*. This membrane is by M. *Ruyſch*, and ſince by other anatomiſts, divided into two; the outermoſt of which retains its old name, and the innermoſt is called †*tunica ruyſchiana*. Both theſe coats are often conſidered as one, under the old name choroeides; the back part of which in the human eye is of a ruſty dark colour, occaſioned by a black *mucus* wherewith it is covered ‖.

Uvea and iris. 4. Adjoining to the choroeides at BB, is ‡ a flat membrane called by moſt authors the *uvea*, and by ſome the *iris*; but generally by the *iris* is meant only the fore and circular coloured part of the

* It is not to be expected, that the account we are to give, ſhould be altogether perfect. For as it is impoſſible to examine the internal parts of the eye, whilſt it is in its natural and vivid ſtate; ſo is it difficult after the eye is taken out of its ſocket, to preſerve the figures of its ſeveral parts entire: and this is an excuſe for ſome diſagreements we find among *anatomiſts*. However, the accounts we have, are ſufficiently accurate for our purpoſe.

† This coat is deſcribed as leaving the choroeides at B, B, and as paſſing behind the proceſſus ciliares, and round the edge and back part of the cryſtalline humour.

‖ Anatomiſts obſerve, that in brutes of prey the middle part of this membrane is perfectly white; and in brutes that feed on graſs, the ſame part is of a bright green. The whiteneſs in the one is ſuppoſed to enable them to diſcover their prey better in the night; and the greenneſs in the other, is alſo ſuppoſed to be a help to them to find their food: whilſt men have the advantage of the dark colour ſuffocating the rays, to prevent an indiſtinctneſs of viſion by their reflection back. But theſe Hypotheſes for explaining the Uſes of the different colours of the choroeides in the eyes of different animals, are not ſo ſatisfactory as at firſt they may ſeem to be. For if viſion is owing, as I think it is now univerſally acknowledged, to the impulſe of light upon the retina (or to its impulſe upon the choroeides, as ſome have imagined) there does not ſeem to be any neceſſary connection betwixt viſion, and the colours of either the choroeides or retina; unleſs it be ſuppoſed, that according to their colours, they are more ſenſible of the impulſes of the rays of light of theſe colours, than of any other: But ſuch an hypotheſis ſeems very precarious. Perhaps the eyes of ſome animals tranſmit more of ſome kind of the coloured rays, than of others; and that this may be the cauſe of the diverſities of colours above obſerved: And admitting this, it will appear very probable, that theſe different conſtitutions of the eyes of different animals, are intended for the purpoſe above aſſigned.

‡ This membrane was formerly ſuppoſed to be ſpherical, and of much the ſame curvity with the choroeides; but Monſ. *Petit*, a ſkilful anatomiſt at *Paris* in the *Mem. de l'Acad. Royale*, for the Year 1728, ſhews that in the human eye, the uvea is flat. In delineating the eye, I have followed this gentleman's meaſures, as he ſeems to have taken more than ordinary pains in the examination of its ſtructure.

the ſaid membrane. This membrane is opaque, and of different colours in different eyes, as *grey*, *black*, *hazel*, &c. and being ſeated behind the *cornea*, its colour is ſeen through it; whence the eye is accordingly denominated *grey* or *black*, &c. The uvea ariſes from a circular ridge or protuberance running all along the inſide of the cornea, at its juncture with the ſclerotica at BB.

5. In the uvea, a little nearer the noſe than the middle, is †a round hole *aa*, for the admiſſion of light into the eye, called the *pupil*. ‖The pupil may be enlarged or contracted for the admitting of more or leſs light, as the circumſtances of viſion require: for which purpoſe the uvea is furniſhed with ſtrait muſcular fibres, proceeding from its circumference towards the center, and terminated in the orbicular limb of the pupil; which limb is alſo ſuppoſed to be muſcular **, that ſo by the contraction and dilatation of theſe ſtrait and annular fibres, like the cloſing and opening of a purſe, the pupil may be dilated and contracted. Pupil.

6. At the back part of each eye, a little nearer to the noſe than the middle, there enters a nerve as V, called the *optic nerve* ‡. Optic nerve.

7. The fibres of the optic nerve, after their entrance into the eye at V, ſpread themſelves like a fine web or net over the choroeides or *Ruyſchiana*; and ſo form a very thin membrane, called *Tunica Retina*, repreſented in the figure by the etched circle BFB. The retina is reckoned to extend all round as far as BB; and is of a *darkiſh colour, occaſioned by its immerſion in the [a] black *mucus* adhering to the *tunica ruyſchiana*. But the optic nerve is white like the brain, from whence it is propagated. Retina. [a] 3. II.

† *Anatomiſts* obſerve, that in animals which are the prey both of birds and beaſts, the pupil is round as in man, which fits them to ſee every way alike. But in large animals which feed on graſs, the pupil is oblong horizontally, which ſuits them for ſeeing a large circular ſpace upon the earth. And in animals of the cat-kind, which climb trees, the pupil is oblong the contrary way, which fits them beſt for looking both upwards and downwards: And it is farther remarkable, that cats in the dark have their pupils prodigious large, which enables them to ſee their prey with a very little light.

‖ The latitude of contraction and dilatation of the pupil is pretty conſiderable; and it is very admirable, that while the pupil changes its magnitude it preſerves its figure.

** It is ſuppoſed neceſſary that the uvea ſhould be furniſhed with theſe two ſorts of fibres. Anatomiſts are agreed as to the ſtrait ones; and it is allowed that there is a muſcular ring round the pupil, into which the ſtrait fibres are inſerted: but the exiſtence of this ring is rather inferred from the reaſon of the thing, than proved from experiments. *Albini* thinks, that the cryſtalline by preſſing againſt the uvea, changes its figure and widens the pupil.

‡ The ſclerotica and the choroeides enter the orbit of the eye along with the optic nerve: the origin of the ſclerotica is the *dura mater*, a thick tough membrane lying immediately under the *ſkull*; and the choroeides is derived from the *pia mater*, which is a fine thin membrane adhering cloſely to the brain. Through the middle of the optic nerve, there runs a pretty conſpicuous artery, which goes ſtrait through the vitreous humour, and ſpreads itſelf upon the membrane that covers the back-ſide of the cryſtalline.

* This membrane is ſaid to be quite black in infants; not ſo black at the age of twenty; of a greyiſh colour about the thirtieth year; and in very old age almoſt white.

Cryſtalline humour.

8. Within the cavity of the eye, a little behind the pupil, is a tranſparent ſubſtance EE, for the moſt part of about the conſiſtency of hard jelly, which it nearly reſembles; but from the middle outwardly, it grows gradually ſofter. This is called the *cryſtalline humour*; it is contained in a thin membraneous and tranſparent capſula †. The figure of the cryſtalline reſembles that of a double convex lens of unequal convexities, the flatteſt ſide being that next the pupil. The circular edge EE is flattiſh, or as it were rounded off, as repreſented in the figure. The cryſtalline till about twenty or thirty years of age is ſaid to be colourleſs; after which time it begins to be ſlightly tinged with yellow, which increaſes with age, till about eighty it is as yellow as amber.

Proceſſus Ciliares.

9. The cryſtalline is ſuſpended within the eye by a muſcle B*b b*B called ‖ *Proceſſus Ciliares*, and by ſome authors ‖ *ligamentum ciliare*. This muſcle is a little convex towards, and much weaker than, the uvea, and ariſes behind it at BB where the ſclerotica and cornea join together; at *bb* it enters the capſula, and thence ſpreads over a great part of its anterior ſurface. The anterior ſurface of the proceſſus ciliares is beſmeared with a black unctuous matter, as is alſo the back ſide of the uvea: this black is, with good reaſon, ſuppoſed to be for preventing the reflection back of any of the rays of light, that may happen to reach ſo far.

Aqueous and vitreous humours.

10. The cryſtalline humour with the proceſſus ciliares divide the cavity of the eye into two unequal portions; in the firſt and ſmalleſt of which BAB*b*C*b*, is contained a tranſparent fluid like water, called thence the *aqueous humour*. The hinder and larger part of the concavity of the eye B*b*D*b*BF is filled with a tranſparent ſubſtance called the *vitreous humour*: This humour is not ſo hard as the cryſtalline, or ſo fluid as the aqueous *. Again, the cavity containing the aqueous humour,

† That part of the capſula which inveſts the hinder ſurface of the cryſtalline humour, adheres to the membrane incloſing the vitreous humour, yet ſo as to be ſeparated from it without cutting; but all along the limb or edge EE of the cryſtalline, theſe two membranes adhere ſo cloſely together, as not to be parted without the knife. The cryſtalline has no viſible communication with its capſula, and conſequently with no part of the body; for as ſoon as the capſula is opened, the humour within ſlips clean out: whence it is imagined by ſome that the cryſtalline has only a vegetable kind of life; it having, as above obſerved, no viſible continuity with the adjacent parts by any fibre or blood-veſſels: but although no ſuch fibres or ſmall veſſels have been yet diſcovered, there ſeems to be but little room to doubt of their exiſtence. M. *Petit* in he *Mem. de l' Acad. Royale* for 1730 has given a table of the dimenſions, *&c.* of the cryſtalline humour in 26 human eyes.

‖ Both theſe names are uſed for the ſame thing by different authors, even of good repute. Mr. *Sharp*, in his treatiſe of ſurgery, *p.* 169, chuſes the firſt, and calls the circular ridge where all the coats unite at BB (which he deſcribes as whitiſh, and denſer than any of the coats) *Ligamentum Ciliare*.

* The vitreous and aqueous humours, are each contained in ſo very thin and tranſparent films or bags, as ſcarce to be perceivable till they are broke, and the liquid let out. The aqueous humour is more ſpirituous and viſcous than common water: and the vitreous appears ſomewhat like to ſoft jelly; but is ſuppoſed nevertheleſs to conſiſt of a liquid like the other, contained in a great number of delicate veſicles or cellules, which are eaſily broke; for upon touching it always appears moiſt and wateriſh. It is not doubted, but that both theſe humours are nouriſhed through proper veſſels communicating with the adjacent parts.

humour, is divided by the uvea into two unequal portions; the firſt and largeſt of which next the cornea is called the *anterior chamber*; and the other next the cryſtalline, is called the *poſterior chamber*.

11. It is agreed, that the ſeveral coats, and the ſurfaces of the different humours of the eye, are ſo ſituated as to have one ſtrait line perpendicular to them all. This imaginary line ACDF is called the *axis of the eye*. And the axes of both eyes produced, are called the *optic axes*. Axis of the eye. Optic axes.

12. Having now deſcribed thoſe parts of the eye, that are more immediately concerned in viſion †; I ſhall next ſubjoin a table of their dimenſions, at a medium, from ſeveral obſervations of Monſ. *Petit*. The *French* meaſures are reduced to my hand to the *Engliſh*, by the learned Dr. *Jurin*, in his curious *Eſſay upon diſtinct and indiſtinct viſion*, publiſhed at the end of Dr. *Smith*'s Optics. In the table the tenth of the *Engliſh* inch is made the unit; and tho' the parts are given to the hundredth part of this unit, or the thouſandth of an inch; yet it is not to be imagined, that we have the dimenſions of the eye to that exactneſs; theſe ſmall fractions having their riſe only by the reduction of the *French* meaſures (which I do not find given to leſs than $\frac{1}{4}$ of a line) to the *Engliſh*.

13.

† For the ſake of having a deſcription of the eye all together, I ſhall borrow from the anatomiſts a brief account of ſome of the other parts.

The ſclerotica is embraced on its out-ſide with ſix muſcles, which ſerve to roll the eye in its orbit all the different ways, for the ſake of ſeeing objects not directly before us without moving the head; and to facilitate theſe motions, it is obſerved that all the vacant places, between the orbit and the ſclerotica, are filled with a looſe fat. Four of theſe muſcles are ſtrait, and the other two oblique; of the ſtrait muſcles two of them are ſituated vertically oppoſite to one another; and the other two horizontally. Each of the ſix, according to its ſituation or office, has a proper name.

In the upper and outer part of the orbit, is ſeated the *lachrymal gland*, whoſe uſe is to furniſh at all times water enough to keep the outer ſurface of the eye moiſt, without which the cornea would be leſs pellucid. And that this liquor may be rightly diſpoſed of, we frequently cloſe the eye-lids, even when we are not conſcious of it. At the inner corner of the eye between the eye-lids ſtands a caruncle, whoſe office ſeems to be to keep that corner of the eye from being totally cloſed, that any tears or gummy matter may flow from under the eye-lids when we ſleep, or in o the puncta lachrymalia, which are little holes one in each eye-lid near this corner, for carrying into the *ductus ad naſum* any ſuperfluous tears.

Both the eye-lids are lined with a membrane, which alſo folds ſo much of the eye as is called the white. This membrane is named *conjunctiva*, alſo *adnata*; and beſides other uſes it may have, one very conſiderable is, to hinder any duſt or extraneous particles from getting behind the eye into the orbit.

To give a more minute account of theſe, and ſome other parts of the eye, which have yet been here mentioned, is the proper province of anatomiſts.

13. A table of the * dimenſions of ſome of the parts of the human eye, at a medium from ſeveral eyes.

	Tenths.
The axis AF of the eye from out to out, is about	9.5
The radius of the outer ſurface of the cornea	3.35
The radius of the *ſclerotica*	4.61
And the diſtance of its center from the cornea	4.89
Thickneſs of the ſclerotica	0.25
Whence the diameter of the eye from the outer ſurface of the cornea to the choroeides	9.25
Diſtance between the outer ſurface, of the cornea and cryſtalline	1.06
Radius of the anterior ſurface ECE of the cryſtalline	3.31
Thickneſs or axis CD of the cryſtalline	1.85
Radius of the hindermoſt ſurface EDE of the cryſtalline	2.50
Breadth or chord EE of the cryſtalline	3.73

Let *r* be the center of the cornea, *s* the focal center of the cryſtalline; then we ſhall have $Cs = {}^{a}1.05$, $Ds = {}^{a}0.8$; and conſequently $As = 2.11$, and $sr = 1.24$

[a] 207. I.

14. The diameter *aa* of the pupil, varies according to the degree of light the eye is expoſed to; † it being bigger or leſs, as the light is fainter or brighter, that ſo in all caſes the quantity of light entering the eye, may be the better ſuited for the purpoſe of viſion. The pupil alſo contracts a little in the ſame degree of light, when we look near at a ſmall object; but the limit of its aperture, in the eyes of adult perſons, is, I think, from about $\frac{1}{4}$ to $\frac{1}{10}$ or $\frac{1}{12}$ of an inch.

15. *Of the refractions of the different humours:* It is very difficult, I think, to determine accurately the meaſures of theſe refractions; but from ſuch ‖ experiments as could be made, it has been found, that the refractive

* I found it neceſſary to qualify ſome of theſe numbers a little, that the parts of the eye might the more uniformly join; ſome incoherence in the meaſures unavoidably ariſing from ſome diſtortion at the time they were taken: however, upon the whole, their agreement is greater than could be expected.

† Of this any one may be ſatisfied, by looking in a glaſs whilſt a lighted candle is moved tranſverſely before him. At the approach of the candle the pupil will contract, and enlarge as the candle is moved ſideways. If inſtead of the looking glaſs be uſed a ſmall concave ſpeculum, the change in the pupil will be more ſenſible, as the eye will be then magnified. The experiment may be alſo made, without the candle, by turning to and from a window.

‖ This experiment was made, by preſſing the humours of an ox's eye into a hollow priſm, formed by two glaſs planes cemented together in a given angle; and this ſeems to be the beſt method of making it: or it might be made by preſſing the humour into the ſhape of a lens, between two glaſſes properly figured. But the difficulty lies, to know what changes theſe humours may undergo after the animal is dead, and alſo after they are bruiſed, and taken out of their proper vehicles; and whether the humours of different animals refract alike. If an eye be placed in a ſmall hole in the window-ſhutter of a darkned chamber, and the bottom of the ſclerotica be pared till it becomes tranſparent, the pictures of objects that are on the outſide will be ſeen on the bottom of the eye: and if while the eye is in this poſition, the cryſtalline be couched or depreſſed, and a paper be held behind, the focus of the two humours that remain may be found: and thence knowing the figure of the cornea, the refractive

powers

refractive powers of the aqueous and vitreous humours, are each of them much the ſame with that of common water; and that of the cryſtalline is a little * greater: That is, the proportion between the ſines of incidence and refraction out of air into the cornea or † aqueous humour, is as 4 to 3; out of the aqueous humour into the cryſtalline, as 13 to 12; and out of the cryſtalline humour into the vitreous, as 12 to 13.

16. The cornea and the aqueous and vitreous humours being ſuppoſed to have the ſame refractive power; all three may be looked upon as one denſe medium, whoſe refractive ſurface is the cornea; and the cryſtalline humour may be conſidered as a convex lens, placed in a given poſition within the ſaid medium; whence the humours of the eye all together make a kind of a compound lens, whoſe effect in refracting rays, having a given focus of incidence, may be found as by the following

PROPOSITION I.

Having the focus of incidence of rays upon the cornea of the eye, and the refractive powers, radii, and poſitions of the cornea and of each ſurface of the cryſtalline humour; to find the focus of the rays, after their laſt refraction, by the poſterior ſurface of the cryſtalline.

Let the refractive ſurfaces of the eye be as above ſuppoſed in *Art.* 13, and 15; and let GF be the axis of the eye; F the principal focus of the cornea; G the focus of rays parallel to the axis within the cryſtalline, after refraction by the ſurface C at their entrance into the aqueous humour; and H the focus of rays coming parallel to the axis out of the vitreous, upon the ſurface D of the cryſtalline humour, after refraction by that ſurface. Then we ſhall have Fig. 2, 3.

AF = [a] 13.4; CG = [b] 39.72; and DH = [a] 32.5.

[a] 126. I.
[b] 127. I.

17. CASE 1. *To find the focus of parallel rays, after all their refractions by the three refracting ſurfaces of the eye.*

1. Having found the focus F of parallel rays refracted by the cornea, it will be, Fig. 2.

2. powers of the aqueous and vitreous humours taken together will be known; and ſuppoſing the refractive powers of theſe two to be the ſame, that of the cryſtalline may now be eaſily found.

* It is obſerved, that thoſe who have been couched for a cataract (which is a depreſſion of the cryſtalline humour to the bottom of the eye) are obliged to uſe convex glaſſes for obtaining diſtinct viſion; which is a proof that the cryſtalline refracts light more than the adjacent humours.

† It is ſuppoſed that the refractions of the cornea and aqueous humour, are much the ſame: But if it was otherwiſe, no ſenſible difference could thence ariſe, becauſe the cornea is nearly of the ſame thickneſs.

[c] 142. I. 2. FG : FC :: [c]FC—3.31 : F*b*; (see *fig.* 47. Book I.) that is, 52.06 (=39.72+13.4—1.06 or CG+AF—AC) : 12.34 (13.4—1.06) :: 9.03 : 2.14.

3. To find the focus Y, after the last refraction by the surface D; say H*b* : HD — 2.5 :: [d] D*b* : DY. See *fig.* 45. Book I.

[d] 144. I. Now D*b* (=AF—AD—F*b*=13.4—2.91—2.14) = 8.35; and H*b* (=HD+D*b*=32.5 + 8.35) = 40.85.

Wherefore 40.85 : 30 :: 8.35 : 6.13=DY; whence AY (=AD+DY) = 9.04. *Q. E. I.*

18. Case 2. *Having the focus of rays diverging or converging upon the cornea, to find their focus after the last refraction by the posterior surface of the crystalline.*

Fig. 3. Let Q be the focus of incident rays, and *f* the focus of rays coming parallel to the axis out of the aqueous humour upon the cornea; say,

1. Q*f* : QA :: [c]QA+3.35 : Q*a*. Let QA be 8 inches or 80 tenths; then 69.95 (=80—10.05) : 80 :: 83.35 : Q*a*=95.325. See *fig.* 43. Book I.

Now C*a* (=95.325—80—1.06) = 14.265, and *a*G (=14.265+. 39.72) = 53.985.

2. Say *a*G : *a*C :: [c]*a*C—3.31 : *ab*. (see *fig.* 47. Book I.) that is,

53.985 : 14.265 :: 10.955 : *ab* = 2.895—.

Now D*b* (=C*a*—CD—*ab*=14.265—1.85—2.895) = 9.52 and H*b* (=32.5 + 9.52) = 42.02.

3. To find the focus *q* after the last refraction at D, it will be, H*b* : HD—2.5 :: [d] D*b* : D*q*. (see *fig.* 45. Book I.) that is,

42.02 : 30 :: 9.52 : 6.797=D*q*. *Q. E. I.*

After the same manner, by 142 to 144. I. may be found the focus *q* of the refracted rays, wherever the focus of incidence Q is placed. *Vide fig.* 43 to 48, Book I.

19. *Schol.* 1. Let *t* be the bottom of the retina; then D*t* = [a] 6.34, Y*t* = [b] 0.21, *tq* = [c] 0.45, and Y*q* = 0.66. And hence if it be supposed that parallel rays, and rays diverging from a focus at the distance of 8 inches from the cornea, are equally suited for distinct vision; the retina should bisect Y*q*; and therefore the axis of the eye should be either a little longer, or its refractive power a little greater, than what has been above

[a] 13. II.
[b] 17. II.
[c] 18. II.

above ſuppoſed. However, if we argue from the places of Y, *t* and *q*, the meaſures of the eye are pretty conſiſtent.

20. *Schol.* 2. By ſuppoſing *t* a focus of incidence, we may, after the manner ſhewed in the preceding *Prop.* find a focus Q, from whence rays diverging upon the eye would be united at the retina; and according to the numbers in *Art.* 13, 15, AQ = 23.956 or 24 inches nearly. Whence, if our meaſures of the eye could be depended upon, it might reaſonably enough be concluded, that 24 inches is the natural diſtance of diſtinct viſion; or that diſtance at which the eye would ſee an object perfectly diſtinct, without any ſtraining or effort of any of its parts. And if the axis of the eye be ſuppoſed a little longer, ſo as the retina may biſect Y*q*, the ſaid diſtance AQ will be leſs than 24 inches. But to aſſign that diſtance from the meaſures of the eye, would be a vain attempt; for a ſmall error in theſe meaſures, would produce a very great one in the ſituation of the point ſought, Q.

21. *Schol.* 3. If it be ſuppoſed that the pictures of objects, when ſeen diſtinctly, are formed upon the retina; it is manifeſt that a different conformation of the eye is neceſſary, for obtaining diſtinct viſion at different diſtances, either as to the length of its axis, or as to the figure or poſition of the cornea or cryſtalline. But which way ſoever this different conformation is effected, as ſmall a change in the eye as will ſuffice for procuring a diſtinct viſion of objects, from a moderately ſmall diſtance, to double that diſtance; will alſo ſuffice for all diſtances, from that double diſtance to infinity.

Let AQ be the axis of the eye, AB its ſemi-aperture; *p* the place of the neareſt object, Q a ſecond object; ſo that *p*Q = *p*A or *p*B, and let *k*B be a ray parallel to the axis. Now becauſe B*p* = *p*Q, the ∠ *p*BQ = BQ*p* = *k*BQ; whence it is ſufficiently manifeſt, that if the eye is ſo adapted as to unite a ray *p*B with the axis, at the point where the retina interſects it; as great a change in the eye is requiſite to bring a ray QB to unite with the axis and the retina, as will afterwards be neceſſary to bring the ray *k*B to an union with the axis of the eye and retina. Fig. 3.

22. *Schol.* 4. The focal center of the eye, (or that point O in its axis, at which an object, and its image upon the retina, form both the ſame angle) is nearly, where the hindermoſt ſurface of the cryſtalline interſects the axis. Fig. 2, 3.

The focal center of the cornea alone, is its center *r*; and the focal center of the cryſtalline alone is the point *s*, ſo that *s*D = [a] 0.8, and *sr* = [a] 1.24. Whence it is ſufficiently manifeſt, that the focal center of the [a] 13. II.

eye, or of the cornea and cryſtalline taken both together, muſt be ſomewhere betwixt *s* and *r*. Let *f* be the principal focus of rays coming out of the aqueous humour upon the cornea, *g* the principal focus of the cryſtalline humour, and let the ſpace *sr* be ſo divided in O, that *s*O : O*r* :: *sg* : *rf*; it is pretty evident that the point O thus found, will be the focal center of the eye, at leaſt nearly *; to find which we ſhall
[a] 185. I. have *rf* = 13.4, *sg* = [a] 17.56, and *sr* = 1.24; whence *s*O = 0.7. But *s*D = 0.8, and D*t* = 6.34, and therefore O*t* = 6.44, and AO = 2.81. *Q. E. I.*

Fig. 4. 23. *Corol.* 1. In every oblique pencil which enters the eye, that ray is to be eſteemed the axis, which interſects the axis of the eye in O, as repreſented by the black lines in the annexed figure. And therefore in an oblique pencil, the axis is not the middlemoſt ray; and the greater is the obliquity of the pencil, the more remote will its axis be from the middle; and when the obliquity is very great, the axis will be excluded from entering into the pupil.

24. *Corol.* 2. Having the angle ſubtended by any object at the eye, by making O*t* radius, we ſhall have the diameter of its image upon the retina. Thus, a circle facing the eye, and ſubtending an angle thereat of 1 *min.* will have for its image a circular ſpot upon the retina, whoſe diameter is about $\frac{1}{340}$ part of an inch.

§. 2. *Of the manner of viſion.*

The nerves the organs of ſenſation. 25. It is ſuppoſed, without any controverſy, that the nerves (which are diſtributed all over the body, and center in the brain) are the immediate organs of ſenſation. And there ſeems no room to doubt, but that our ſenſations are immediately cauſed by impulſes from external bodies, either by touching our bodies in the groſs, or by ſome ſmall material particles ſome how propagated from them. Of the firſt ſort, is the ſenſe of *feeling*, ſo called; and not unlike that, is alſo the ſenſe of *taſting*, the bodies we taſte being in immediate contact with the palate, tho' diſſolved into ſmaller particles, which either by their different texture, figure or ſizes, give us the ideas of different taſtes. *Smelling* is cauſed by the impulſes of ſmall particles, or effluvia, propagated from the odoriferous body: While the idea of ſound, or of *hearing*, is excited by

* Authors differ very much about the ſituation of the focal center O; ſome placing it in the center of the eye, others in the vertex of the cornea; ſome in the middle of the cryſtalline, and others in the middle of the pupil; but for the reaſon above aſſigned, the point O cannot be far from where we have placed it. The method of finding the focal center of two or more lenſes, or refracting ſurfaces any how combined, will be ſhewed hereafter; but for our preſent purpoſe, the ſummary explanation above is ſufficient.

by impulſes of the air (put into an undulating motion, by a tremor or vibrating motion of the ſonorous body,) on the drum of the ear *.

Light, the mediate cauſe of viſion.

26. And we can make no doubt, but that light propagated from viſible bodies, by ſtriking upon the proper organ, is the mediate cauſe of viſion.

Fig. 4.

Light, how refracted by the eye, a picture formed upon the retina.

Thus, for example, the pencils of light BPB, BQB, BRB propagated from the points P, Q, R of an object PQR, are firſt refracted by the cornea, ſo as to belong to the focus's *a*, *b*, *c* reſpectively, at ſome certain diſtance behind the eye; then by the anterior ſurface of the cryſtalline humour, they are again refracted towards the focus's *h*, *i*, *k* nearer to the eye than before; and laſtly, in going out of the cryſtalline into the vitreous humour, they are refracted ſo as to unite in the points *p*, *q*, *r*. In like manner, the pencils of rays coming to the cornea from every phyſical point of the object PQR, are, by the different refracting ſurfaces of the eye, brought orderly to unite upon the retina, and there form as it were an image *p q r*, of the object; but in an inverted poſition, the rays belonging to P uniting at *p*, &c.

Thus, the cavity of the eye is a kind of † *Camera Obſcura*; the cornea and cryſtalline making a ſort of compound lens, whoſe aperture is limited by the breadth *aa* of the pupil.

Sir *Iſaac Newton*'s thoughts concerning the manner of viſion.

27. Sir *Iſaac Newton* ſeems to think that ſenſations are excited in the brain by a vibrating motion of the nerves. *Newton*'s *Optics*, Q. 12. pag. 319. "Do not the rays of light in falling upon the bottom of the eye excite vibrations in the *tunica retina*? Which vibrations, being propagated along the ſolid fibres of the optic nerves into the brain, cauſe the ſenſe of ſeeing. For becauſe denſe bodies conſerve their heat a long time, and the denſeſt bodies conſerve their heat the longeſt, the vibrations of their parts are of a laſting nature, and therefore may be propagated along ſolid fibres of uniform denſe matter to a great diſtance, for conveying into the brain the impreſſions made upon all the organs of ſenſe ‖."

* How exquiſitely ſmall muſt be the effluvia of bodies, and how delicate our nerves, to be moved by ſuch tender impreſſions; and not only that, but to be affected differently by ſuch an innumerable variety of them! How vaſt are the variety of ſmells, and what a long time will ſome odoriferous bodies, preſerve their odour, without a ſenſible diminution of their weight! What an immenſe ſpace an inch of tallow is capable of filling! For a lighted candle may be ſeen at the diſtance of ſome miles, and the motion of light is in a manner inſtantaneous. What an infinite variety of ſounds can we diſtinguiſh; and how ſurpriſing is the effect of muſick! Beſides theſe effluvia's of bodies that immediately affect our ſenſes, there may be a great many of other ſorts that are conſtantly emitted: Magnetiſm is one remarkable inſtance; and electricity is a wonderful phænomenon. The flaſhes of lightning from rubbing a glaſs tube in the dark; the ſpray of the ſea againſt the ſides of a ſhip under a freſh gale; the ſhining of touchwood; are inſtances, among ſeveral others, that bodies may emit light without being in what we call a burning ſtate. And how does that light differ from that of fire, except in degree?

† For the theory of the *Camera Obſcura*, ſee *Art*. 243, to 246. Book I.

‖ See alſo Q. 16. latter end.

Of colours, how perceived; by the ſame author.

28. In the next *Query* (13.) the ſame great author propoſes the following moſt beautiful hypotheſis, as a ſolution of the phænomena of colours. "Do not ſeveral ſorts of rays make vibrations of ſeveral bigneſſes, which according to their bigneſſes excite ſenſations of ſeveral colours, much after the manner that the vibrations of the air, according to their ſeveral bigneſſes, excite ſenſations of ſeveral ſounds? And particularly, do not the moſt refrangible rays excite the ſhorteſt vibrations for making a ſenſation of deep violet; the leaſt refrangible, the largeſt, for making a ſenſation of deep red; and the ſeveral intermediate ſorts of rays, vibrations of ſeveral bigneſſes, to make ſenſations of the ſeveral intermediate colours?"

Sir *Iſaac Newton*'s opinion, that viſible ſenſations are communicated to the brain by the mediation of a fine æther.

29. In a manuſcript of Sir *Iſaac Newton*'s, a copy of which I was favoured with by the curious and learned *William Jones* Eſq; that great author's opinion is, that the ſenſations cauſed by the impulſes of light upon the retina, are communicated to the brain or ſenſorium, by the mediation of a fine æther: His words ſee below *.

Structure of the eye wiſely contrived.

30. In order to form diſtinct viſion, the ſtructure of the eye itſelf plainly indicates, that a certain quantity of rays from every viſible point of an object, are neceſſary to be united at the fund of the eye; and that the points of union of the rays of the different pencils, muſt be diſtinct and ſeparate from one another, at leaſt that there ſhould be but as little mixture of them as poſſible. For the effecting of theſe purpoſes, the ſtructure of the eye is admirably contrived: All the rays coming from any viſible point of an object, that can enter the pupil, are united cloſely together upon the retina, and thereby make a vaſtly greater impreſſion, than a ſingle ray alone could do; and for this end, the retina is placed at a proper diſtance behind the refracting ſurfaces. In like manner, each pencil of rays is refracted orderly into diſtinct focus's,

* 'Light ſeldom ſtrikes upon the parts of groſs bodies, (as may be ſeen in its paſſing through them;) its reflection and refraction is made by the diverſity of æthers; and therefore its effect upon the retina can only be to make this vibrate: which motion then muſt be either carried in the optic nerves to the ſenſorium, or produce other motions that are carried thither. Not the latter, for water is too groſs for ſuch ſubtile impreſſions; and as for animal ſpirits, tho' I tied a piece of the optic nerve at one end, and warmed it in the middle, to ſee if any airy ſubſtance by that means would diſcloſe itſelf in bubbles at the other end, I could not ſpy the leaſt bubble; a little moiſture only, and the marrow itſelf ſqueezed out. And indeed they that know how difficultly air enters ſmall pores of bodies, have reaſon to ſuſpect that an airy body, tho' much finer than air, can pervade and without violence (as it ought to do) the ſmall pores of the brain and nerves, I ſhould ſay of water; becauſe thoſe pores are filled with water: and if it could, it would be too ſubtile to be impriſoned by the *dura mater* and ſkull, and might paſs for *æther*. However, what need of ſuch ſpirits? Much motion is ever loſt by communication, eſpecially betwixt bodies of different conſtitutions. And therefore it can no way be conveyed to the ſenſorium ſo entirely, as by the æther itſelf. Nay, granting me, but that there are pipes filled with a pure tranſparent liquor paſſing from the eye to the ſenſorium, and the vibrating motion of the æther will of neceſſity run along thither. For nothing interrupts that motion but reflecting ſurfaces; and therefore alſo that motion cannot ſtray through the reflecting ſurfaces of the pipe, but muſt run along (like a ſound in a trunk) entire to the ſenſorium. And that viſion thus made, is very conformable to the ſenſe of hearing, which is made by like vibrations.'

focus's, that the whole object may at once be diſtinctly ſeen. If the rays of light were not thus refracted, very few of them would ſtrike upon the leaſt ſenſible point of the viſionary nerve, wherever that was placed, and the rays from different objects, or from different parts of objects, would ſtrike on the ſame place at once, which muſt have created an indiſtinctneſs equal to blindneſs.

When the light is weak, or ſtrong, the pupil is accordingly enlarged or contracted, for the admiſſion of more or fewer rays; that the impreſſions upon the retina, in both caſes, may be the more ſuitable.

The cryſtalline humour being denſer towards the [a]middle than elſewhere; it is reaſonable to ſuppoſe, that this humour is not equally refractive, the denſer, or middlemoſt part, being more refractive than the reſt *. This different denſity, of the conſtituent parts of the cryſtalline, is admirably contrived for correcting the aberration from its figure, as well as that of the cornea †. The rays in each pencil that are more and more remote from the axes, by paſſing through a medium gradually diminiſhing in denſity, from the middle towards the extremes, have their focus's gradually lengthened, which corrects the aberrations of the figure, that ſo they may all unite nearer together. The concave figure of the retina is alſo ſerviceable in this caſe. [a] 8. II.

31. It is indiſputably true, that objects are ſeen by means of the impulſes of the rays of light, propagated from them upon the viſionary nerve; and we have the concurrent teſtimony of all our ſenſes, that viſible objects, or ſuch of which we have viſible ideas, do really exiſt ſomewhere, that is, at ſome, may be uncertain, diſtance ||, in the direction of thoſe rays, or of the principal ray in each pencil. And hence it neceſſarily Why objects are ſeen erect.

* It is conſtantly obſerved, that in mediums of the ſame kind, but of different denſities; that which has the greater denſity, has the greater refractive power.

† This conjecture appears ſo natural, that it looks as if this important end be one main deſign of the cryſtalline. For if we conſider it as a mere lens, for ſhortening the focus's of rays refracted by the cornea; that end might have been obtained by a much leſs apparatus, by only increaſing a little the convexity of the cornea, and the denſity of the humour behind it: Perhaps it alſo helps to accommodate the eye for diſtinct viſion, at different diſtances. In the eyes of fiſhes the cryſtalline is globular, having in the middle a ſmall nucleus or kernel, much harder than the reſt. This figure cauſes a greater aberration than that of a thinner lens, to correct which, the kernel in the middle is accordingly harder in proportion, than the middle of the cryſtalline in the eyes of other animals. A due conſideration of the ſtructure of the eye, may furniſh uſeful hints for the improvement of optical inſtruments.

|| It is almoſt unneceſſary to advertiſe the reader, that what is here meant, is light coming directly from the object to the ſpectator, without being bent by reflection or refraction. Images, as thoſe in a looking-glaſs, &c. excite the ſame kind of ideas, and that by the ſame laws of viſion, as real objects do; and it is very true, that our ideas themſelves have no ſimilitudes, or perhaps any neceſſary connection, with material bodies. And hence ſomewhat like an argument has been advanced, to ſhew that a material world does not exiſt, or at leaſt does not, as to any thing we know, neceſſarily exiſt. But to wave metaphyſical arguments, which perhaps are more ſerviceable for maintaining of feigned hypotheſes, than for diſcovering truth; it is ſufficiently manifeſt that ſomething does really exiſt, which is inactive, ponderous, endued with ſome peculiar qualities, which men have learned to modify various ways for their conveniency. But if theſe refined logicians will not allow us the teſtimony of our ſenſes, they will not, 'tis to be hoped, deprive us of our religion too; and does not that ſuppoſe the exiſtence of a material world?

neceſſarily follows, that we ſhould ſee objects erect, and all their parts in their due order and ſituation; that is, thoſe parts that are to the right and to the left, we perceive in theſe ſituations. For though inverted images of objects are, as it were, formed at the bottom of the eye, yet it cannot be ſuppoſed that the eye, or the mind ſees theſe images, as ſuch: For the eye ſees no part of itſelf, and even external objects muſt be at a competent diſtance from it, to become diſtinctly viſible.

What is uſually meant by an image upon the retina.

32. The impreſſions upon the retina, of the ſeveral pencils of light propagated from an object, will be neceſſarily more or leſs diſtinctly perceived, according as the imaginary image there formed by them, is more or leſs perfect; and ſo for ſhortneſs of expreſſion, an image upon the retina is often ſpoke of, to denote the degree of diſtinctneſs of a viſible object; and alſo to denote in general, the impreſſions of the pencils of light upon the retina, when the diſtinctneſs of viſion is not particularly conſidered.

Ideas accompanying different ſenſations acquired by experience.

33. As all our ſenſations are ſome how cauſed by impreſſions from external bodies, ſo particular ſenſations never fail to excite in us the ideas of particular bodies; and theſe ideas, or ſomething at leaſt in the compoſition of them, we undoubtedly acquire by experience. How elſe could a particular ſound produce the compound idea of a bell, a drum, a man, an acquaintance? And language is a ſurpriſing inſtance of the great ſtrength and quickneſs of our retentive faculties, in aſſociating together a vaſt variety of ideas, which before were learned and treaſured up, as the proper ſignifications of ſuch particular words or ſounds. In like manner, different ſmells and taſtes are accompanied with ideas, which before had been treaſured up in the memory. Our ſenſations themſelves are probably coeval with us, and perhaps they differ but little all our lives, except in degree of quickneſs or perfection: It is the ideas that accompany them, which are acquired; and theſe for the moſt part are got ſo early, that we forget our having ever learned, as it may not improperly be called, to ſee and to hear, *&c.* And yet ſo true it is, that we have as it were learned ſeeing and hearing, *&c.* that an adult perſon, who had been blind or deaf from his infancy, would not be able immediately to make any great uſe of his new acquired ſenſe, till he had time and opportunities to make his proper obſervations, as others had done in their childhood *. The theory of viſion, as little as we know of it, is yet better underſtood than that of either of our other ſenſes. And it is very manifeſt, that the ſame object placed in a different light, in a different poſition with reſpect to the optic axes, or at a different diſtance from the eye, will have a different image upon the retina;

* See *Phil. Tranſ.* N° 402; and *Tatler*, N° 55.

retina; and yet innumerable variety of theſe circumſtances will cauſe no difference in the viſible idea; experience having taught us, that theſe different ſenſations proceed from thoſe different circumſtances, we inſtantly, and imperceptibly, form the ſame idea of the object that we were uſed to do.

Like ſenſations, tho' from different cauſes, excite like ideas.

34. There ſeems but little room to doubt, but that like ſenſations do always excite like ideas, although the cauſes of theſe ſimilar ſenſations may be different. The flaſhes of lightning ſometimes perceivable from a ſudden blow upon the eye, or other part of the head, are probably owing to the ſame kind of motions being excited in the optic nerve by the blow, as by the impreſſions of real lightening. After the ſame manner we may make ſome probable conjectures, as to the cauſes of other phænomena of the like ſort. As the circle of colours, like thoſe in the feather of a peacock's tail, that appear upon preſſing in the dark one corner of the eye with the finger, are probably owing to the like motions being excited at the bottom of the eye, by the preſſure of the finger, as at other times are excited there by light for cauſing viſion *. And theſe phænomena do ſomewhat corroborate the hypotheſis, of viſion being cauſed by ſome motion excited in the optic nerve, by the impulſes of light upon the bottom of the eye.

Viſible impreſſions have ſome duration.

35. The ideas we have from the impreſſions of light, do not inſtantly vaniſh, but remain for ſome ſmall time. This muſt undoubtedly be true, with reſpect to all our ſenſations; for all motions muſt be performed in time, though imperceptible to us. The phænomenon of a burning coal appearing like a ring of fire, when whirled nimbly about in the circumference of a circle, ſeems to prove that viſible ſenſations have a ſenſible duration. For the impreſſions of light from the coal in every part of the circle, do probably remain entire on the eye, till they are renewed by the arrival of the coal at the ſame part again; otherwiſe, inſtead of a ring of fire, we ſhould ſee the coal diſtinctly in the ſucceſſive places, as the caſe is when it is moved but ſlowly. In like manner, in going ſuddenly out of a ſtrong light into the dark, the impreſſions of the light are perceptible for ſome time; and the uneaſineſs we feel at ſuch times, ſhews that a ſudden change, even from light to darkneſs, is hurtful to the eyes: The ſame kind of uneaſineſs is likewiſe felt, in going ſuddenly out of a dark room into a ſtrong light. Theſe phænomena do alſo ſhew, that the eye requires ſome time to adapt itſelf to different degrees of light. Theſe and ſome other phænomena, † Sir *Iſaac Newton* brings to corroborate his hypotheſis, that the motions excited by the impreſſions of light upon the retina, are of a vibrating nature;

And laſt about a *ſecond* of time.

* *Newton*'s Optics, *Quer.* 16. † Ibid.

ture; and moreover, he deduces from them, that motions once excited at the retina, continue there about a *ſecond* of time before they ceaſe.

§. 3. *Of apparent motion.*

Motion how perceived.

36. In like manner, as the different ſituations of ſeveral bodies, or of the ſeveral parts of the ſame body, whilſt at reſt, are perceived by the impulſes of the light proceeding from them upon correſpondent parts of the retina; ſo alſo are the motions of bodies, perceived by the ſucceſſive progreſs of theſe impulſes over different parts of the retina. Thus, if an object moves from P along the line PQ, its image upon the retina will alſo move from *p* along the line *pq*.

Fig. 6.

A ſpectator in motion, ſees an object at reſt move the contrary way.

37. The ſame phænomenon, as above, will happen, if while the object P is atreſt, the eye be carried parallel to PQ the contrary way; for the imageof the object P will in both caſes be moved alike along the line *p q*. And in the two caſes, if the velocity of P and the eye be the ſame, the apparent velocity of P will be the ſame alſo *.

This accounts why a paſſenger carried in a coach or boat, ſees, or imagines he ſees, the adjacent trees or buildings move, juſt at the ſame rate, as a perſon at the place he looks at would ſee his vehicle move. And this deception, if it may be ſo called, will be the ſtronger, as he is the leſs ſenſible of his own motion, and the quicker it is; for then he has the leſs incitement, and the leſs time, to inform his judgement of the true cauſe †.

Fig. 6.

Phænom. of a ſpectator and an object both in motion.

38. *Corol.* An object moving along a line PK, will appear at reſt, to a ſpectator moving with the ſame velocity along the line QG, parallel to PK. And if the motion of the object, is quicker or ſlower than that of the ſpectator, it will have an apparent motion, which will be direct or retrograde accordingly. If the two motions are contrarily, the apparent motion of the object will be ſo much the greater, in proportion to the real motion of the ſpectator. To paſſengers in a boat on a broad river, the phænomena here explained do frequently occur; and they are perhaps more agreeable to theory, becauſe we are leſs uſed to them, than to objects at reſt.

PROP.

* Hence, if the earth be ſuppoſed to move round its axis from weſt to eaſt, while the heavenly bodies are at reſt, they will appear to us, to move the contrary way from eaſt to weſt; there being nothing in this caſe to indicate to us our own motion. And therefore no argument drawn from the apparent diurnal motions of the ſtars and planets, can be of any ſupport to either the *Ptolemaic* or *Pythagorean Syſtems*.

† Thus a perſon walking or riding ſlowly, tho' he perceives the change of ſituation of adjacent bodies, yet being ſenſible of his own motion, and having time ſufficient to reflect in the intervals of theſe apparent changes, thoſe bodies appear nevertheleſs to keep in their places. But if he runs or rides ſwiftly, he cannot help fancying the bodies he looks at, to move towards him. The deception is ſtill ſtronger, when one ſits at eaſe. In coming up the river on board a ſhip, I could not help fancying that the veſſels a-head came at a good rate to meet us, and being in ſome dread of their running foul upon us; tho' their want of ſails, was an indication to undeceive me.

PROP. II.

39. *The ſame degree of velocity appears greateſt, when the motion is in a line perpendicular to the optic axis; and when the motion is in other directions, the apparent velocity will be, as the coſines of the angles of inclination to the ſaid perpendicular.*

Apparent velocity in oblique directions to the optic axis.

Thus, if two bodies ſet out at the ſame time from P, the one moving along the line PQ perpendicular to the optic axis Q*q*, and the other along the line PS obliquely to it; and if their velocities be ſuch, as to paſs over the lines PQ, PS in the ſame time, it is manifeſt that their apparent velocities will be the ſame; for the images of each, will paſs over the ſame ſpace *pq* at the bottom of the eye, in the ſame time. Their real velocities in this caſe being as PQ to PS; it is manifeſt that when the velocities are equal, *the apparent velocity of the body which moves in* PQ : *the apparent velocity of the body moving in* PS :: PS : PQ :: *Radius : Coſine of the* ∠ QPS. Therefore, *&c. Q. E. D.* Fig. 6.

40. And we can no otherwiſe perceive motion directly towards or from the eye, than by the change of the apparent magnitude or brightneſs of the object: For in that caſe, the image of the moving body, keeps its ſituation upon the retina invariable.

Motion in the optic axis how perceived.

PROP. III.

41. *If two or more objects at different diſtances from the eye, move in parallel lines nearly at right angles to the optic axis; and if their velocities are proportional to the ſaid diſtances, their apparent velocities will be all equal: And if their real velocities are equal, their apparent velocities will be reciprocally as their diſtances from the ſpectator *.*

1. Let an object move from Q to P, in the ſame time that another moves from G to H, and it is manifeſt that their apparent velocities will be equal; for the ſpace *qp* upon the retina, will be paſſed over in the ſame time, by the image of each; and their real velocities are, as the lines QP, GH; but QP : GH :: QO : GO. *Q. E. D.* 2. Let the velocities of the objects G and Q, be equal; then will the object G arrive at K, and its image deſcribe the ſpace *qk* upon the retina, in the ſame time that the image of the object Q deſcribes the ſpace *qp*; and therefore the apparent velocities of theſe two objects are as *qk* and *qp*, or as GK (=QP) and GH; that is, the apparent velocity of the object G, is to that of Q; as QO to GO. *Q. E. D.* Fig. 6.

P PROP.

* It is here ſuppoſed, that the ſpectator makes no allowance for the different diſtances; when he does, the phenomena may diſagree with the theory.

PROP. IV.

Apparent velocities at different distances.

42. *The apparent velocities, of bodies moving in parallel lines at different distances from the eye, are directly as the real velocities, and reciprocally as the distances; that is, as the velocities divided by the distances.*

Fig. 6. Let two bodies move from Q, G, in the parallel lines QP, GH; and let the velocity of the object Q, be called V, and that of G, v; and let their apparent velocities be called A, a. I say, that

$$\frac{V}{QO} : \frac{v}{GO} :: A : a :: V \times GO : v \times QO.$$

If the two objects be conceived to move in the same line, suppose GK, then whatever be their velocities, it will be $V : v :: A : a$; and supposing the velocity of the object Q to be the same in QP, as before in GK, it will be A : A in GK :: [a] GO : QO;

[a] 41. II.

And A in GK : a :: V : v; and compounding these, it will be $A : a :: GO \times V : QO \times v :: \frac{V}{QO} : \frac{v}{GO}$. *Q. E. D.*

Objects at rest may have different apparent motions.

43. *Schol.* 1. After the same manner, as objects in motion will have different apparent velocities, according to their different distances from a spectator; so to a spectator in motion, objects at rest will have different apparent velocities. Thus, a passenger in a coach sees the trees in the next hedge move swiftly backwards, whilst those in the field beyond move slower, and those beyond these still slower; and so on, those that are very remote being scarce perceived to move at all. And if a spectator in motion keeps his eye fixed upon an object at some distance, objects which are pretty near that, will appear at rest, as in this case, their images upon the retina will remain fixed; whilst nearer objects will seem to go backwards, and more remote ones progressively forwards, the same way with the spectator.

Objects at rest may seem in motion, and the converse, when the spectator is at rest.

44. *Schol.* 2. If two or more objects, having the same apparent velocity, move all the same way; an object at rest, by which they pass, may appear to move the contrary way, whilst the objects in motion may seem at rest. For no motion among themselves is perceivable, since their images upon the retina keep the same distances from each other; and if the spectator insensibly moves his eye, so as to keep these images in the same place, the image of the object at rest will pass successively over them, after the same manner as if that object had been in motion the contrary way. The same phænomenon may happen, if the single object be in motion either the same or contrary way; only its apparent motion will be quicker or slower, direct or retrograde, according to the different circumstances. Thus, when the clouds move successively over the

the moon, ſhe ſeems to go with their velocity the contrary way, whether that be eaſtward or weſtward.

45. *Schol.* 3. From the preceding propoſitions, it appears how difficult it is, to form a juſt eſtimation of the real velocities of objects, from the apparent ones; ſince we ought to know both the directions and diſtances of the moving objects, neither of which in many caſes, can be gueſſed at with any tolerable accuracy; and tho' we had them both given, the calculus is too perplexed to be made *extempore*. We are in ſome caſes, as above obſerved, even liable to miſtake the way towards which an object ſteers.

Degrees of velocity difficult to be eſtimated.

46. Apparent motion, or the motion of the images of a moving object upon the retina, muſt have a certain limited degree of velocity to become perceptible; that is, the ſpace deſcribed upon the retina in a given time, muſt be neither leſs than ſome given ſpace, nor greater than ſome other determinate ſpace.

Viſible motion limited within certain degrees of apparent velocity.

47. If the motion of the ſun's ſhade, at the diſtance of five feet from the gnomon that caſts it, be obſerved at about a foot diſtance from the ſhade, when the ſun changes his azimuth about ten degrees in an hour; the apparent motion of the ſhade will be then as ſlow as can well be perceived, and the ſpace deſcribed by it upon the floor, will be about * $10\frac{1}{2}$ Inches in an hour; and the motion of the image of the ſhade, upon the retina of an eye looking at it at the diſtance of 11 Inches, will be at the rate of about $\frac{6}{10}$ Inch in an hour, or .01 of an inch in a minute.

Of the ſloweſt apparent motion.

48. By the above ſuppoſition, the ſpace deſcribed in an hour by an object having the ſloweſt perceivable motion, and the diſtance of the eye from the object, are nearly equal: And therefore the ſpace deſcribed in a minute of time, by an object having ſuch a degree of motion, ſubtends an angle at the eye of about one degree: Or the ſpace deſcribed in a min. in a line perpendicular to the optic axis, by an object having the leaſt perceivable motion, is about $\frac{1}{60}$ of the diſtance between the object and the ſpectator. Hence, we ſee why the heavenly bodies are not perceived to move; the ſpaces deſcribed by them in a minute, not ſubtending an angle of above a $\frac{1}{4}$ degree, when their apparent motion is greateſt.

The angular ſpace deſcribed in a given time, by an object having the leaſt perceptible motion.

49. It is difficult to aſſign the greateſt angular ſpace, that can be deſcribed in a given time, by an object having the greateſt perceptible motion; and the difference may be very great, betwixt that degree of velocity with which ſomething may be perceived to croſs the optic axis,

Of the quickeſt perceptible motion.

* The motion of the ſhade would not be perceivable, if we did not attend to fix marks upon the floor; and when we cannot compare the ſituations of moving bodies with others at reſt, a motion much ſwifter than that of the ſhade will be imperceptible.

and that, with which the ſame object may be diſtinctly ſeen, as it moves: Of this latter ſort, let that of a falling body at the diſtance of 20 feet from the ſpectator, and in that part of its fall when its velocity is at the rate of 19 feet in a ſecond, be the quickeſt perceivable motion; the motion of the image upon the retina at that inſtant, will be at the rate of about $\frac{6}{10}$ of an inch in a ſecond; and therefore upon this ſuppoſition, the quickeſt perceivable motion is to the ſloweſt, as an hour is to a ſecond of time, or as about 3600 to 1.

50. We do not here pretend to aſſign the preciſe limits of either the leaſt or greateſt perceivable velocity; what has been obſerved above, is ſufficient to ſhew that they have their limits, and perhaps theſe limits are not far from thoſe above ſuppoſed; and perhaps alſo, the ratio of the greateſt to the ſloweſt of velocities upon the retina, that are diſtinctly perceivable, is not greater than about 2000 to 1. We have ſeen [a] 48. II. [a]above, that a prodigious great velocity, ſuch as is the diurnal motions of the heavenly bodies, may be yet too ſlow to be perceived; and the flight of a ball out of a gun, is much too quick for an eye not far off.

§. 4. *Of viſion with both eyes.*

What is ſaid below about the manner of communication between both eyes, and of viſion (diſtinguiſhed by the inverted comma's) is taken *verbatim* from the manuſcript of Sir *Iſaac Newton*'s, refer'd to in *Art.* 29.

Deſcription of the optic nerves and their juncture in the brain, by Sir *Iſaac Newton*.

51. " The *tunica retina* grows not from the ſides of the optic nerve " (as the other two * which riſe one from the *dura*, the other from the " *pia mater:*) but it grows from the middle of the nerve, ſticking to it all " over the extremity of its marrow. Which marrow, if the nerve be any " where cut croſs-wiſe betwixt the eye and the union of the nerves, appears " full of ſmall ſpots or pimples, which are a little prominent, eſpecially " if the nerve be preſſed, or warmed at a candle; and theſe ſhoot into " the very eye, and may be ſeen within ſide, where the retina grows to " the nerve; and they alſo continue to the very juncture EFGH. Fig. 5. " But at the juncture they end on a ſudden into a more tender white pap, " like the anterior part of the brain: and ſo the nerve continues after " the juncture into the brain filled with a white tender pap, in which " can be ſeen no diſtinction of parts as betwixt the ſaid juncture and " the eye †."

52.

* Meaning the choroides and ſclerotica.

† The object P Q R S, with the pencils of rays ſuppoſed to flow from it, together with the fore-part of the eyes, are here added to the figure in the Manuſcript.

Sir *Iſaac Newton*'s explanation of the manner of viſion with both eyes.

52. "Now I conceive that every point in the retina of one eye, hath "its correſpondent point in the other *; from which two very ſlender "pipes filled with a moſt limpid liquor do, without either interruption "or any other unevenneſs or irregularity in their proceſs, go along the "optic nerves to the juncture EFGH, where they meet either be-"twixt G,F or F,H, and there unite into one pipe as big as both of "them; and ſo continue in one, paſſing either betwixt I,L or M,K into "the brain, where they are terminated perhaps at the next meeting of "the nerves betwixt the *cerebrum* and *cerebellum*, in the ſame order that "their extremities were ſituated in the retina's. And ſo there are a "vaſt multitude of theſe ſlender pipes which flow from the brain, the "one half through the right-ſide nerve IL, till they come at the junc-"ture GF, where they are each divided into two branches, the one "paſſing by G and T to the right ſide of the right-eye AB, the other "half ſhooting through the ſpace EF, and ſo paſſing by X to the "right-ſide of the left eye $\alpha\beta$. And in like manner the other half "ſhooting through the left-ſide nerve MK, divide themſelves at FH, and "their branches paſſing by EV to the right eye, and by HY to the left, "compoſe that half of the retina in both eyes, which is towards the "left ſide CD and $\gamma\delta$ †."

Our author proceeds, and draws the following concluſions.

Why objects appear not double.

53. "Hence it appears, 1. Why the two images of both eyes, make "but one image *abcd* in the brain."

Why objects appear double when one eye is diſtorted.

54. "2. Why, when one eye is diſtorted, objects appear double; "for if the image of any object be made upon A in the one eye, and "β in the other, that object ſhall have two images in the brain at *a* and "*b*. Therefore the pictures of any object ought to be made upon the "correſponding points of the two retina's; if upon A in the right eye, "then upon α in the left; if upon B, then alſo upon β. And ſo ſhall "the motions concur after they have paſſed the juncture GH, and "make one image at *a* or *b* more vivid than one eye alone could "do."

55.

* An hypotheſis ſomewhat like this we have in *Rohault*'s *Phyſicks*, Part I. chap. 31. *Art.* 20. that is, Mr. *Rohault* ſuppoſes with our Author, that the two optic nerves have their correſpondent fibres which unite in the brain; and ſo accounts why objects ſeen with both eyes appear not double. But theſe two authors differ in their anatomy of the optic nerves; *Rohault* continuing them ſeparate, without any union at EF, till the ſympathetic fibres, as he calls them, of each nerve, ultimately meet in the brain in ſo many points.

† The ſubſtance of this elegant hypotheſis, is contained in *Qu.* 15. annexed to our Author's Optics. It is agreed, that the union of the optic nerves at EGFH is ſo cloſe, that their ſubſtances ſeem to be confounded: But notwithſtanding this ſeeming confuſion, ſome anatomical obſervations render it at leaſt very probable, that the ſubſtance of each of the two nerves, continue diſtinct without being any ways mixed or united together. *Vide Edinb. Med. Eſſ.* Vol. 3. pag. 198. We have other hypotheſes by other authors; nor is it a wonder that men ſhould differ in opinions, where they have ſo little light to guide them. However, the reader cannot be diſpleaſed to have Sir *Iſaac Newton*'s opinion upon this difficult ſubject, in his own words.

Why two things cannot appear in one place.

55. "3. Why, tho' one thing may appear in two places by diſtorting "the eyes, yet two things cannot appear in one place. If the pic- "ture of one thing fall upon A, and of another upon α, they may both "proceed to *p*, but no farther; they cannot both be carried on the "ſame pipes *pa* into the brain; that which is ſtrongeſt or moſt helped "by phantaſy will there prevail, and blot out the other."

56. "4. Why, if one of the branches of the nerve beyond the "juncture, as at GF or FH, ſhould be cut, that half of both eyes "towards the wounded nerve would be blind, the other half remaining "perfect *."

Sir *Iſaac* proceeds to account for the ſtructure of the optic nerves, as in the note below †.

57.

* Such an experiment, if it could be made, would very much corroborate the author's hypotheſis; or, what would come to the ſame thing, if upon diſſection it be found, that a perſon who ſaw only at the right or left ſide of each eye, had the nerve cut or otherwiſe rendered ineffectual beyond FH or GF; it would go a great way towards proving a kind of coaleſcence, or union of the fibres of the two optic nerves, ſomewhere between that ſection and the eyes.

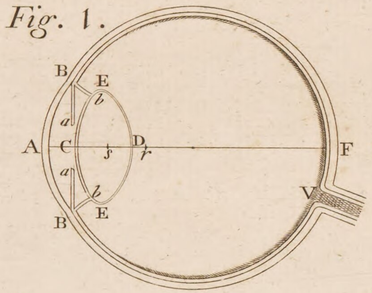
† 5. "Why the juncture is almoſt as broad again betwixt G and H, as between E and F; becauſe all the tubuli of both eyes paſs between G and H, and but half of them betwixt E and F."

6. "Why the nerve GILF buts not directly upon the nerve XEHY, but deviates a little towards TV; becauſe its tubuli are to paſs only into that ſide of the nerve EHXY towards EX. The like of FMKH."

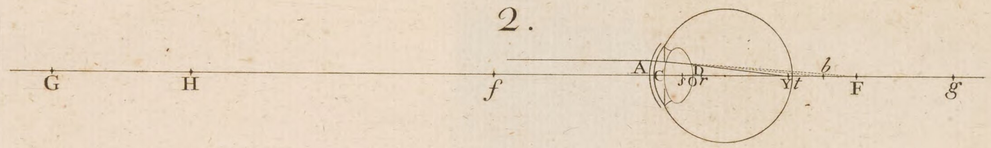
7. "Why the marrow of the nerve TVEG grows ſoft on a ſudden, when it comes at the juncture EF, and more ſuddenly on that ſide towards G than towards E. And the like of the nerve EXYH: For it being neceſſary that the nerve TVEG ſhould be ſtretched and bended ſeveral ways by the motion of the eye; therefore the tubuli are involved or wrought up within the ſubſtances of ſeveral tough ſkins, which being folded up together, compoſe the marrow of the nerve pretty ſolid and flexible, leſt the tubuli ſhould be prejudiced by the ſeveral motions of the nerve. And thoſe ſmall pimples or prominencies which appear in the nerve cut croſswiſe, I conceive to be made by the foldings of thoſe craſſer ſkins. But the nerve at the juncture EGFH, being well guarded from all violence and motion by the bones into which it is cloſely adapted; 'tis not neceſſary the ſaid membraneous ſubſtance ſhould be continued any farther than EG: therefore the tubuli there on a ſudden unſheath themſelves, that thoſe on the inner ſide of the nerves towards VE and XE may ſeverally croſs 'twixt E, F, and be united with their correſpondents on the other ſides YH and TG. Now becauſe the inner tubuli muſt firſt croſs, before they can concur with the outmoſt tubuli of the oppoſite nerve; hence it is, that the nerves grow ſoft ſooner on the inner ſide at E, than on the outer ſide at G and H."

"8. Why the two nerves meet a ſecond time in the brain: becauſe the two half images carried along IL and MK may be united in one compleat image, in the ſenſory. *Note*, that the nerves at their meeting, are round about disjoined from the reſt of the brain; nor are they ſo thick there, as a little before their meeting. But by their external figure, they ſeem as if the *capillamenta* concentered like the radii of a hemiſphere to a point in the lower part of the juncture. And 'tis probable, that the viſive faculty is there: for elſe why do the nerves ſwell there to ſo great a bulk, as it were preparing for their laſt office? Why do they run directly croſs from either ſide the brain to meet there, if the deſign was to have the motions conveyed by the ſhorteſt cut from the eye to the ſenſorium, before they grew too weak. If they were to proceed farther, they might have gone a ſhorter cut, and in a leſs channel. There is indeed a marrow ſhoots from under them towards the *cerebellum*, to which they are united; but the greateſt part of their ſubſtance, if not all of it, lies above this marrow, and alſo ſhoots croſs beyond it to the center of the brain where they meet. Laſtly, the ſubſtance here is moſt pure,

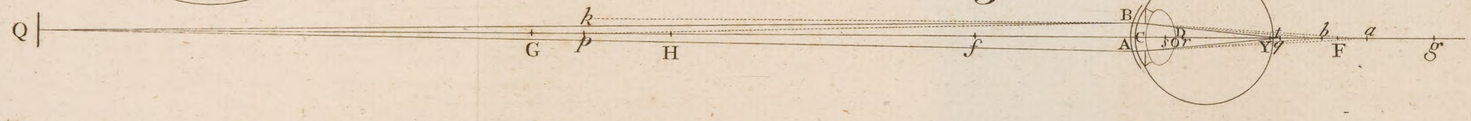
Fig. 1.


B
E
b
a
A
C
s
D
r
F
a
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B
E
V

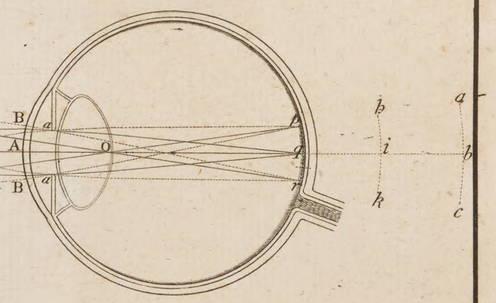
2.


G
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3.


Q
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4.


B
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5.

57. *Defin.* Let *a*AF, *b*BF produced till they meet in F, be the axis of the right and left eyes A, B reſpectively; the angle AFB contained between theſe two axes, is called the *optic angle*. And a plain PR touching the point F, ſo as to be perpendicular to the plain of AFB, and alſo make equal angles with the axes AF, BF, is called the *Horopter*. The angle P*v*R, *fig.* 7. or POR, *fig.* 4. contained between two lines P*v*, R*v*, or PO, RO drawn from the extremities of an object to the focal center of the eye, is called the *viſual angle* of that object; the viſual angle then is that, which any object, however ſituated, ſubtends at either eye of the ſpectator. Fig. 7, 8, 9. Optic angle, horopter, and viſual angle.

58. So far as obſervation can reach, it is univerſally confirmed, that when we look attentively upon any object, the axes of both eyes are directed towards it, or towards that part of it which we more particularly look at; and we are ſo habituated to turn both our eyes towards the ſame place, that if one eye be ſhut, its motion will follow that of the other: As in the figure, the axes of both eyes are directed to the object F, where they both meet and concur. This concurrence of the optic axes, is the reaſon aſſigned by ſome, why an object ſeen with both eyes appears ſingle; it being impoſſible for us to conceive two objects exiſting in the ſame place, at the ſame time. To this it may be anſwered, that the eyes ſee not the concurrence of theſe imaginary axes; and tho' we can make ſome eſtimation of diſtances by ſight, yet it is ſcarce allowable that each eye can determine the diſtance of an object F ſo preciſely, as the caſe before us requires. Whatever be the cauſe of it, we are aſſured of the fact, that when both eyes look at the ſame object, the mind ſees that object ſingle. And it is very certain, that in at leaſt abundance of caſes, the viſive faculty is aſſiſted by a kind of reaſoning, drawn from former experience, tho' perhaps unattended to at the time we uſe it. And whether the phænomenon before us, is owing to experience, or to ſome ſympathetic correſpondence in the parts of the eye, is a very difficult query. But whatever kind of reaſons are admitted, to ſhew why the point F appears not double, may be alſo eaſily enough admitted, to ſhew why the lateral points P, R appear likewiſe ſingle. Another cauſe aſſigned why objects appear not double. Fig. 7.

59.

pure, the ſituation in the midſt of the brain, conſtituting the upper part of that ſmall paſſage 'twixt all the ventricles, where all ſuperfluous humours have the greateſt advantage to ſlide away, that they may not incumber that precious organ."

This elegant account is an inſtance of the great ſagacity and penetration of its author, and very much ſtrengthens his hypotheſis. Our author proceeds next to give his opinion, of the manner that the impreſſions of light upon the retina, are communicated or carried to the brain, as in *Note**, *Art.* 29.

When objects appear double.

59. When the axes of both eyes are directed to ſome determinate point F, an object G within the optic angle, *fig.* 8, or its oppoſite, *fig.* 9, will appear double; that is, inſtead of being ſeen at G, it will appear ſomewhere in the lines *r*G, *s*G; as at *m* and *n*, either farther or nearer than its true place G, according as G is nearer or farther than the horopter PR. And if either eye A or B be ſhut, while the other is kept open, the object *m* or *n* will accordingly vaniſh, and ſo alternately. Whatever hypotheſis be admitted, for ſolving the phænomenon of the ſingle appearance of an object F, to which both the eyes are directed, the ſame will alſo ſerve to ſolve that of the double appearance of G. The images *r*, *s* of the object G, are now made on contrary ſides of the optic axes A*a*, B*b*; and this does not ordinarily happen, but when there are really two objects, as *m* and *n*, on a different ſide of the mark F. Thus, in *fig.* 8, the image *r* being to the right of the axis A*a*, the mind judges the object to be to the left of that axis, as in the like caſe of ordinary viſion; and for the ſame reaſon, the left eye ſees the object to the right of its axis. The ſame argument is alſo applicable to the caſe of *fig.* 9; and generally, as near as can be eſtimated, the apparent diſtances of *m* and *n* from F, in both caſes, are meaſured by the viſual angles FAG, FBG; or the apparent diſtance *mn*, is as the ſum of theſe angles, or as the ſum of the arches *ar*, *sb*. A double appearance may alſo happen, when G is in one of the optic axes, or even without them both: In the one caſe, that eye in whoſe axis G is, will have but one image; and in the other, both the images *r*, *s* will be to the ſame ſide of the axes A*a*, B*b*, but the arches *ar*, *sb*, will be unequal; and the apparent diſtance *mn* will be as the difference of the viſual angles, or of the arches *ar*, *sb*.

Fig. 8, 9.

Seeing two objects, and ſeeing one double, a different ſenſation.

60. The double appearance of an object, does perhaps as frequently happen, as we ſee an object before us that we do not look at, or before we have directed our eyes properly towards it. But we are in no danger of miſtaking one object for two, for the ſenſations we have when we ſee two objects, and ſee one double, are very different: In the one caſe, there are two images as *p*, *r*, fig. 7. formed upon each retina; in the other, there is but one image in each eye, and theſe images being on parts of the retina's which do not correſpond, may be the cauſe of that painful ſenſation we generally have, when we ſee an object double. The two falſe objects are always indiſtinct, have a kind of a tremulous motion, approach nearer together, retreat back, or advance forwards, as the caſe is; and at length, as ſoon as the eyes have conformed themſelves properly for ſeeing the real object diſtinctly, the two falſe objects vaniſh or coaleſce into one, at that determined place where the real object is.

61.

61. Two ſimilar objects, not far aſunder, if they happen to be one in each optic axis, may alſo appear as one object; and that for a like cauſe, that one object may appear double. But this phænomenon ſeldom happens; and when it does, the miſtake is ſoon found, as in the caſe of an object ſeen double; and the eyes preſently adapt themſelves, to the proper diſtance of the two objects.

Two objects may appear in one.

62. When the eyes are directed to a determinate mark, as F, we generally regard only thoſe objects that are pretty near to F, or to the plane of the *horopter* PR, and not thoſe that are remote from that plane. And hence it is, that double appearances of objects do not more frequently happen.

Why objects do not often appear double.

63. An object that is a little out of the plane PR, may yet appear ſingle: But the object will not be ſo diſtinct, as after the eyes have properly conformed themſelves to it; and till then, it will alſo ſhift its place by winking either eye, and looking at it with both; but afterwards, it will remain fixed to either eye alone, or to both open.

An object may appear ſingle, tho' out of the horopter.

64. By an inſpection of the figures, it is manifeſt enough, that a larger ſegment of a round object can be ſeen with both eyes open, than with one eye alone: And the reaſon is likewiſe manifeſt, why when we would bring a third object into a ſtrait line with two given ones, as in *levelling*, *ſhooting*, &c. we muſt take our aim, or look by two of them with one eye only; and to prevent miſtakes, it is beſt to ſhut the other eye, as people in ſuch caſes generally do.

Fig. 8, 9.
Why an aim muſt be taken with one eye only.

65. Perſons that ſquint have not the axes of both eyes directed to the ſame object, and yet they ſee objects ſingle. Squinting may proceed from ſeveral cauſes: one may be, that the correſpondent fibres in the two retina's are differently ſituated, which makes it neceſſary to turn the eyes different ways, that the images of objects may be received upon the correſponding fibres in both: And this alſo accounts, why perſons having ſuch eyes, ſee objects ſingle. It is ſuppoſed, and that with great probability, that there is a ſmall part in the retina of each eye, which is more ſenſible than the reſt; and that we naturally turn our eyes ſo, that the impreſſions of the light may be received upon this moſt ſenſible part: Thoſe who have this part awry, or to one ſide of the optic axes in one or both eyes, neceſſarily ſquint; and yet ſuch Perſons ſee objects ſingle like other men, either becauſe they have learnt by experience, that ſuch is the natural direction of their eyes for looking at the ſame object; or becauſe correſpondent fibres of the optic nerves, proceed to thoſe moſt ſenſible parts in both eyes. It is manifeſt, that ſquinting from either of theſe two cauſes, is neceſſary, and incurable.

Of ſquinting.

Sometimes a cold in the head, a lowneſs of ſpirits, or long poreing with the eyes, will cauſe people to ſquint, that never do at other times. This is a fact I am well ſatisfied of; and the cauſe of it probably is, a kind of ſoreneſs or tenderneſs of the retina, which make the impreſſions of light offenſive, or hurtful; to avoid which, the perſon ſo troubled turns his eyes about different ways, without being able to fix them attentively to any one object: that is, he avoids as much as he can ſeeing at all, and then he ſees but very obſcurely. This diſtemper vaniſhes with the cauſe, and may be remedied generally by uſing green ſpectacles with flat glaſſes, or ſome other contrivance, to leſſen the light coming to the eyes. This ſoreneſs may be a fixt diſtemper in one eye only, and then the perſon troubled with it, turns this eye awry, not for the ſake of ſeeing better with it, but to avoid ſeeing with it at all. People may acquire a habit of ſquinting, eſpecially children, when there is no defect in either of their eyes, by uſing themſelves to look at particular objects with one eye only, till by degrees they acquire the habit of always turning obliquely the other eye. The beſt method of cure in this caſe, is for them to look at ſome object ſtedfaſtly with the diſtorted eye, while the other is ſhut, then try to look at it with both eyes: And ſo repeat theſe trials, till they can look with both eyes at objects in different poſitions. Fowlers can take their aim better with one eye, than the other; and but few people can ſee equally well through a teleſcope with one eye as they can with the other: If they look with the eye they have not been accuſtomed to, they are obliged to ſhut the other, before they can ſee at all. This is entirely owing to practice, as one hand becomes more expert or dextrous than the other.

A circular ſpace at the bottom of each eye inſenſible of light.

66. Monſieur *l'Abbe Marriote* is the author of a curious experiment, by which he diſcovered, that there is a certain ſpace at the bottom of each eye, where the optic nerves enter, which is inſenſible of light. The experiment is to be made in this manner: Upon an obſcure wall, at about the height of the eye, faſten two round pieces of white paper; or if the wall is of a bright colour, let the papers be dark; the papers may be a foot or two, more or leſs aſunder: Then placing your ſelf before one of the papers, ſhut the correſpondent eye, that is, if you ſtand before the left-hand paper, ſhut the left eye, and looking at this paper ſteadily with the right eye; if the other paper is viſible, move ſlowly to or from the paper you look at, and at a certain ſtation the other paper will totally diſappear, tho' objects, that are more remote from the axis of the eye, are viſible at the ſame time. Thus, if the left eye be ſhut, the obſerver muſt place himſelf ſo as to bring the left object F into the axis *a*A of his right eye; then moving to and fro in the line AF, at a certain ſtation the object P will diſappear, that is, when its image falls upon the optic nerve *p*.

Fig. 10.

The paper looked at ſhould be ſmall, that the eye may be the more ſteadily fixed upon it; the other may be two or three or more inches diameter, according as the papers are leſs or more aſunder. Or the experiment may be made with only one paper, and from any ſtation, by turning the axis of the open eye gradually towards the noſe. In this manner, I have loſt a ſegment of a ſteeple, whilſt I could ſee the top and bottom of it.

This experiment, which ſucceeds univerſally, proves undoubtedly that there is a certain ſpace in the eye, which is inſenſible of the impreſſions of light; and this inſenſible ſpace being ſituated about the places where the optic nerves enter, it is very probable that it is ſituated at thoſe very places; or, that the optic nerves, or ſome part of the middle of them, at their entrance into the eyes, are inſenſible.

Situation and magnitude of this inſenſible ſpace determined.

67. In order to form an eſtimate of the ſituation, and magnitude, of the inſenſible ſpot in the eye, Mr. *Short* (well known for the excellent performance of his reflecting Teleſcopes) and myſelf, made the following experiment, thus: Having fixed upon a light wainſcot, at a proper height, a dark-blue circular paper of three inches diameter, and another paper of the ſame colour of about $\frac{1}{8}$ inch diameter, to the right ſide of the firſt, at the diſtance of 30 inches, reckoning from their centers: At a medium of ſeveral obſervations, which agreed pretty near together, looking with the left eye on the ſmall paper, the other being ſhut, Mr. *Short* continued to loſe ſight of the large paper, from the diſtance of 7 feet 8 inches, to the diſtance of 11 feet 2 inches, from the wainſcot; and I loſt ſight of the ſame paper, from the diſtance of 9 feet 4 inches, to the diſtance of 11 feet 2 inches. The ſame papers being placed 18 inches aſunder, Mr. *Short* loſt the biggeſt paper, from the diſtance of 4 feet 9 inches, to 6 feet 5 inches: And being placed $7\frac{1}{3}$ inch aſunder, he loſt the biggeſt at the diſtance of 2 feet 3 inches; but by the leaſt motion from this place, recovered part of it again. A paper of 4 inches diameter, was loſt by Mr. *Short* at the diſtance of 3 feet from the wainſcot, and recovered again at the leaſt motion of the head from this place; the diſtance between the centers of the papers being 10 inches. The paper being placed one diameter higher or lower, was ſeen at all diſtances, which ſhews that the inſenſible ſpot in the eye is round.

Taking a medium of the ſeveral obſervations; the $\angle avp$ contained between the optic axis, and a line paſſing through the focal center of the eye, to the center of the inſenſible ſpot, is in Mr. *Short*'s eye about $15\frac{1}{3}$ *deg*. and in mine about $13\frac{1}{2}$ *deg*. The greateſt ſpace that can be loſt by Mr. *Short*, ſubtends an angle of about 6 *deg*. or the diameter of the greateſt circle that can be loſt by him, is ſomewhat greater than Fig. 10.

the $\frac{1}{100}$ part of its diſtance from his eye. The greateſt angle I can loſe does not exceed 4 *deg*. If we ſuppoſe the diſtance *vp* in each eye to be $6\frac{1}{2}$ *tenths*; the diameter of the inſenſible † hole will be, in Mr. *Short*'s eye about 0.68 *tenths*, and in mine about 0.45 *tenths*.

Why we are not ordinarily ſenſible of this defect in our eyes.

68. In looking with one eye, at a large object of an uniform colour and ſurface, we are not ſenſible of this defect, or of the loſs of one part: The reaſon may be, becauſe objects near the place where the defect of viſion happens, being remote from the optic axes, are but obſcurely ſeen; and ſo, a particular part is not miſſed, till it becomes a remarkable object, by its being, as it were, detached from the reſt by a diſtinct figure and a different colour: Nor in viewing objects of uniform ſurfaces and of like colours, do we uſually attend to their parts, but take the whole figure in the lump. To theſe reaſons may be added, that we ſeldom fix our eyes long to one point, but we nimbly, and inſenſibly, turn them towards different parts.

An object cannot be loſt to both eyes at once.

69. It is manifeſt, that an object cannot be loſt to both eyes, at the ſame time; for when the image of an object falls upon the optic nerve in one eye, it will fall at ſome diſtance from it, in the other. See *fig*. 7.

Objects appear a little brighter when ſeen with both eyes than with one.

70. An object ſeen with both eyes appears a little brighter, or more luminous, than it does when ſeen with one eye alone; as will be evident, by looking alternately with both eyes, and with one only: And the difference of brightneſs will be ſtill more manifeſt, if at the ſame time that a part of a flat object, of an uniform colour, is ſeen with both eyes, the light from the adjacent part is excluded from one of them; which may be done by applying a book to one ſide of the head, and ſo as it may reach a little forwarder than the face. But although the difference of brightneſs, in the two caſes, is very perceptible, yet is it not very conſiderable; nor is it eaſy to determine it accurately. Dr. *Jurin*‖ has given a pretty method of making this experiment, thus: In a room illuminated only with two candles, placed at about the ſame height, and kept burning as near as may be with equal brightneſs, fix a ſlip of white paper flat againſt the wainſcot, at about the ſame height with the flame of the candles, as they ſtand upon the tables: Having placed one of the candles at a determinate diſtance from the paper, as a yard; place the other at a greater diſtance, as ſuppoſe two yards, and a little ſide-ways of the firſt; then place a book, or ſome ſcreen, ſo before this hindermoſt candle, as to prevent its light from falling

† M. *Marriote*, from the bigneſs of this hole, thinks that viſion is performed in the choroides, and not in the retina; and gives ſeveral ingenious arguments to ſupport his hypotheſis: But therein he has had but few followers, and the argument drawn from the obſervation of the nerves being the organs of all our other ſenſations, makes againſt him. But after all, this queſtion ſeems too difficult, to be determined with certainty.

‖ Dr. *Smith*'s Optics, Remarks p. 108.

falling upon one half of the paper. In this caſe the light of the firſt candle being reckoned 1, the light of the ſecond candle, which is at twice the diſtance from the paper, will be only $\frac{1}{4}$; and therefore the light upon that half of the paper which is illuminated by both candles, is to the light of the other half illuminated by one alone, as 5 to 4. Things being thus diſpoſed, apply a book to one ſide of the head, ſo as to hide that half of the paper, which is illuminated by both candles, from one eye; and looking at the paper, if both halves of it appear equally luminous; the degree of brightneſs of an object ſeen with both eyes, and with one eye alone, will be as 5 to 4. But if, as will be the caſe to moſt people, the half illuminated by both candles appears brighter, than the other half of the paper; remove the ſecond candle ſtill farther and farther, till by repeating the experiment, the ſcreen being placed before the ſecond candle, and to one of your temples, as before directed, the two halves of the paper appear equally luminous: Then compute as before; as if the ſecond candle is diſtant three yards, and the firſt one yard, the light of the brighteſt half of the paper will be to the light of the other, as $1\frac{1}{9}$ to 1, or as 10 to 9; and that will be the proportion of the brightneſs of an object ſeen with both eyes, to its brightneſs when ſeen only with one.

71. The ingenious author above quoted, found that an object ſeen with both his eyes, appeared brighter than when ſeen with one eye alone, by about a thirteenth part. By this experiment I find, that my eyes are not both equally good, for the difference will be greater or leſs as I exclude the light from one or t'other eye. But at a medium, I gueſs the difference above to be about $\frac{1}{10}$ or $\frac{1}{12}$, it being very difficult to make the experiment to great exactneſs.

Objects appear of the ſame bigneſs to both eyes, and to one alone.

72. In viſion with both eyes, and with one eye alone, the apparent magnitudes of objects are, as near as can be perceived, the ſame in both caſes. Of the truth of this any one may eaſily ſatisfy himſelf, either by looking alternately at the ſame object with both eyes, and with one only; or clapping a book to his temple, as in the preceding experiment, and looking at two or three parallel lines, placed ſo before him as to be equidiſtant from both his eyes; if the lines appear ſtrait throughout, it is a proof that the ſpace between them, appears of equal breadth to both eyes, and to one alone.

Advantage of both eyes.

73. It may be wonder'd, that there ſhould be ſo little difference betwixt viſion with both eyes, and with one only. Perhaps we beſtow a greater attention in one caſe, than what ſuffices in the other; and that this may be the reaſon, of the ſo little difference in the apparent brightneſs: And the idea of the diſtance of the object, being previouſly fixed

fixed in the mind, looking at it with one eye, or with both, does not, it ſeems, alter this idea; which may be the reaſon, of the apparent magnitude being the ſame in both caſes. Theſe are only conjectures; however, the fact is rather advantageous to us, than otherwiſe; that if one eye be hurt or loſt, the other may, as it were, perform the office of both, with very little diſadvantage.

Field of viſion.

74. *Defin.* That conical ſpace indefinitely extended before us, and ſpreading ſo wide, as to include all objects as can be ſeen at one view, or without turning the eyes, is called the *Field of viſion.*

Its extent.

75. It is not eaſy to aſcertain the preciſe limits of the field of viſion, or the greateſt angle that can be ſeen at one view. When we look attentively at a ſmall diſtance before us, we have an imperfect glimpſe of objects for almoſt the ſpace of half a ſphere, or at leaſt for above 60 degrees each way from the optic axes. But towards the extremity of this ſpace, objects are ſeen but very imperfectly; and they are gradually diſtincter, the nearer they are to the point we look at. About this point there is ſome ſmall ſpace, that ſeems at the ſame time equally diſtinct; but to aſcertain exactly the quantity of this ſpace is difficult, becauſe the indiſtinctneſs round it, is ſo very gradual: And this difficulty is increaſed by that of keeping our eyes fixed for any time, on the ſame object; we are apt to move them nimbly, even when we are not conſcious of ſo doing, over the circumjacent objects, and the impreſſions upon the retina[a], having ſome duration, we thence imagine the field of diſtinct viſion to be much larger than it is. But, I think, the diameter of this field does not ſubtend an angle of above five degrees at moſt; whence the diameter of a diſtinct image upon the retina, is leſs than .06 of an inch. A ſmall object, as a ſtar ſeen by twilight, is loſt by a ſmall motion of the eye from it; and if we look attentively upon a letter in the middle of a word, conſiſting of capitals of a middling ſize, the adjacent letters on either ſide will be ſomewhat indiſtinct. Theſe are proofs that the field of diſtinct viſion is very ſmall, and probably it does not exceed one degree, or two degrees at the moſt. The limitation of this field may be owing to the retina's being more ſenſible near the optic axes; or, becauſe the points of union of the rays of the oblique pencils fall ſhort of the retina; or both theſe cauſes may conſpire together.

[a] 35. II.

76. To me there is no ſenſible difference in the bigneſs of the field of diſtinct viſion, when I look with either eye alone, and when I look with both together; only the field is a little brighter in one caſe than in the other.

SECTION

SECTION II.

Of the minimum viſibile; *diſtinct and indiſtinct viſion; of long and ſhort-ſighted eyes, and of ſpectacles; of apparent diſtances, and magnitudes.*

§. 1. *Of the extent or limits of viſion.*

77. AS in a dark chamber, a very ſlender beam of light is perceptible, ſo in all caſes, when the medium about us is very dark, objects are perceivable by ſmall quantities of light; and the darker is the medium, or the freer the eye is from extraneous light, the leſs light ſuffices for viſion. Thus, in a dark night, the feeble light of a candle is perceptible at a great diſtance; and the fixed ſtars, though they have no ſenſible diameters, are yet very viſible; and the darker the night, the more of them are ſeen: The eye being now free from other impreſſions, the mind perceives, and attends to, theſe delicate ones of thoſe very ſlender pencils. But even in this caſe, a certain quantity of light is requiſite for viſion, without which objects cannot be diſcerned. The impreſſions of light from the Satellites of *Jupiter* and *Saturn*, are too feeble to be perceived without the aſſiſtance of teleſcopes, although then theſe Satellites have ſenſible diameters; and a ſmall flame will not viſible beyond a certain limited diſtance. Luminous objects by night.

78. At the approach of day, and as the twilight increaſes, the eye begins to be enlightened by the reflection of the atmoſphere, and then gradually more and more; ſo the ſtars grow fainter, and diſappear by degrees; firſt the ſmaller ones, and ſo on. As the day advances, the moon herſelf loſes of her luſtre, and at length her light is overpowered by that reflected from the clouds, and ſhe is no longer ſeen. Thus, weaker impreſſions upon the retina, are obliterated and rendered inſenſible, by thoſe that are much ſtronger; and this is an eaſy explication of the above phænomena, and of others of the like kind. Very ſmall particles are ſeen floating in a beam of light let in to a darkned room; but as ſoon as the room is enlightened, theſe particles diſappear. A man in a darkiſh room eaſily ſees thoſe that are in broad day-light, but cannot be ſeen by them; the light reflected out from the room, compared to the open day-light, being inſenſible. Stars, why inviſible by day-light.

79.

Objects distinguished by their diversities of colour and splendour.

79. The light of the ambient medium, constantly intrudes into the eye, along with the pencils of light proceeding from bodies, having sensible magnitudes; but such bodies, if they are not too far off, are nevertheless very perceptible, as well by their different colour and splendour, or by the different degrees and sort of light reflected by them; and also by their intercepting the light of the ambient medium. A shade becomes an object, or objects are perceivable by their want of light, as a hole in a board, or a dark spot on a light body, are perceived by the unenlightened chasm in the eye, within the illuminated field; and the figure and magnitude of the hole or spot, are distinguished by those of the said chasm. So a steeple afar off is distinguished by its covering a part of the sky, of such a figure and apparent magnitude.

Opacity of the atmosphere, a cause of indistinctness and invisibility of remote objects.

80. It is manifest from many observations, that there are heterogeneous particles constantly floating in the atmosphere, and that these by their opacity and reflexibility, make a kind of veil that obscures the vision of remote objects: And the more the medium is loaded with these particles, and the more remote an object is from the spectator, the more indistinct and obscure it will be, and the less will be the latitude of vision, or that distance beyond which no objects can be perceived. Every one knows that objects at a given distance, are more distinct, and others are visible at a greater distance, in clear than in foggy weather. And early in a clear morning, when the air is pretty free from vapours, and not much enlightened, a hill or head-land is visible at a great distance, but as the day advances, the land becomes more obscure; and so by degrees, by the greater opacity of the long tract of vapour between, and the greater light reflected by it to the eye, the object becomes less and less perceptible, and at length totally disappears.

Another cause that limits the extent of vision.

81. Another cause of the invisibility of objects, and to remove which optical instruments are more properly adapted, is their smallness in proportion to their distances. For, excepting in the cases of luminous objects seen in the dark, an image upon the retina, to become perceptible, must not be less than some determinate magnitude. To prove this, we do not want for instances: A house may be seen at a considerable distance; but we must approach nearer, before the windows are discernible; and nearer still, to discern the bricks and tiles. At a certain distance, I see a white paper stuck to the wall; coming nearer, I perceive it is printed; at a nearer appproach, I can read the large letters; and at length the smallest letters are legible.

The *minimum visibile*.

82. The quantity of the *minimum visibile*, or the angle subtended by the least visible object, cannot very precisely be ascertained; but in order to form some estimate thereof, I made the following experiment. A parallelogram

parallelogram one inch long and half an inch broad, drawn upon clean white paper, I divided into little ſquares, whoſe ſides were each $\frac{1}{10}$ of an inch; then blacked every other ſquare, very black, with ink. This chequer'd paper being ſtuck to the wainſcot, I retreated a good diſtance from it, and then advancing ſlowly, I began to diſcern ſome imperfect ſpecks of black and white, at the diſtance of 25 feet; and therefore from this place, the ſides of each ſquare ſubtended an angle of $1' 9''$ nearly. But the ſquares were not defined and perfect, till I came within 7 feet of the paper; and therefore I could not ſee them diſtinctly, till they each formed angles of $4' 5''$. I placed by the firſt another white paper, which was divided into ſquares of the ſame dimenſions, as above; and the alternate ſquares, inſtead of being very black, were only tinged lightly with *Indian* ink: The different colours of the ſquares upon this paper, were not perceivable at a greater diſtance than 16 feet; but at the diſtance of 7 feet, they were as well defined, as in the former paper.

Upon the ſame paper, was drawn a black ſquare by it ſelf; a white ſquare within a black circle; a ſquare, and a circle, with *Indian* ink; each the $\frac{1}{10}$ inch as before: Alſo a black ring and parallelogram, each of the ſame breadth as above.

The white ſquare within the black field, and the black ſquare, were each perceivable as ſome white and black ſpeck, at the diſtance of 40 feet; but were not well defined, till I came within 7 or 8 feet. The ſquare ſhaded with *Indian* ink, and the white one within the circle ſhaded with that ink, could not be perceived at a nearer diſtance than 25 feet; but were well defined at the ſame diſtance with the reſt. The black ring and parallelogram, could be perceived at a greater diſtance than I had room to retreat, which was 51 feet: But a parallelogram of the ſame breadth, lightly ſhaded with *Indian* ink, became inviſible at the diſtance of 40 feet. The papers were placed in an advantageous light, and the experiments ſucceeded, much the ſame, to other ſpectators.

A ſimple object is perceivable under a leſs angle, than the parts of a compound one.

83. Hence it appears, that a ſimple object, as the white or black ſquare upon the oppoſite colour, can be ſeen under a leſs angle than the parts of a compound object, as the chequered paper; and that this leaſt angle, in moſt caſes cannot be leſs than $40''$; and at a medium, I think, it is not leſs than 2 minutes. But by the above experiment, it appears that ſeveral contiguous objects, are ſcarce diſcernible one from another, unleſs they each ſubtend angles that are not leſs than about 4 minutes. The difficulty of keeping the eye perfectly ſteady, may be one cauſe why a ſimple object can be diſcerned under a leſs angle, than the parts of a greater one; and it is very natural to ſuppoſe, that the fewer objects we contemplate, and the more they differ in colour, the eaſier

we can diſtinguiſh their ſeveral impreſſions upon the retina. But there is no aſcertaining exactly, the leaſt angle under which an object can be ſeen; for the ſame object ſhall be viſible at a leſs or greater diſtance, as the circumſtance of light is different.

84. Some have imagined, that the impreſſions of viſible objects, are received upon *villi* of the optic nerve, ſtanding erect upon the retina, like the pile upon cloth or velvet; and that the *minimum viſibile* is determined by the angles ſubtended by theſe *villi*, at the focal center of the eye. There is no difficulty, in ſuppoſing the retina to be covered with ſenſible *villi*, having inſenſible pores between them: But from the above argument, it would ſeem, that the *minimum viſibile* ſhould be the ſame in all caſes, which is contrary to experience. But, however the retina is conſtituted, it is manifeſt that the cauſe of the non-viſibility of objects, is the want of ſufficient light in the pencils proceeding from them; and that more or leſs light is requiſite for viſion, according as the eye is more or leſs illuminated.

A long ſlender object, is viſible under a ſmaller angle, than a ſquare object of the ſame breadth.

85. Upon a white paper, I drew black lines or parallelograms of different lengths, and each $\frac{1}{120}$ inch broad; and the paper was paſted to the wainſcot, ſo that ſome of the lines were vertical, and ſome horizontal. The longer lines were viſible at the diſtance of 50 feet: A line $\frac{1}{4}$ inch long was juſt perceptible, at the diſtance of 45 feet; and one of half that length, was viſible at no greater diſtance than about 20 feet. I was not ſure of any difference, in the appearance of the vertical and horizontal lines, of the ſame length; but was inclined to think, that the vertical lines were the diſtincteſt. Two parallel black lines, with a white line between them, each the $\frac{1}{120}$ inch broad, did not appear ſeparate at a greater diſtance than 20 feet. In like manner, a piece of fine thread or wire is viſible at ſuch a diſtance, that a globulous object of the ſame diameter is totally inviſible.

A ſlender object, as a phyſical line, may be conſidered as conſiſting of ſeveral ſquares joined together; and though one of theſe ſquares, or a parallelogram conſiſting of two or more of them, may be too ſmall to be ſeen; yet the pencils of light coming from each of them, being contiguous and ſtriking at the ſame time upon the retina, are capable by their united ſtrength to awaken the viſive faculty, and ſo to render viſible the object from whence they came; juſt as in mechanics, the united force of many agents will have its effect, when the effort of a few can avail nothing. And from what experiments I have made, I am inclined to think, that the areas of the leaſt viſible objects are pretty much the ſame, though they differ in figure, all other circumſtances being

being the ſame alſo; that is, the area of the leaſt viſible line, whoſe length does not extend without the field of diſtinct viſion, is nearly equal to the area of the leaſt viſible ſquare or circle; and indeed this ſeems natural enough to be imagined. The phænomenon of the two black lines, with the white line between them, is analogous to that of the chequered table before explained.

86. *Schol.* The ſhadows of ſmall bodies, as hairs, wires, threads, or ſcratches in poliſhed glaſs, *&c.* placed in a beam of the ſun's light, will be much broader than they ought to be, if the rays of light paſſed by theſe bodies in right lines: And a beam of light, let through a ſmall ſlit not above the $\frac{1}{400}$ of an inch broad, will be ſplit into two, ſo as to have a dark ſpace or ſhadow in the middle. The cauſe of both theſe phænomena is, undoubtedly, an inflection of light out of its rectilineal courſe, at or near the ſides of bodies, in its paſſage by them *. Sir *Iſaac Newton* hath alſo ſhewed, that light which differs in refrangibility, differs likewiſe in flexibility.

It has been obſerved, that ſmall wires or threads, placed on or before white paper, or ſuſpended in the air in certain ſituations with reſpect to the light, will be viſible under very ſmall angles, as only 2 or 3 ſeconds: But in theſe caſes, is not the light in paſſing by the ſides of the wire, inflected ſo as to paint upon the retina, the images of theſe ſides much wider aſunder, than according to the angle which the wire ſubtends at the eye? And ſo an image is formed in the eye, equal to that of a *minimum viſibile* in other caſes; or an image that ſubtends, at the focal center of the eye, an angle not leſs than a minute: and if ſo, then in the preſent enquiry, theſe extraordinary inſtances are not to be regarded.

87. A ſmall object in motion is eaſier diſcerned, than if at reſt; and may be viſible in the one caſe, though it would not be ſo in the other. A ſmall ſtar by day or twilight, that cannot be eaſily ſeen through a teleſcope directed to it, will be the ſooner perceived, if the teleſcope be a little ſhaked or moved; but after it is once diſcovered, it will continue very perceptible.

A ſmall object, the eaſier diſcerned when in motion.

The reaſon of this phænomenon, ſeems to be much the ſame, with that of the viſibility of a long ſlender object. The image of the ſtar upon the retina, although too feeble to be perceived, while the eye is at reſt, and the point where it falls is not yet diſcovered; yet by its gradual motion over different parts, and the impreſſion upon each having [a] ſome duration; theſe ſeveral impreſſions being all joined together, before the firſt is obliterated, they form a kind of a linear image,

[a] 35. II.

* See *Newton*'s Optics, Book III. Part I.

which being once perceived, we quickly diſcover the ſtar; and having now found its true place, we afterwards keep ſight of it with eaſe. In like manner, a ſmall hair ſhaked by the air, is viſible at a greater diſtance, than the hair at reſt could be ſeen.

Extraordinary quickneſs of ſome eyes.

88. There are great differences to be obſerved, in the degrees of quickneſs, or ſenſibility, of the eyes of different people; ſome having them more acute, and others leſs ſenſible than common. We have heard of perſons ſeeing a ſatellite of *Jupiter*, without the aid of glaſſes, which to many ſeems incredible. But when we conſider, how much the various circumſtances of light affect viſion, and how much farther our viſion is extended at ſome favourable opportunities, than it is at other times; the extraordinary accounts above, may the eaſier gain belief.

Neareſt diſtance of diſtinct viſion.

89. If an object be held too cloſe to the eye, it becomes indiſtinct; and the more ſo, the cloſer it is held, notwithſtanding its apparent magnitude is thereby increaſed; and a very ſlender object will become totally inviſible. To the generality of eyes, the neareſt diſtance of diſtinct viſion is about 7 or 8 inches: At this diſtance, they commonly read a ſmall print, and examine all ſorts of minute objects, as the diviſions upon a plotting ſcale, the fineneſs of the threads of linnen, *&c.* It is true that ſome eyes can ſee ſmall objects beſt, at the diſtance of ſix, four, or even three inches; and ſome again, at no leſs diſtance than twelve, fifteen, or twenty inches: But theſe are uncommon caſes, and do not affect our preſent enquiry.

Smalleſt viſible object.

90. Hence, and from *Art.* 83, a globular object that is leſs than $\frac{1}{6\,5\,0}$ inch in diameter, is, to the generality of eyes, totally inviſible: And excepting in few caſes, an object cannot be ſeen that is leſs than $\frac{1}{4\,3\,0}$ inch in diameter; an object of that breadth ſubtending an angle of 1 *min.* at the diſtance of 8 inches from the eye. But an object, placed on a field differing inſenſibly from it in colour, is not perceivable under a leſs angle than about 4 *min.* and in ſuch circumſtance the ſmalleſt viſible object, is not leſs than about $\frac{1}{1\,0\,0}$ inch in diameter. At a medium, the breadth of the leaſt globular object that is diſcernible by the naked eye, is perhaps about the $\frac{1}{3\,0\,0}$ of an inch.

91. Thus, we ſee naked viſion is limited within certain boundaries, and that by various cauſes. Remote objects become inviſible, as houſes, trees, cattle, *&c.* becauſe of their ſmallneſs, in proportion to their diſtances; and others, tho' they ſubtend angles ſufficiently big, as mountains, *&c.* are viſible or inviſible by turns, as the circumſtances of light vary. Very minute objects, again are never viſible to the naked eye, becauſe we cannot approach them ſufficiently near, to increaſe their magnitudes.

§. 2. *Of diſtinct and indiſtinct viſion.*

Objects when bright and obſcure, diſtinct and indiſtinct.

92. *Defin.* An object is ſaid to be *bright* or *clear*, when the eye receives from it a ſufficient number of rays; and *obſcure* or *dark*, when it receives too few, or when the light is faint and weak. And an object is ſaid to be *diſtinct*, or *ſeen* diſtinctly, when its out-lines appear clear and well defined; and the figures, ſizes, and colours, of its ſeveral parts, if not too ſmall, are plainly diſtinguiſhable, ſo as they may be eaſily compared one with another. It has been already obſerved, that in order to obtain diſtinct viſion, it is requiſite that the object be ſufficiently bright, or have a diſtinguiſhable colour; and that the ſeveral pencils on their falling on the retina, ſhould be ſeparate, and not mixed one with another: And when this is not the caſe, the out-lines of the object, and likewiſe its parts, will appear faint, hazy, and ill-defined; and this is called *indiſtinct*, or *confuſed* viſion. Juſt as in the common *camera obſcura*, the pictures of the objects without, will be more or leſs diſtinct, as the objects are more or leſs illuminated; and as the paper-ſcreen is nearer to, or farther from the focus's of the reſpective pencils: So in a duſkiſh light, the letters of a book will be confuſed and not legible; and in a clear light they will be alſo confuſed, if the book be held too near the eyes.

Chief requiſites for diſtinct viſion.

93. From the above general obſervations, it appears, that in order to have diſtinct viſion, the following are the chief requiſites:

1. The objects ſhould be ſufficiently illuminated: And all other things being the ſame, the nearer is an object, and the brighter is its colour, the more light the eye receives from it; and this is one cauſe, why near objects, are ſeen more diſtinctly, than thoſe that are remote.

2. The geometrical images of objects, ſhould either fall upon the retina, or very near it: And theſe images ſhould be ſufficiently large, otherwiſe the parts of the object cannot be diſtinctly perceived; and this is another cauſe of the indiſtinctneſs of remote objects.

3. A good conſtitution of the eye is a requiſite, without which the others will have leſs effect; the more tranſparent are the cornea, and the humours of the eye, the more light will be tranſmitted upon the retina, and the more lively will the impreſſions there be; and the more tender and ſound is the retina, and alſo the whole optic nerve from thence to the brain, the more diſtinct will the impreſſions of light from objects be perceived.

When all the above requiſites conſpire together, viſion is the moſt diſtinct; and it will be the leſs ſo, as one or more of theſe is leſs perfect.

94.

94. In a given eye, and a given diſpoſition of that eye, an image upon the retina will be moſt perfect, when the object is at ſome determinate diſtance from the eye; and it is at about that diſtance therefore, that objects, if they are not too ſmall or obſcure, will be ſeen moſt diſtinctly: An object at a leſſer, or a much greater diſtance than this, will have its image accordingly either behind, or before the retina; and in both caſes, if the diſtances of the images from the retina be conſiderable, viſion will be indiſtinct. See *fig.* 12, 13, 14. in which *q* is ſuppoſed the focus of rays, diverging from a point Q, after their refraction by the humours of the eye.

Diſtinct viſion, when the images are not preciſely upon the retina.

95. The chief requiſites for diſtinct viſion, have been already [a] conſidered: But in order thereto, it is not neceſſary that the images of objects, or the points of union of the rays in the ſeveral pencils, be preciſely upon the retina; there being ſome ſmall latitude both before and behind the retina, within which, wherever the images are formed, viſion will be equally diſtinct; and this latitude will be greater or leſs, according as the viſual angles ſubtended by the reſpective objects, are greater or leſs. Let a printed page, in which there are letters of three or four different ſizes, be placed at ſuch a diſtance, that every ſort of print may, without any ſtraining or effort of the eye, appear perfectly diſtinct: In this caſe it may reaſonably be preſumed, that the images of the ſeveral letters do fall upon the retina: If the printed leaf be brought gradually nigher and nigher, the ſmalleſt print will firſt begin to be confuſed, whilſt all the larger ſort are yet as diſtinct as before; and by advancing nearer, the ſmalleſt print will be more confuſed, the next ſize above that will be a little confuſed, whilſt the larger print is ſtill as legible as at firſt; and ſo through ſeveral degrees, till at length the whole is in confuſion. The experiment may be made the contrary way, by uſing a pair of ſpectacles of a proper convexity: Thus, firſt place the paper at ſuch a diſtance, that all the ſeveral ſorts of print, may appear as diſtinct as poſſible; then moving the paper farther and farther, the ſmaller prints will, as before, one after another, according to the ſeveral ſizes, become confuſed, whilſt the larger ſtill preſerve their diſtinctneſs. By the two ſorts of experiments, it is manifeſt, that we may have diſtinct viſion when the focus's of the pencils are at ſome diſtance, either beyond or before the retina; and alſo that the larger the object, the greater will be the latitude the image has to move from the retina, before we can be ſenſible of any indiſtinctneſs *.

[a] 30 & 93. II.

96.

* It has been thought by others, that a preciſe union of the reſpective rays upon the retina, is not neceſſary to diſtinct viſion; but, I think, the firſt author that publiſhed the above obſervations, which, ſeem to be deciſive, and to ſet this affair above all doubt, is Dr. *Jurin*, in the *Eſſay* already quoted.

96. Let BAC be an object, whose image after refraction by the humours of the eye, is *bac*: Now if the image *bac* coincides with the retina, the object BAC must needs appear as distinct as it is possible; and so must also all objects that are at the same distance, if they are sufficiently bright; the image of every physical point, being also a physical point upon the retina. But if *fg* or *hk* be the retina, that is, if the distance of the object and the constitution of the eye be such, that the image falls beyond the retina, as when objects are too near; or before the retina, as when the rays have not a sufficient divergency; in either case, the rays of each pencil will occupy a circular space upon the retina. This circle may be called the *circle of aberration*; and all other circumstances being alike, the said circle will be greater or less, according to the greater or lesser distances from the retina, of the focus's of the refracted rays Fig. 11.

Circle of aberration.

97. From *Art.* 95, it is manifest, that a circle of aberration causes no perceptible difference in the distinctness of vision, while that circle is not greater than some determinate magnitude; but as soon as it exceeds that magnitude, we begin to perceive an indistinctness, which increases as the circle of aberration increases, and at length the object is lost in confusion. The greatest circle of aberration, that causes no perceptible indistinctness, is also greater or less, according to the greater or less magnitude of the visible object: It is difficult perhaps, to assign the diameter of the said circle; but it seems very probable, that vision continues distinct for all such distances, or so long, as the circles of aberrations of the least visible objects, or the pencils of light from them, do not touch one another upon the retina; and that indistinctness begins, when the said circles begin to interfere. By the least visible object here, is meant the least object we regard at the time, and not the least object that we are capable of perceiving. To a person that reads, the thickness of a letter is properly then his *minimum visibile*; and so long as the pencils of light from the several letters are not mixed upon the retina, so long he reads with ease, and the letters appear distinct. The larger are the least visible objects, the larger will be the circles of aberration before they touch; and hence it is, that a larger print is legible at a nearer distance, than a smaller.

Greatest circle of aberration, that impedes not the distinctness of vision.

Let the visual angle of the least visible part of an object, be equal to the angle AEC, and let *bac* be the retina: Now, upon the above supposition, the object will continue distinct at all such distances, as its image will fall between *hk* and *fg*; for between *hk* and *fg* the pencils of light proceeding from the points A, C do not touch one another. If the focus's of the pencils are farther from the retina than *hk* or *fg*, the object will be indistinct, and the indistinctness will be the greater, according Fig. 11.

according to the greater diſtance from the retina of the ſaid focus's. If the leaſt viſual angle be much leſs, as the caſe generally is, than the angle AEC, as the $\frac{1}{2}$ or the $\frac{1}{4}$, *&c.* of AEC; then will the limited diſtance of a diſtinct image, be accordingly the $\frac{1}{2}$ or the $\frac{1}{4}$, *&c.* of the diſtance between *hk* and *fg*.

Why a ſingle object ſhall be diſtinct, when ſeveral objects are in confuſion.

98. Hence we have the reaſon, why a ſingle object ſhall be viſible at ſuch a diſtance, that the parts of another object, each as big as the ſingle object, ſhall be in confuſion; as the caſe of the ſingle ſquare, and chequer'd table, *Art.* 82.

99. Perhaps the humours of the eye cannot, in any caſe, perfectly unite, into a point, all the rays in one pencil; but that there will always be ſome aberration from the focus, as in the caſe of glaſs lenſes: If this be the caſe, it may be a reaſon why the *minimum viſibile* is limited to a particular angle; and there will be no need of ſuppoſing the retina to be covered with *villi* of a certain ſize, but the whole ſurface of it may be equally ſenſible.

Viſion equally diſtinct within a certain limit.

100. But whatever it is that limits the *minimum viſibile*, it ſeems very probable, as has been before obſerved, that in all caſes viſion will be diſtinct, ſo long as the circles of aberrations upon the retina, are not greater than the geometrical image of the *minimum viſibile*: And it is alſo very probable, that in all ſuch caſes, viſion is equally diſtinct; for within the above limits, the pencils of light from the leaſt viſible parts of the object, are ſeparate one from another; and the quantities of light received upon the retina, are alſo the ſame. The principal, and may be the only, perceptible difference here will be, that the nearer the geometrical image is to the retina, the ſmaller will be the *minimum viſibile*, or the angle ſubtended by the leaſt viſible object.

Natural diſtance of diſtinct viſion.

101. It is difficult to aſcertain preciſely, the natural diſtance of diſtinct viſion; or that diſtance, at which the eye when in a ſtate of reſt, without any ſtraining or effort of any of its parts, is ſuited to ſee objects diſtinctly. If we ſuppoſe this diſtance to be that, at which we uſually read a fair large print; this to the generality of eyes will be about 15 or 16 inches: And leſs than that, I think, it cannot be; for we are rather more concerned, with objects that are bigger than the letters of a book, than we are with thoſe that are leſs; and when we view objects nearer, it is on account of their ſmallneſs, that they may ſubtend larger angles: Nor is it probable, that the ſaid diſtance is many feet; for we chuſe to view objects pretty near, excepting when they are very large, as buildings, *&c.* and then we retreat to a greater diſtance, for the ſake of comprehending more of them at one view. I am inclined to think that to moſt eyes, the natural diſtance of diſtinct viſion is about 2 or 3 feet.

By the preceding [a] articles, it appears, that the ſame conformation of the eye, will ſuit for diſtinct viſion at different diſtances, within ſome certain limits; excepting perhaps only in that caſe, when the object we look at is a *minimum viſibile*.

[a] 95 *to* 100.

II.

The eye changes its conformation, for viſion at different diſtances.

102. None can be ignorant, that when ſeveral objects are at different diſtances before us, they will not all appear equally diſtinct, at the ſame time. But, leſt it ſhould be ſuſpected, that the indiſtinctneſs in this caſe, may be owing to the impreſſions not being made upon correſpondent fibrils of the two retina's; let us make a trial with one eye alone, while the other is ſhut: Thus, place two ſmall objects, as two pins, one behind the other; and let the one be at about a foot, and the other at about ſix foot diſtance from the eye: Either of theſe objects that we look at, will appear diſtinct; but the other, at the ſame time, although it be in the axis of the eye, will be confuſed. And this, I think, is an undeniable proof, that the ſame conformation of the eye is not ſuited for diſtinct viſion at all diſtances; and alſo that the eye ſome how changes its conformation, ſo as it may be the better ſuited for viſion at different diſtances. In like manner as above, if the eye looks attentively upon the little ſcratches, or particles of duſt upon a window-glaſs, the objects without doors will be indiſtinct; and when we look at the external objects, the opaque particles upon the glaſs, which before were diſtinct, will now be confuſed. It frequently happens, that when we firſt look at an object, it will appear very confuſed, which confuſion will vaniſh by degrees, and preſently the object will become quite diſtinct: And how can this phenomenon be better explained, than by ſuppoſing that at firſt, the eye was not properly adapted for viſion at ſuch a particular diſtance, and that afterwards, a ſuitable change was wrought in it accordingly.

103. In like manner, if after poreing ſome time on a book, we ſuddenly look at objects farther off, they will at firſt appear confuſed, and become diſtincter by degrees: The like indiſtinctneſs happens, when from looking at remote objects, we ſuddenly look at one that is near. This phenomenon will happen, when both objects are in the ſame room, and have the ſame degree of light; and what elſe can this be owing to, but that the eye changes its conformation for viſion at theſe different diſtances, and takes ſome ſmall time ſo to do? It cannot be altogether owing, to the laſt impreſſions on the eye being yet not quite obliterated; otherwiſe the ſame confuſion would be obſervable, upon ſhifting the eye from one page of a book to the other, which does not happen, at leaſt in the ſame degree. Theſe phenomena are ſtronger, when they happen without our thinking of them; for when we make the experiment on purpoſe, and the mind is already prepared for what is to happen, it has time in part to fruſtrate our deſign; theſe things being a little

painful to us. The ſame obſervation is juſt, with reſpect to ſome other experiments of this kind.

It may be owing, at leaſt partly, to the ſame cauſe, that when we ſuddenly ſee an object which we did not before look at, we ſometimes have a *painful ſenſation; and we immediately ſhut our eyes, till we have time, as it were, to prepare them for ſeeing at ſuch a diſtance. It is probable, we may have theſe ſenſations when we do not attend to them; and that they ſerve as monitors to inſtruct the mind, that the eyes are not properly ſuited for viſion at ſuch a diſtance.

104. Before we proceed any farther, it may not be amiſs to take notice of the following experiment, and the inferences that have been drawn from it. Let r, s, be two ſmall holes made in a card, ſo near to each other, that when the card is held cloſe to the eye, the light coming through both holes may enter the pupil; and let Q be any ſmall object, whoſe focus or image, after refraction by the humours of the eye, is q; the two ſlender pencils Qr, Qs, which enter the eye through the holes in the card, being both united at q; if q falls upon the retina, as in *fig.* 12, the object will appear ſingle, and in its true place; the only difference cauſed by the interpoſition of the card in this caſe, is, that the object is fainter for want of the light contained in the ſpace between Qr and Qs. But if q falls either beyond or before the retina, in both caſes, the object ſeen through the two holes in the card, will appear double: Thus,

Fig. 12, 13, 14.

Fig. 13, 14. The ſmall pencil of rays Qr, being refracted to a, and the pencil Qs, to b; the object inſtead of being ſeen at Q, is ſeen in the two places A, B; on different ſides, and a little wide, of the true place Q. For the impreſſions at a, b, being received on the ſame parts of the retina, on which would be formed the images of two objects, ſituated in the directions aA, bB, and ſeen by diſtinct viſion without the interpoſition of the card; the mind has the ſame perception, as it would have, if two objects were really placed in theſe directions; and therefore the object Q appears double. The phantaſms A, B, are ſimilar to the object Q, and alſo pretty diſtinct; becauſe the rays contained in the ſlender pencils Qr, Qs, are united ſufficiently cloſe at a, b; only they appear faint for want of light.

That the cauſe above aſſigned, for the double appearance of the object, is the true one, is farther evinced from hence: If when the object is very near, as in *fig.* 13, the right hole s be covered, the left object B will vaniſh; and if the left hole r be ſtopped, the object A on the right hand, will vaniſh: And if, in *fig.* 14, where the object is ſuppoſed to be at

* This ſort of painfulneſs in the eyes, does alſo ſometimes proceed from a different cauſe; as when we ſuddenly go out of a light room into the dark, or out of darkneſs into a ſtrong light.

at a conſiderable diſtance from the eye, either of the holes *r* or *s* be ſtopped, the object A or B on the ſame ſide, will accordingly vaniſh. And hence it is manifeſt, that the focus *q* in the firſt caſe, is beyond the retina; and in the latter, before it; that is, the rays in the one caſe diverge too much, and in the other too little, for the humours of the eye to unite them all upon the retina. If three holes be made in the card, within the compaſs before limited, then the object will be ſeen in three places.

105. *Schol.* 1. If the lens of a *camera obſcura*, be covered with an opaque paper, having in it two holes or ſlits at a little diſtance from each other; and a white paper be held behind for receiving the ſun's image, or that of any bright object, without doors; by moving this paper along the axis, we may have all the phenomena of the preceding experiment. When the paper is in the proper focus, there will be only one image of the object; the pencils of light tranſmitted through each hole, uniting in this caſe, upon the paper: But as ſoon as the paper is moved from the focus, the pencils through the different holes will begin to ſeparate, and the image will begin to widen; and preſently it will be ſplit into two, which will be the wider aſunder, according as the paper is more remote from the focus. When the paper is before the focus, the images will be on the ſame ſide of the axis, with the reſpective holes; and on the contrary ſide, when the paper is beyond the focus. The ſame experiment may be made, by covering the lens only with patches.

106. If the above lens be covered only with one large patch, and a paper be held in the focus, the images of ſmall objects without, will be free from any ſpeck; but if the paper be without the focus, each image will have a dark ſpot in it. In like manner, if a ſmall opaque object be held near the eye, before which is placed many candles, at the diſtance of 10 or 20 yards; if the eye looks attentively on the ſmall object, it will be ſeen multiplied, according to the number of candles; that is, it will be ſeen like a dark ſpot in each candle. The reaſon of which is, becauſe in this caſe the eye having conformed itſelf, as much as it could, to the near object, the geometrical images of the candles are before the retina; and ſo the rays from each candle, ſpreading themſelves into wide ſpaces upon the retina, the ſhade of the opaque object falls within them, and cauſes the dark ſpots above noted. But when the eye looks at the candles, no ſpots will be ſeen in them, and the opaque body will not be multiplied: For the light which paſſes by the opaque body, being now united into the proper focus's upon the retina, there is no other imperfection in the images of the candles, excepting their being leſs bright,

on account of the loſs of the intercepted light; and the ſmall opaque body itſelf, ſeems tranſparent.

107. *Schol.* 2. The ſmaller is the object, and the wider the holes in the card are aſunder, ſo that they are not wider than the breadth of the pupil; the leſs ought to be the diſtance between thoſe two places, at which the object begins to appear double, from a too great and too little divergency of the rays. A needle held vertically, or a black line drawn upon white paper, makes a better object than a point of the ſame diameter, becauſe they are eaſier ſeen; alſo two parallel ſlits in the card, will do better than the holes, becauſe they admit more light to paſs through them. A ſlit made in the ſide of a dark lanthorn, having a lighted candle in it, is a proper remote object. In making the experiment, care muſt be taken, that the object is ſeen through both holes or ſlits, which may be known by moving the card a little ſide-ways.

But after all the care that can be taken, in making this experiment, it will be frequently attended with inconſiſtencies; and an object will appear ſingle or double, as it were caſually: Sometimes it will be double at all diſtances; ſometimes it will be ſingle for all diſtances beyond a certain one; and frequently an object will appear double at ſuch a diſtance, that with an open eye we might be ſure of having diſtinct viſion.

[a] 104. II. 108. From the [a] experiment with the card, M. *de la Hire* concludes, that no change is made in the eye for viſion at different diſtances, excepting only as to the ſize of the pupil: But herein, I think, this author has had no followers. Dr. *Jurin* *obſerves, that if M. *de la Hire*'s opinion was juſt, the *minimum viſibile* would, in moſt caſes, be much greater than what we find it by experience. And Dr. *Porterfield* †, who has wrote very largely about the above experiment with the card, ſhews, that when the ſmall object appears double, at a proper diſtance for diſtinct viſion, it is becauſe the eye is not properly ſuited for diſtinct viſion at that diſtance; which he proves by ſhewing, that if in ſuch a caſe we look at the object with the other eye, it will appear double, on account of its being out of the plane of the horopter. And this ingenious author gives good reaſons to ſhew, why we are ſo liable to miſtake the diſtance of an object, ſeen through the two pin-holes in the card: But nevertheleſs he proceeds afterwards to aſſign, from this experiment, the leaſt and greateſt diſtance of diſtinct viſion.

Experiment with the card not concluſive, for ſettling the limits of diſtinct viſion, &c.

109. The above experiment, I think, is very little to be depended upon, either for proving that the eye does, or does not, change its conformation

* Eſſay upon diſtinct viſion, p. 136.

† See *Edenburgh* Medical Eſſays, Vol. III. and IV.

formation, for viſion at different diſtances; or, for aſſigning the limited diſtances of diſtinct viſion. For in this caſe, the ſlender pencils of rays, tranſmitted through the pin-holes, are united ſo cloſe upon the retina, that no perceptible difference is obſervable in the diſtinctneſs of the object, by changing the conformation of the eye; and foreign rays, which otherwiſe, by mixing with theſe pencils, might have created an indiſtinctneſs, are here excluded: And ſo in the preſent caſe, we have no incitement to change that conformation, which the eye happens to have, when the card is placed before it; for why ſhould we alter our eyes, unleſs it be for ſeeing more diſtinctly? If it be ſaid that the double appearance of the object, is it ſelf in ſome caſes, an indication that the eye is not adapted to the proper diſtance; yet this being an uncommon phenomenon, and not attended with ſuch a ſenſation as we have, when an object is either too near or too remote; we are at a loſs what motion we ſhould give our eye, and alſo where to ſtop. What paſſes in our eye, with reſpect to viſion at different diſtances, is rather involuntary, than dependent on our will; the eye adapts itſelf by a kind of habit, unknown to us, ſo that we may ſee an object the moſt diſtinctly. And this accounts for the inconſiſtencies of the experiment; and alſo ſhews, that the concluſions drawn from it muſt be weak and precarious. An object much too near for diſtinct viſion, ſeen through the two pin-holes, always appears double; but for the reaſons above aſſigned, I think, the experiment is not concluſive for ſhewing even the leaſt diſtance of diſtinct viſion, and much leſs for ſhewing the greateſt; for in this latter caſe, the card being near the eye, may be a prejudice that deters us from exerting our utmoſt, as we ordinarily do, when we look at remote objects.

But, without regarding the experiment with the card, we may reſt the argument for proving that the eye changes its conformation, for viſion at different diſtances, upon what has been already offer'd, in *Art.* 102, 103.

Of the change of conformation in the eye.

110. Authors are divided in their opinions, concerning that change which is made in the conformation of the eye, in order to obtain diſtinct viſion at different diſtances: Some, thinking it to be a change in the length of the eye; others, a change in the figure or poſition of the cryſtalline humour; others, that it is a change in the cornea. Theſe opinions by being ſplit, make no leſs than ſix different ones; according as the eye when in a ſtate of reſt, and inaction, is ſuppoſed to be accommodated for diſtinct viſion, at the leaſt or greateſt diſtance, it is capable of. The authors of each opinion have their objections to the reſt, and perhaps the truth may lie in a mean betwixt them all. Dr. *Jurin** ſuppoſes

* Eſſay upon diſtinct and indiſtinct viſion, p. 136, to 144.

ſuppoſes, that the change which is made in the eye, for obtaining diſtinct viſion at different diſtances, is in the figure of the cornea; and this learned author, ſupports his opinion by very ingenious arguments. And as the rays of light ſuffer a greater refraction at the cornea, than they do afterwards; it is manifeſt, without entering into particulars, that a leſs change, as to quantity, in the radius of the cornea, will effect the buſineſs, than will ſuffice in any other part of the eye. But if it be true, as ſome authors affirm, that perſons who have been couched for cataracts, are obliged to have convex-glaſſes of different focus's, in order to have diſtinct viſion at different diſtances; it ſeems neceſſarily to follow, that the cryſtalline humour is ſome how concerned, in changing the conformation of the eye: And the manner that this humour is ſuſpended in the eye, ſeems to intimate as if it might be ſubſervient for ſuch a purpoſe. Perhaps, the cornea and the cryſtalline, if not ſome other parts of the eye beſides, may contribute their ſhares together: And that in order to obtain diſtinct viſion at a nearer diſtance, at the ſame time, as the cornea is rendred more convex; the axis of the eye may be a little lengthened; the cryſtalline may be rendred a little more convex, and alſo brought a little forwarder; all which changes conſpire to the ſame end: And the contrary, for obtaining viſion at a greater diſtance. But this matter ſeems to be too far out of our reach, to be decided with certainty.

Of indiſtinct viſion.

111. An object that is far without the limits of diſtinct viſion, will not only be confuſed, but alſo its figure, unleſs it be circular, will appear too broad in proportion to its length.

Fig. 15.

Let ABC be an object, ſo near the eye, that its geometrical image is at *pqr*, beyond the retina. Now the light contained in the pencils proceeding from the phyſical points A, B, C, is ſpread upon the retina, over the circular ſpaces *a*, *b*, *c*, reſpectively; which circular ſpaces, anſwer in bigneſs to the images of circles A, B, C, ſeen by diſtinct viſion; as appears by the dotted lines drawn through the focal center of the eye. And as the rays from every phyſical point of the object, are, in like manner, ſpread over circular ſpaces upon the retina, theſe circles run into one another; and ſo the object, inſtead of appearing lineal, is confuſed, and as broad as the circles A, B, C, as repreſented in *fig.* 16.

Circle of aberration.

112. *Defin.* The leaſt circle upon the retina, into which all the rays coming from a phyſical point of an object ſeen by indiſtinct viſion, can be collected, we ſhall call the *circle of aberration*; as the circle *a*, or *b*, or *c*, fig. 15.

PROP.

PROP. V.

113. *A circular object, ſeen by rays diverging either too much or too little for diſtinct viſion, has alſo a circular image upon the retina; and the diameter of this image, exceeds the diameter of the true image, by the diameter of the circle of aberration.*

Let the circle RABC be the true image, or that circle formed upon the retina, by the axes of all the pencils that come from the object; and let the circles of aberrations be each equal to the circle *rabc*, whoſe center *r* is here in the periphery of the circle RABC; and let RDEF be a circle, whoſe periphery touches the outermoſt part of the periphery of the circle *rabc*. I ſay, that the whole image upon the retina is the circle RDEF. Fig. 17, 18.

For, the axes of all the pencils of light coming from the periphery of the object, do [a] fall in the periphery ABC; and the circles of aberrations of theſe pencils, are [a] each equal to *rabc*. Therefore, *&c. Q. E. D.* [a] Hypoth.

PROP. VI.

114. *In the middle of the image of a circular object, ſeen by indiſtinct viſion; there is a circular ſpace, whoſe diameter is the difference of the diameters of the true image and of the circle of aberration, which is all over equally enlightened; and from the circumference of this equally enlightened circle, the light continually diminiſhes towards the periphery of the image, ſo as at that periphery to be infinitely rare.*

Retaining the ſame references as in the preceding *Prop.* let RGHI be a circle touching the circle of aberration *rabc*, in that point *c*, which is neareſt to the center R; let the radius of the true image RABC be called R, and the radius of *rabc*, call *r*; then it is manifeſt that the radius of RGHI $=$ R $\sim$ *r*. I ſay, that the circle RGHI is all over equally enlightened, and that without this circle, the light is continually diminiſhed from GHI to DEF. Fig. 17, 18.

1. Let R be greater than *r*, and in the line *r*R, take *cs* $=$ *r* $=$ *cr*. Now, if the lines *rc*, *sc*, be alike divided from *r* to *c*, and from *s* to *c*, and theſe diviſions be ſuppoſed the centers of circles of aberrations, whoſe radii are each equal to *r*; it is manifeſt then, that an indefinite ſmall ſector R*c*, will receive as much light, from the circle of aberration whoſe center is next to *r*, as it will be deprived of, by the extenſion beyond *c* of the circle whoſe center is next to *s*: And ſo the correſpondent circles will all the way countervail each other, till their centers unite in *c*; ſo that the point *c* loſes none of its light, and all the points from Fig. 17.

from *c* to *s*, are equally enlightened: And this is true, of all the indefinitely ſmall ſectors, whoſe center is R. Thoſe circles of aberrations whoſe centers are not more remote from R, than *s*, are all included within the circle RGHI; and by the preceding argument, it is manifeſt, that the light is equally denſe in every point of the ſaid circle. But as no circle of aberration can have any return, for ſo much of its light as falls without ABC, the light without GHI muſt continually decreaſe, till at the periphery DEF it is infinitely rare; the ſaid periphery, receiving only one ray from each of the outermoſt circles of aberration.

Fig. 18. 2. When R is leſs than *r*. It is manifeſt that every point in the circle RGHI, is equally enlightned; for the whole of that circle, falls within every one of the circles of aberrations: And by the ſame argument as above, it appears that the denſity of the light continually diminiſhes, from the periphery GHI to DEF. Therefore, *&c. Q. E. D.*

115. *Schol.* 1. When R is greater than *r*, as in *fig.* 17. the circle RGHI receives the ſame quantity of light, as it would if there was no aberration: This is manifeſt from the preceding. When the circle of aberration as CKRL, is equal to the true image, only the center of that image has its full ſhare of light; and in all other places the light decreaſes in denſity, according to the diſtance from the center. But when R is leſs than *r*, as in *fig.* 18. no part of the image has its full quantity of light: For ſince every pencil, whoſe axis falls within GHI, diffuſes its light into a circle as large as *rabc*, and the points where theſe axes fall, receive light only from pencils whoſe axes fall within the circle RABC; it follows, that the circle GHI diſperſes more light than it receives, and that in proportion of r^2 to R^2; and by the preceding, the light in every point without that circle, is diminiſhed ſtill more.

116. *Schol.* 2. The image at the circumference DEF will be imperceptible, becauſe there the light is [a] infinitely rare: And to a given circle of aberration; the more luminous the object is, the image will be ſenſible, ſo much the nearer to the circumference DEF.

[a] 114. II.

For the denſity of the light decreaſes in equal proportion from GHI to DEF, whatever be the quantity of light in the whole image [a]; and therefore the more luminous the object is, the more ſenſible muſt the image be, at any given diſtance from the edge DEF; and it will be equally ſenſible, ſo much the nearer to that edge.

117. *Schol.* 3. The three preceding articles, may be alſo applied to objects of other figures, as parallellograms *, *&c.* The principal difference in the phenomena of circular and oblong objects, is, that the falſe images of the latter, will be [b] broader in proportion to their lengths, than their

[b] 111. II.

* This Subject has been conſidered very minutely by Dr. *Jurin*, in the *Eſſay* before quoted.

their true images, and alſo blunt at the corners. But the phenomena above ſeldom happen, excepting when we look at objects that are too near, or through lens's at objects that are out of the proper focus; and when the aberration is conſiderable, the confuſion is generally ſo great, that the appearance, at leaſt in many caſes, will not anſwer to the theory; and it would be needleſs, here, to purſue this ſpeculation any farther.

A ſmall bright object, by cloſing the eye-lids, becomes double.

118. A ſmall bright object, by cloſing the eye-lids a little, will have two ſimilar images incroaching one upon another; and the cloſer the eye-lids are brought together, the more detached will be the images, or the farther aſunder will be their centers: Both the images will be diſtinct, but one will be brighter than the other; and that part, which is common to both, will be brighter than either. Thus, let A be a round hole in the window-ſhutter of a dark room; if this hole is not too big, upon cloſing a little the eye-lids, there will appear another hole, as B; and ſometimes there will be three images, one above, and the other below the true image, as in *fig.* 20. The true image A will be brighter than B; and the part *ab*, as receiving light from both images, will accordingly be brighter than the reſt of A. There will be the ſame appearance, if one eye be totally ſhut; and therefore, it cannot be owing to the object being out of the plane of the horopter, and beſides the images are here always in a vertical ſituation; nor can it be owing to indiſtinctneſs of viſion, becauſe in that caſe, inſtead of two images, [a] we ſhould only have one broad and confuſed image.

Fig. 19.

[a] 113. II.

Beams of light ſhooting from candles.

119. If the eye-lids be pretty much cloſed, the two falſe images above deſcribed will vaniſh; and they will be ſucceeded by luminous beams, as ABC, ADE, *fig.* 21, projecting towards the eye; juſt after the ſame manner with thoſe beams, that, in a like diſpoſition of the eyes, appear to ſhoot from the flame of a candle. If the head be inclined upwards or downwards, the upper or lower beam will accordingly vaniſh; and ſo, by a like inclination of the head, will alſo vaniſh the reſpective falſe images in the preceding *Art.*

Cauſe of the above phenomena.

120. The phenomena of the falſe images, and the beams, muſt undoubtedly be both owing to a like cauſe; and that cauſe, muſt be ſome inflection of the rays at the edges of the eye-lids. If a ruler be held againſt the face below the eye, and then moved upwards, the uppermoſt beam or image will firſt vaniſh; and if the ruler be placed above the eye, and moved downwards, it will be the lowermoſt beam that will vaniſh: And this ſhews, that the rays which cauſe the appearance of the upper and lower beams, are inflected reſpectively at the lower and upper eye-lid. M. *Rohault* ſuppoſes, that the rays are re-

flected at the edges of the eye-lids, as by a ſpeculum; but the phenomenon of the ruler juſt mentioned, invalidates this hypotheſis. M. *de la Hire* thinks, that the water which ſerves to moiſten the eye, is ſo ſqueezed by the preſſure of the eye-lids, as to form between them a concave ſurface; and by this ſurface, the rays which fall on the lower lid, are refracted downwards; and thoſe falling on the upper, are refracted towards the upper part of the retina: And ſo by the greater refraction of the rays, as they fall nearer to the edges of the eye-lids, the beams of light appear extended to a greater diſtance from the object.

Fig. 21. Thus, one of the outermoſt rays, in the pencil of light proceeding from A, is refracted by the concave water adhering to the eye-lid G, ſo, that after its refraction by the cornea and cryſtalline humour, it falls upon the retina at the point *e*; and ſome of the other rays that are nearer the axis, after all the refractions, fall at the point *d*, between *e* and *a*. In like manner, the extreme ray which comes to the eye-lid H, is, by the concave ſurface of the water adhering to it, ſo refracted as to fall upon the retina at *c*; and the reſt of the rays, from this laſt ray to the axis, ſtrike orderly upon the retina from *c* to *a*; as if they proceeded from ſome luminous beam of light ABC. And ſo the rays which fall betwixt *a* and *e*, ſeem to have proceeded from another luminous beam ADE.

121. *Schol.* 1. From the above ſolution, it ſeems neceſſary it ſhould follow, that the beams of light ſhould appear coloured, as in the caſe of viſion through a priſm: But this not being the caſe, it may ſeem more likely, that the rays inſtead of being refracted, are inflected at the eye-lids, as in *Art.* 86 *; the upper eye-lid, inflecting the rays upwards, and the lower lid inflecting them downwards upon the retina, ſo as to cauſe the appearance of the luminous beams, for the reaſons aſſigned in the preceding *Art.* Perhaps both the above cauſes may conſpire together, as each of them is equally ſuited for diſperſing the rays over the ſame parts of the retina: But by either of them, it ſeems difficult to account for the falſe images in *Art.* 118. The middlemoſt and true image, is undoubtedly cauſed by rays that freely enter into the eye, as uſual; and the rays that cauſe the falſe image, muſt be ſome how regularly or equally inflected. If this is done by refraction, it muſt be by a thin wedge of water, whoſe thickeſt ſide adheres to the eye-lid; and this refracting but little, the falſe image is not ſenſibly diſtorted, nor moved far out of its true place.

The phenomenon above conſidered, does not happen, but when the object is much brighter than the medium round it; which ſhews, that the inflected rays are but thinly ſpread upon the retina, and that perhaps.

* See *Newton*'s Optics, p. 300.

haps may be one reaſon, why the falſe images do not appear coloured. The rays which enter the pupil laterally, or at the ſides, meet with no obſtruction from the eye-lids; and this is the reaſon why, the falſe images, and the luminous beams, appear always one above, and the other below, the object.

122. *Schol.* 2. Two phyſical points, or ſmall objects, ſeen by indiſtinct viſion, will have two large circular images, as A,B; and if they are ſo near as to have a part *ab* common to both, that part will be more luminous than either A or B, and the whole will form a kind of an oblong image. But in this caſe, theſe images are confuſed, ill defined, and very unlike the phenomenon above explained. Fig. 19.

PROP. VII.

123. *All other things being the ſame: The indiſtinctneſs of an object whoſe geometrical image falls either behind or before the retina, will be, as the area of a circle of aberration; or, as the area of the leaſt circle upon the retina, into which all the rays of a ſingle pencil can be collected.*

Let RABC be the geometrical image, or that ſpace upon the retina which contains the axes of all the pencils of light coming from the object, and let the circles of aberrations be each equal to *rabc*: I ſay the indiſtinctneſs of the object, will be as the area of *rabc*. Fig. 17, 18.

For every ſenſible point upon the retina, within the ſpace RABC, will be a center of a circle of aberration, and will at once be affected by a mixture of the rays of as many different pencils, as there are ſenſible points within that circle: But the indiſtinctneſs is neceſſarily, as the number of pencils which at once affect the ſame point of the retina, and that number is as the area of a circle of aberration. Therefore, *&c* *Q. E. D.*

PROP. VIII.

124. *All other things being the ſame: The indiſtinctneſs of an object having its geometrical image at a given diſtance from the retina, will be as the area of the pupil.*

Let *q* be the focus of a pencil of rays, after refraction by the eye; now the ſituation of the point *q* being invariable, it is manifeſt that the circle of aberration *b*, will be as the circle whoſe diameter is *st*, that is as the area of the pupil. The ſame demonſtration takes place, if the focus *q* be before the retina; and in both caſes, it is ſuppoſed that the pencils of light at their entrance into the eye, are each at leaſt as broad as the pupil. But the indiſtinctneſs, is [a] as the area of the circle *b*. Therefore, *&c.* *Q. E. D.* Fig. 15.

[a] 123. II.

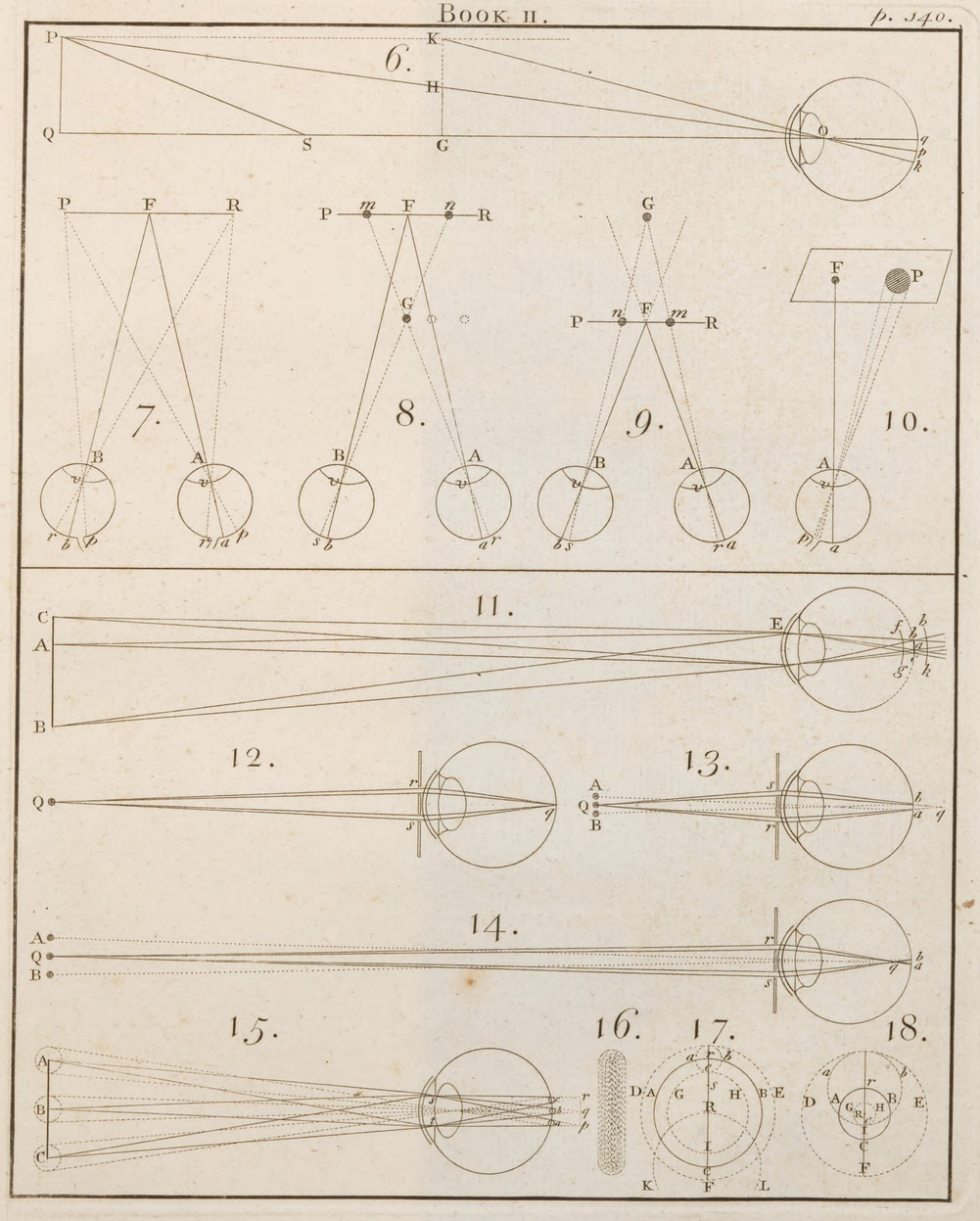
125. *Schol.* 1. If we had a power at will of contracting the pupil, an indiſtinctneſs of viſion might be thereby leſſened: But this is not the caſe; the motions of our eyes are rather ſubſequent to particular ſenſations, than ſubject to our direction; and a greater degree of light is a much more powerful cauſe of contracting the pupil, than an indiſtinctneſs of viſion. The eye looking at a ſtar will have its pupil much wider, than when it looks at an object at a moderate diſtance by day-light. And a book may be read at a nigher diſtance by a ſtrong light, than it can by a faint light; the ſtrength of the light contracting the pupil into a ſmaller ſize, than the ſenſation of indiſtinctneſs alone could do. It is eaſy to make a kind of artificial pupil, of what ſize we pleaſe; but this will not help us much, as is ſhewed in the following.

Viſion through a pin-hole.

Fig. 22.

126. *Schol.* 2. If we look through a pin-hole, made in a card or thin plate, held cloſe to the eye, an object will appear diſtinct at a ſomewhat leſs diſtance than it will to an open pupil. Thus, if AQ is the leaſt diſtance at which the eye with an open pupil, can ſee a ſmall object as Q, diſtinctly; if the eye approaches nigher to the object, as to B, and no more light be admitted through the hole *a*, than what before entered the pupil at A, the object will be ſtill pretty diſtinct; becauſe the outermoſt rays in the pencil, have now no greater degree of divergency than they had before. In like manner, if the hole in the card be leſſened, in proportion to the diſtance, the eye may approach ſtill nigher to the object, as to C: However, there is a limit as to this approach, otherwiſe we ſhould have a ready way of making a kind of microſcope; and although it ſeems, as if the eye received the ſame rays of light from the object, at the different diſtances; yet the object does not appear equally bright, and diſtinct, at thoſe ſeveral diſtances. It is to be obſerved, that an image upon the retina is more or leſs vivid, according to the quantity of light received from the leaſt viſible part of an object, and not according to the quantity received from the whole object: And the diameter of the *minimum viſibile* is increaſed, as the diſtance from the object is leſſened. And therefore in the caſe above, if the hole be leſſened in proportion to the diſtance, the object, as it is approached nigher will be darker and darker, and ſo by degrees grow indiſtinct for want of light; and if the hole be bigger than in the above proportion, there will be an indiſtinctneſs from a too great divergency of the rays: And in both caſes, the indiſtinctneſs may be increaſed, by [a] an inflection of ſome of the rays at the edge of the hole; and the ſmaller the hole, the greater proportion will theſe inflected rays bear to thoſe that are freely tranſmitted to the eye. So that the above experiment,

[a] 86. II.


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periment, though it may help a little in ſome caſes, yet is it of much leſs uſe than ſome have imagined *.

127. *Schol.* 3. In all that has been ſaid about an indiſtinct image upon the retina, no regard has been had to any aberrations of the humours of the eye; for although it is very probable that there may be ſuch aberrations, yet it is impoſſible to diſcover them by any experiment; we cannot ſee what paſſes in the eye, and light after it is once entered there, can be traced no farther.

Latitude of diſtinct viſion.

128. Before we conclude this head, we ſhould make ſome enquiry about the extent of diſtinct viſion, ſo far as diſtinctneſs is dependent on the different diſtances of objects, or the degrees of divergencies of the rays of light proceeding from them. It is agreed that, to the generality of eyes, the neareſt diſtance of diſtinct viſion is [b] about 7 or 8 inches. But the greateſt diſtance of diſtinct viſion to a common eye, is perhaps not ſo eaſily ſettled: Dr. *Porterfield* determines this greateſt diſtance for his eye, to be 27 inches; and this concluſion he draws from the experiment, *Art.* 104. But that experiment is [c] too precarious to be relied on: And I ſee no room to doubt of our being able to ſee objects diſtinctly, if they are ſufficiently large, at a much greater diſtance than two or three feet. A large print at the diſtance of five or ten feet, is, I think, every whit as legible, as a print that is five or ten times ſmaller, at the diſtance of one foot. And cannot a perſon, having common eyes, ſee a large object, as the front of a houſe, *&c.* as diſtinctly at the diſtance of 20 or 30 feet, as if that object was nigher to him?

[b] 89. II.

[c] 109. II.

I am inclined to think, that a common eye can adapt itſelf ſo, as to have diſtinct viſion at the greateſt diſtance, or with parallel rays; or that in ſuch a caſe, the geometrical image is not ſo remote from the retina, as to cauſe any indiſtinctneſs upon that account. For,

1. It is natural to ſuppoſe, that the eye can alter its figure as much, from that conformation it naturally has when in a ſtate of reſt and inaction, one way, as it can another; or, that the eye can change its conformation as much for viſion with rays diverging leſs, as it can for rays diverging more, than that degree of divergency which is proper for viſion when the eye is in its natural ſtate of reſt. Upon this ſuppoſition, and admitting the natural diſtance of diſtinct viſion is not leſs than 16 inches; it will follow, that the image of an object at an infinite diſtance will be as near the retina, [a] as the image of an object at no greater diſtance from the eye, than about 8 inches; and ſo in both caſes, the images will be equally diſtinct.

[a] 21. II.

2.

* A ſlender object, as a needle, placed between the eye and a ſmall hole in a card held pretty near, will appear inverted; as will be manifeſt, by moving the needle ſlowly over the hole; but in this caſe, it is not the needle, but its ſhadow on the other ſide, that is ſeen, and that is the cauſe of the inverted appearance.

2. Without regarding what is the natural diſtance of diſtinct viſion, let us ſuppoſe 10 feet and 8 inches to be the greateſt and leaſt diſtance, at which a common eye can ſee an object diſtinctly, with eaſe, and without much ſtraining of its parts. Now if 2 tenths be ſuppoſed the breadth of the pupil, no greater change in the eye will be requiſite for procuring diſtinct viſion from 10 feet to an infinite diſtance, than is ne-
Fig. 23. ceſſary from 8 inches to $7\frac{1}{2}$ inches. Let AB be the ſemi-aperture of the pupil $=$ 1 tenth, AC the axis of the eye produced, EB a ray parallel thereto, DB the axis of a pencil whoſe focus is 10 foot from the eye, and let AC $=$ 8 inches. The $\angle$ EBD is $=$ 2′ 52″, and by making the $\angle$ CBF $=$ $\angle$ EBD, AF will be $7\frac{1}{2}$ inches. It cannot be doubted, but that we can ſee an object diſtinctly at the diſtance of 10 feet; and it is as certain, that we can have diſtinct viſion, at leaſt half an inch nearer than that diſtance, at which we commonly look at ſmall objects.

3. But if it be doubted, whether we can change the conformation of our eyes, as much for procuring diſtinct viſion farther, as we can for procuring it nigher: Let a perſon having common eyes, after looking attentively at a remote object, as the moon, look at the ſame object through a concave lens, having what focal length he may think beſt; and I believe by this trial, he will be convinced that a paralleliſm of the rays of light, is not an impediment to diſtinct viſion, and that therefore the indiſtinctneſs of remote objects is to be referred to ſome other cauſe. By looking through a convex lens having a long focus, it will appear, that a common eye may even have diſtinct viſion with rays a little converging.

Cauſe of indiſtinctneſs of remote objects.

129. *Schol.* 1. The chief cauſes of the indiſtinctneſs of remote objects, I take to be thoſe [a] already aſſigned: And it may be farther obſerved, that we uſually diſtinguiſh near objects, by certain marks, as the figures, ſizes and colours, of ſome eminent parts; and when theſe marks, by reaſon of their ſmallneſs, are loſt; the object, by wearing a quite different face from what it uſed to have at a nearer view, becomes neceſſarily indiſtinct. For in this caſe, we not only loſe ſight of our principal marks, but alſo the out-line becomes blunt, faint, and imperfect, as the exterior parts of the object are not diſtinguiſhable one from another; and ſo there being nothing viſible, but the groſſer parts, the whole appears confuſed, and ill defined. The features of a man, the buttons on his coat, *&c.* are not viſible beyond a certain diſtance: And at a greater diſtance, his head and ſhoulders will ſeem contiguous, and no marks will be left, whereby he may be diſtinguiſhed from the ſtump of a tree.

[a] 80, 81, & 93. II.

130.

130. *Schol.* 2. Let AB be the ſemi-diameter of the pupil, *b*AC pro- Fig. 23.
duced the axis of the eye, EB a ray parallel thereto, *e*C*g* an indefinite line perpendicular to AC; and let the conformation of the eye be ſuch, as to ſee moſt diſtinctly at the diſtance AC, or let *b* be the geometrical image of the point C: Now, if innumerable pencils of rays, be ſuppoſed to enter the eye, from innumerable focus's taken in the axis AC, from C onwards to infinity; the diameter of the leaſt circle upon the retina, which will contain all the rays that enter the pupil, will be equal to that of the geometrical image of a circular object, whoſe ſemi-diameter is the line CE.

In the line *eg*, take CG=CE, and let *a* be the geometrical focus of a pencil of rays flowing from E: Now it is manifeſt, that a ray coming parallel to the axis, from an infinite diſtance, through the point E, will thenceforward be carried to the eye, along with the ray EB, and that both theſe rays will fall upon the retina, at the ſame point *a*; *ba* being the geometrical image of CE: And all the rays, whoſe focus's are any where in the axis beyond C, will fall ſomewhere within the circle whoſe diameter is GE; and conſequently will fall upon the retina, within the image of that circle. Therefore, *&c.* *Q. E. D.*

131. *Corol.* Let BG be a line cutting the axis AC in H: Now the focus of a pencil of light diverging from H, will fall at a point *y*, about as much beyond the retina, as the focus *x* of parallel rays, is before the retina; and all the pencils of light, whoſe focus's are any where in the axis betwixt G and H, will be contained upon the retina, within the circle whoſe ſemi-diameter is *ba*; for theſe rays produced backwards, will fall within the circle whoſe diameter is GE.

And univerſally, if C be ſuppoſed that point in the axis, whoſe geometrical image falls upon the retina; and innumerable pencils of rays be ſuppoſed to enter the pupil, ſome whereof are converging and others diverging, to or from, certain focus's in the axis of the eye: Produce backwards that ray, as IB, which converges the moſt, and that ray as *b*B which diverges the moſt, till they meet with the perpendiculars to the axis C*e*, C*g*, in *e* and *g*; and the longeſt of theſe two perpendiculars C*e* or C*g*, will be the ſemidiameter of that circle, whoſe image is the leaſt circle upon the retina, which can comprehend all the rays that enter the pupil.

§. 3.

§. 3. *Of long and ſhort-ſighted eyes, and of ſpectacles.*

Of long-ſightedneſs. Fig. 24.

132. If the cornea and cryſtalline, or either of them, be too flat, a pencil of light coming from an object at an ordinary diſtance, will have its focus at ſome point *y*, beyond the retina; and therefore viſion will be indiſtinct, as in the caſe of an object too near for a common eye. This defect commonly comes on with old age, and is thought to be occaſioned by a ſhrinking of the humours, which flattens the cornea, and the coats of the cryſtalline: And hence, theſe kind of eyes are called by authors, *præſbytæ*, *viſus ſenilis*; alſo *long-ſighted*, becauſe they cannot ſee near objects diſtinctly. See alſo *fig.* 13.

133. Long-ſighted people, in reading, place the book at a greater diſtance than ordinary, as two or three feet; that being their neareſt diſtance of diſtinct viſion: They alſo commonly write a large hand, and always chuſe a book with a large print; for being obliged to read at a great diſtance, ſmall letters will not ſubtend angles ſufficiently large, and upon that account will be confuſed, as to a common eye, at a like diſtance. The long-ſighted alſo require more light for reading a given print, than others do for reading the ſame print; for they receive leſs light than others, in proportion as the object or book is held at a greater diſtance.

Of ſhort-ſightedneſs. Fig. 25.

134. When the cornea and cryſtalline are ſo convex, that the images of objects, at an ordinary diſtance, fall before the retina, as at *y*; the pictures upon the retina, and viſion cauſed thereby, will be confuſed: Perſons having ſuch eyes, can ſee no objects diſtinctly, but ſuch as are very near; and hence they are called *ſhort-ſighted*; alſo *purblind*, and *Myopes* *. See *fig.* 14.

135. Short-ſighted people, commonly chuſe a ſmall print, and write a ſmall hand; for to them, the letters are magnified by their nearneſs, ſo as to appear as big as thoſe of a middling ſize do to a common eye; and the letters being ſmall, they have the advantage of ſeeing more of them at one view. It has been remarked, that *Myopes* do not uſually look at the perſon they converſe with; for as they cannot obſerve his looks, and the motions of his eyes and features, they are attentive to his words only.

136.

* There are other cauſes that limit the extent of viſion: As, when the pupil is more than ordinary ſmall, the humours turbid, or leſs tranſparent than common; the retina ſomewhat callous or inſenſible: People labouring under theſe diſorders, require more light for viſion, than ordinary; and therefore they cannot ſee ſo far as other people, although their eyes be of proper figures, for uniting the rays from remote objects upon the retina.

136. Thoſe who are ſhort-ſighted, can ſee ſmaller objects, and alſo ſee clearly a ſmall given object with leſs light, than other people can: For by viewing them nearer, they increaſe their viſible magnitudes, and alſo receive more light from a given object, in proportion to that nearneſs. And hence it is, that *Myopes* can read by moonſhine or twilight, when a common eye is not able to diſtinguiſh one letter from another: They alſo can ſee ſomewhat farther off, by a ſtrong light, than they can by a weak light; for the ſtrength of the light contracts the pupil, and that leſſens the indiſtinctneſs. Some advantages of ſhort-ſightedneſs.

137. *Schol.* If to a given eye, the neareſt diſtance of diſtinct viſion be four inches; as great a change of configuration will be requiſite to ſuch an eye, for procuring diſtinct viſion from four inches to eight inches diſtance [a]; as will ſuffice a common eye for obtaining diſtinct viſion at all diſtances, from 8 inches to infinity.

[a] 21. II.

There are ſome people that are ſhorter-ſighted than as above ſuppoſed: It is no wonder then, that ſuch cannot ſee a ſmall object at arm's length from them, or to avoid a ſtone at which they are juſt going to ſtumble.

138. It is a common obſervation, that the eyes grow flatter with age; and hence it is, that people who were purblind in their youth, ſee better as they grow in years, and continue to read without ſpectacles to the very laſt. On the other hand, the long-ſighted eye grows worſe and worſe; and at length its refractive coats may become ſo flat, as not to be able to collect even parallel rays upon the retina. And hence all objects will neceſſarily be confuſed; thoſe that are ſmall, as the letters of a book, *&c.* will be altogether imperceptible; and at length by a ſtill greater decay of the humours of the eye, the benefit of naked viſion will be in a manner totally loſt. The eye grows flatter with age.

It cannot be doubted, but that the parts of the eye, like the reſt of the body, grow ſtiffer with years, and leſs pliable; and hence aged perſons, have leſs latitude of diſtinct viſion, than when they were younger. Alſo it is probable, that the optic nerve grows leſs ſenſible with years; and this may be another cauſe of the decay of ſight in old age.

139. It has been obſerved, that people who are frequently uſed to look at remote objects, as country-men, travellers, ſailors, and the like, are generally long-ſighted; and ſo want ſpectacles the ſooneſt: On the other hand, the greateſt number of ſhort-ſighted perſons, are found among ſtudents and mechanics, who are daily converſant with ſmall Short and long ſightedneſs may be acquired.

and near objects. The eye, it ſeems, in both caſes endeavouring to keep that configuration, to which it is moſt accuſtomed.

Sometimes come by accidents.

140. Sometimes both long and ſhort-ſightedneſs, come on ſuddenly by accidents, or illneſs: Colds eſpecially have been productive of both theſe effects; and there have been inſtances of aged perſons, who had uſed ſpectacles many years, having by an accidental cold become ſo ſhort-ſighted, as never to want them again.

Some perſons both long and ſhort-ſighted.

141. There are many inſtances of people who ſeem to be both long and ſhort-ſighted; that is, they can ſee near objects, and ſeemingly alſo remote objects, diſtinctly; but thoſe that are at intermediate diſtances, appear confuſed: This caſe will admit of an eaſy ſolution, if it be ſuppoſed, that ſuch people have one eye ſhort-ſighted, and the other long-ſighted, and that they ſee only with that eye, which is ſuited for diſtinct viſion at the near or remote diſtance *.

Viſion of remote objects by ſhort-ſighted people.

142. But when we conſider what are the [a]chief cauſes of the indiſtinctneſs of remote objects to a common eye; it will ſeem probable, that with reſpect to viſion of diſtant objects, the ſhort-ſighted are not under ſuch great diſadvantages, as at firſt might be imagined. The planets and all the brighter ſtars, are as viſible to the moſt ſhort-ſighted, as they are to any other; and probably their appearances are much alike to all ſorts of eyes. And the greater quickneſs of ſome in diſcerning the ſmaller ſtars, may perhaps be more owing to a better conſtitution of the retina, *&c.* than to any advantage in the figure of the eye. But becauſe of the aberrations upon the retina, a ſhort-ſighted perſon cannot diſtinguiſh beyond a certain ſmall diſtance, ſuch ſmall parts of an object, as another can: Thus, he cannot diſtinguiſh the features of a face a-croſs the room; and as objects in general are diſtinguiſhed by their viſible parts, ſo with reſpect to viſion at moderate diſtances, the diſadvantages of the ſhort-ſighted are very great and manifeſt. But although ſuch a perſon cannot diſtinguiſh parts, unleſs they be very large, or very near him; yet he can perceive an object in the groſs, if it be not too ſmall, at a conſiderable diſtance: He can perceive a man at the diſtance of ſeveral paces, though perhaps he muſt advance within one of him, before he can call him by his name. A large tree he can diſtinguiſh yet farther; and from his experience in like caſes, the large obſcure object which he perceives at a great diſtance off, he concludes

[a] 129. II.

* Some have imagined, that we never ſee but with one eye at a time: If this be the caſe, it may be wondered why in looking we always turn the axes of both eyes towards the object. It is very likely, that ſome people either ſee altogether with one eye only, or ſee better with one eye than they do with the other: But that this is not univerſally the caſe, I think, is very plain, from the following obſervation, *viz.* If, whilſt looking at any object, either eye be covered ever ſo nimbly, we continue ſtill to ſee the object, without any the leaſt intermediate ſtop; which ſeems difficult to be accounted for, if the now naked eye had before been idle.

cludes to be a houſe, to the great ſurprize, perhaps, of his acquaintance who knows his caſe. And hence, without ſuppoſing one eye to be ſhort and the other long-ſighted, we may ſolve that ſeeming paradox; why a perſon who is ſo purblind as not to be able to ſee a ſmall object at arm's length from him, is yet, in many caſes, not apparently under any ſuch great diſadvantages, with reſpect to viſion of remote objects.

Long and ſhort-ſightedneſs, how remedied

143. An eye, that has no other defect but that of being long or ſhort-ſighted, may be helped by a lens of a proper figure, ſo as to be able to ſee diſtinctly at any given diſtance. As, if A*q*, *fig.* 24. be the neareſt diſtance of diſtinct viſion to a long-ſighted eye; A*q*, *fig.* 25. the remoteſt diſtance of diſtinct viſion to a ſhort-ſighted eye; and AQ be that diſtance, at which each reſpective eye wants to ſee an object diſtinctly. Let O be a lens of ſuch a figure, that rays diverging from Q will after refraction, diverge from *q*; an object Q ſeen through ſuch a lens will appear diſtinct. For in both caſes, [a] the rays, by the refraction of the lens, are made to enter the eye with ſuch a degree of divergency as is proper for diſtinct viſion: The want of plumpneſs in the long-ſighted eye, being ſupplied by the convex lens, and the too great plumpneſs of the myopical eye is corrected by the concave lens.

Fig. 24, 25.

[a] Hypoth.

Of ſpectacles.

144. *Defin.* Two glaſſes, having ſuch figures as are requiſite for procuring diſtinct viſion at given diſtances, and fixed in a proper frame for holding them before each eye, are called *ſpectacles*.

Their deſign and uſefulneſs.

145. Convex lenſes magnify, and concave ones diminiſh, the objects that are ſeen through them; but theſe are accidental properties: The end or deſign of ſpectacles, being neither to magnify or diminiſh objects; but to alter the diſpoſition of the rays at their entrance into the eye, as may be moſt ſuitable for procuring diſtinct viſion. And this is an end of ſuch vaſt importance, that the aged is thereby placed, in a manner, upon a level with the young; and enabled to read a common print, when by his naked ſight he can hardly diſtinguiſh one letter from another. By the help of glaſſes, the purblind, whoſe ſphere of diſtinct viſion ſcarce extends the length of his arm, has his ſight extended to the greateſt diſtances: And it is owing to the ſame help, that many of the arts have been brought to that great perfection they now are in.

Præſbytical eye receives more benefit by glaſſes, than the *Myopical*.

146. The long-ſighted eye receives greater benefit from a convex lens, than the ſhort-ſighted does from one that is concave. For an object ſeen through a convex lens, is not only magnified; but alſo the eye receives a larger pencil of light from each viſible point, becauſe the rays enter it leſs diverging. On the other hand, a concave lens not only diminiſhes objects, but alſo renders them darker; for the rays coming

to the eye more diverging, the pupil cannot receive ſo many of them, as otherwiſe it would: And hence it is, that *Myopes* cannot ſee remote objects, unleſs they be very large and bright, ſo well through a concave lens, as the theory of the images being upon the retina, may ſeem to promiſe. For the principal impediments to diſtinct viſion of remote
[a] 129. II. objects, [a] being their want of light and magnitude; both theſe wants are, by the concave lens, ſtill increaſed.

The propereſt ſpectacles.

147. Thoſe ſpectacles are the moſt proper, whoſe glaſſes are the leaſt convex, or leaſt concave, of any that will anſwer the purpoſe. For the leaſt convex will accuſtom the long-ſighted eye, to that conformation which is proper for ſeeing objects as near as it can; and ſo may prevent the eye from growing more long-ſighted. In like manner, the leaſt concave glaſs accuſtoms the ſhort-ſighted eye, for viſion, at as great a diſtance as it can. Having fixed upon the proper focal length of the glaſſes, it is ſcarce needful to add, that the next thing is to chuſe thoſe that ſhew objects the cleareſt; and this is beſt determined by a compariſon of ſeveral together. There are divers methods of fixing ſpectacles to the head; but it ſeems to me, that thoſe which have ſprings for embracing the temples, are the moſt convenient.

Of reading-glaſſes.

148. A *reading glaſs* is a broad lens, fixed in a frame, having a convenient handle to hold it by: The theory of ſuch glaſſes is ſcarce different from that of ſpectacles, excepting what ariſes on account of the one being commonly held about midway between the eyes and the object. But for all purpoſes, ſpectacles are undoubtedly the moſt convenient; not only becauſe the ſpace between the face and the object is left open and free, and alſo each hand is at liberty; both which are advantages in writing, and ſtill more in working: but alſo they are inſenſibly moved, by the natural motion of the head, without any kind of trouble. Spectacles alſo ſhew objects clearer, and diſtincter, than a broad lens; becauſe their glaſſes are thinner, and placed exactly before each eye.

Uſing ſpectacles, not detrimental.

[a] 147. II.

149. It is a common received opinion, that looking through glaſſes is detrimental to the eyes: But, I think, this opinion is ſo far from being true, that to thoſe who want them, ſpectacles having glaſſes [a] of proper figures, may contribute to preſerve the eyes longer, as they take away the neceſſity of ſtraining them too much. However, ſpectacles ſhould not be uſed but in ſuch caſes, that diſtinct viſion cannot be had without them. Without this caution, the eyes may become long or ſhort-ſighted, ſooner than otherwiſe they would; and to prevent this inconveniency, the beſt remedy is frequently to look with the naked eye at objects at all ſuch different diſtances, as we can ſee tolerably diſtinct; and alſo to look ſometimes through glaſſes of different figures.

PROP.

PROP. IX.

150. *To ſuit a given eye with proper ſpectacles: Or, to find a lens that will enable a given eye, to ſee diſtinctly at a given diſtance.*

Let O*q* be the leaſt diſtance of diſtinct viſion to a long-ſighted eye, and the greateſt to a ſhort-ſighted eye; and let OQ be that diſtance, at which each eye would ſee diſtinctly. Fig. 24, 25.

1. To find a convex lens for the long-ſighted eye; ſay, Fig. 24.
$Qq : QO :: QO : QF$; and place F from Q towards *q*.

2. To find a concave lens, for the ſhort-ſighted eye; ſay, Fig. 25.
$Qq : qO :: qO : qF$; and place F from *q* towards Q.

And in both caſes, OF will be the focal length of the lens required. For rays diverging from Q, will after refraction [a] diverge from *q*. Therefore, *&c. Q. E. D.*

[a] 227. I.

151. *Schol.* 1. In the above caſes, OQ is the neareſt diſtance of diſtinct viſion through the convex lens, and the remoteſt through the concave. For Q and *q* [b] moving both the ſame way, and *q* being the neareſt point of diſtinct naked viſion in the one caſe, and the remoteſt in the other: If Q be brought nearer the convex lens, the refracted rays will diverge too much; and if Q be moved farther from the concave lens, the refracted rays will diverge too little; and ſo in both caſes viſion will be confuſed.

[b] 229. I.

But, the *præſbytical* eye can ſee an object diſtinct, at a ſomewhat greater diſtance than Q; and even as far as F, or beyond F, if it can ſee diſtinctly with parallel or converging rays. In like manner, the ſhort-ſighted eye will be able to ſee diſtinctly, at a ſomewhat nearer diſtance than OQ; according to the power it has of changing its configuration.

152. *Schol.* 2. If a long-ſighted perſon wants to ſee at no nearer diſtance than half O*q*; a convex lens whoſe focal length is O*q* will be the propereſt: for then Qq being equal to QO, the point F will fall upon *q*. If the above diſtance be greater than half O*q*, OF will be greater than O*q*. Fig. 24.

153. *Schol.* 3. If, as before, O*q* be the greateſt diſtance at which a ſhort-ſighted perſon can, with his naked eye, ſee an object diſtinctly; then a concave lens whoſe focal length is O*q*, will enable him to ſee objects diſtinctly at the greateſt diſtance. For ſuch a lens will refract parallel rays ſo as to diverge from *q*. Fig. 25.

154.

154. *Schol.* 4. A ſhort-ſighted perſon may eaſily determine by experiment, the fartheſt diſtance at which he can ſee diſtinctly; and ſo, by the preceding theory, he may ſuit himſelf with a lens for any given purpoſe. But it is not ſo eaſy for a long-ſighted perſon to find his neareſt point of diſtinct viſion; for to him the ſmallneſs of objects is a cauſe of indiſtinctneſs, that neceſſitates him to approach them nearer, than according to the theory of aberrations: And therefore he'll be apt to imagine the point q to be nigher than it really is; and ſo the focal length OF, will be greater than that which is beſt ſuited for his purpoſe. In this caſe therefore, it is beſt to lay aſide theory, and to chuſe the glaſſes by trials only, remembring the cautions [a] before given.

Fig. 24.

[a] 147. II.

PROP. X.

155. *Having the neareſt diſtance of diſtinct viſion to each of two given eyes: To find a lens which ſhall put one eye in the ſame ſituation with the other, in reſpect of ſeeing objects at given diſtances; the power of changing the configuration in each being ſuppoſed equal, and neglecting alſo the conſideration of the loſs of light, and other imperfections of the lens.*

Fig. 24, 25. Let Oq and OQ reſpectively, be the neareſt diſtance of diſtinct viſion to the eye A, and to any other given eye; and let it be required to change the condition of the eye A, into that of the other ſhorter or longer-ſighted given eye. By the preceding *Prop.* find the focal length OF, of a convex or concave lens, according as the given eye is ſhorter or longer ſighted than the eye A; and becauſe rays diverging from Q, will after refraction by ſuch a lens diverge from q; the point Q is alſo now the neareſt point of diſtinct viſion to the eye A: Whence A hath now the ſame degree of ſhort or long-ſightedneſs, with the other given eye without a lens. Therefore, *&c. Q. E. I.*

Thus, if Oq is 8 inches, OQ in the one caſe 4 inches, and in the other 20 inches; the focal length of the convex lens that will cauſe the given ſhort-ſightedneſs, will be 8 inches; and the concave lens that will cauſe the given long-ſightedneſs will have its focal length $13\frac{1}{3}$ inches. But though the caſes of the two eyes above, with reſpect to the aberrations upon their retina's, will be the ſame; yet the loſs of light and other imperfections of the lens, which cannot be here accounted for, will make a very great difference.

PROP.

PROP. XI.

156. *Having the limits of diſtinct naked viſion, to a given eye; to find the limits of diſtinct viſion through a given lens, to the ſame eye.*

Let A*q* and A*r*, be the greateſt and leaſt diſtance of diſtinct viſion, Fig.25,26. to the eye A; and let OF be the focal length of the lens O, placed cloſe to the eye. To find QR the limited diſtance of diſtinct viſion through the ſaid lens; ſay,

1. $qF : qO :: qO : qQ$;

2. $rF : rO :: rO : rR$. And placing Q, R, the ſame way from *q*, *r*, that F lies from O; that is, if the lens is concave, place Q, R, farther from it, if it be convex place them nearer to it, than the given points *q*, *r*; and OQ, OR, will be reſpectively the greateſt and leaſt diſtance of diſtinct viſion, through the given lens O. For rays diverging from Q, R, will after refraction by the lens, [a] diverge upon the eye, as from the focus's *q*, *r*, reſpectively; and therefore the ſpace QR is the limited diſtance of diſtinct viſion. *Q. E. I.*

[a] 228. I.

157. *Schol.* 1. Take $Of = OF$: Now if the eye A can ſee diſtinctly, with parallel rays; it will be able to ſee diſtinctly through the lens O, as far as *f*: And if it can ſee diſtinctly with rays converging, as to a point *s*, beyond F; ſay $sF : sO :: sO : sS$; and RS will now be the limit of diſtinct viſion through the given lens O: For rays diverging from S, will after refraction by the lens, [b] converge towards *s*. The nearer *s* is to F, the greater will be OS; and if *s* be in F, an object that is at an infinite diſtance, will appear diſtinct through the ſaid lens. Fig. 26.

[b] 227. I.

158. *Schol.* 2. If the neareſt point *r* of diſtinct viſion to a ſhort-ſighted eye, biſects the line O*q*, the greateſt extent of diſtinct viſion; a concave lens whoſe focal length is O*q*, will enable that eye to ſee diſtinctly at all diſtances, from *q* onwards to infinity. Fig. 25.

For parallel rays, will after refraction by the lens, diverge from *q*; and becauſe $rF\ (rq) = rO$, the point R will fall in *q*.

And if when $Oq : Or :: 2 : 1$, OF be greater than O*q*; then OR will be leſs than O*q*.

159. *Schol.* 3. To the ſame eye, the extent of diſtinct viſion QR, Fig.25,26. through a given concave lens, is greater than through a convex lens of equal focal length.

In *fig.* 25. ſuppoſe a point *f* taken beyond the eye in the axis O*y* produced, ſo that $Of = OF$; then in both *fig.* it will be,

$qF : OF :: {}^{a}OF : fQ$; [a] 227. I.

$rF : OF :: {}^{a}OF : fR$; and therefore $qF : rF :: fR : fQ$; and $qF : qr\ (rF \backsim qF) :: fR : QR\ (fQ \backsim fR)$.

Whence

Whence it is sufficiently manifest, that QR, *fig.* 25. must be greater than QR, *fig.* 26. *Q. E. D.*

160. *Schol.* 4. In all that has been observed in the preceding *propositions* about vision through lenses, the places of the geometrical images were supposed to coincide with the retina; but as a precise union of the rays upon the retina is [a] not necessary for distinct vision; so the extent of distinct vision through lenses, will be somewhat greater than according to the preceding theory.

[a] 95, 97. II.

§. 4. *Of apparent places and magnitudes of objects.*

Apparent distance and magnitude, what.

161. *Defin.* Distance and magnitude, are relative terms; that is, such a distance, and such a magnitude is great or small, according to the relation which each bears to that standard measure, to which it is compared. *Apparent distance* is that visible extension that seems to lie betwixt us and the object, compared to some standard measure; and as it appears in the simple act of vision, abstracted from the judgment we might otherwise make from prænotions of the real distance. The apparent distance of the moon, for instance, is the length of an imaginary line that would reach from that place where the moon seems to be, to the spectator: This apparent extent is certainly not many hundred yards, though we know the moon is many thousands of miles off: And the apparent distances properly so called of remote land-objects, are also much less than at first might be imagined. In like manner, *apparent magnitude* of an object, is its visible size or extension compared to some real measure. Thus, the diameter of the moon seems to be about a foot.

In vision a visible place and extension always joined together.

162. It is very certain, that every visible object appears in a determinate place, and as having a determinate magnitude as well as figure: So that the ideas of locality and extension, are inseparable from that of visibility; and these ideas are so intimately connected together, that a mistake in our estimate of the one, never fails to create a mistake in our estimate of the other. The greater, for instance, is the apparent distance of an object subtending a given angle, the greater also will be its apparent magnitude: For, of two objects subtending equal angles at the eye, we know, as it were intuitively, that the remotest object is the largest; and we instantly, and imperceptibly to ourselves, form an idea accordingly.

Apparent distance, an idea of a definite thing.

163. It has been rightly observed, that distance, of itself and immediately, is imperceptible; for it being only a mathematical line directed endwise to the eye, it can have no image upon the retina; or, if we consider it as a physical line, its image on the retina will be only a point,

point, which point will be invariably the ſame for all diſtances. But it is not diſtance in the abſtract, but the diſtance of objects that we are to conſider; and it is very certain, that all objects have their viſible or apparent diſtance, which we can compare groſsly to ſome known and ſtated meaſure. The ſaid viſible diſtance, during the act of viſion, either remains fixed and unalterable in the mind; or, if it ſeems to enlarge or contract, it will ſeem at the ſame time to have more or fewer parts of certain ſtated lengths. So that viſible or apparent diſtance is an idea of a definite thing; however inadequate it may be in ſome caſes, to the real diſtance.

Ideas of diſtance and magnitude, acquired by experience.

164. It is yet very certain, that the ideas of viſible diſtances and viſible magnitudes, are not innate, but the reſult of experience. As ſuch a particular ſenſation had formerly been obſerved, to be cauſed by an object having ſuch a particular diſtance, magnitude, colour, brightneſs, *&c.* ſo when the like caſe happens again, we form an idea accordingly. Viſible magnitude depends upon the viſible diſtance and the viſual angle, both together: But viſible diſtance ſeems to be independent of both the other two; and as therefore it ſeems neceſſary, that in viſion the idea of viſible diſtance ſhould be formed previous to that of magnitude; it is very likely that the fact is ſo, though theſe two ideas croud ſo cloſely together, that it is impoſſible to diſtinguiſh which of the two was firſt produced. There is yet another reaſon, why we ſhould conſider diſtance firſt; and that is, that we may adapt our eyes properly for diſtinct viſion. But, as above obſerved, theſe things are too quick and tranſient, to be perceived by us.

The different appearances of objects, furniſh us with ſeveral means whereby to form our ideas of their diſtances; and we undoubtedly make uſe of all the helps that occur at the ſame time: But it is convenient to conſider them apart; and I ſhall begin with thoſe that reſult from the motions we give our eyes, in order to ſee as diſtinctly as we are able.

165. *Schol.* But before we proceed any farther, let it be obſerved here, once for all, that when we ſpeak of an *optic angle*, *focus of a pencil of rays*, or *viſual angle*. 1. It is not meant, that we ſee the point of concurrence of the two optic axes, theſe axes being nothing but imaginary lines: But nevertheleſs, as on the account of different diſtances, we give our eyes different inclinations; ſo by experience, we learn from the ſenſations accompanying thoſe motions of our eyes, to judge of ſuch diſtances as are not very great in proportion to the diſtance between the two pupils: And as the above inclinations are always analogous to to the *optic angle*, ſo we expreſs them by that angle. 2. In like manner,

the eye does not ſee a pencil of rays; but whilſt the breadth of the pupil, bears a ſenſible proportion to the diſtance of the focus from whence the rays diverge to the eye, we have peculiar ſenſations, which from the experience we have had, inform us pretty nearly of the place of that *focus*. 3. So when we ſpeak of a *viſual angle*, it is not meant that the eye meaſures that angle, as with an inſtrument: But the magnitude of an image upon the retina, being always proportional to that of the viſual angle of the object; ſo the difference of ſenſations accompanying different degrees of the one, enable us to judge of different degrees of the other. Thus, we ſee that lines and angles in optics, are not only uſeful for communicating our ideas to one another, but they are alſo not without their foundation in nature.

I. *The change of conformation in the eye, is one of the means whereby we are enabled to judge of ſmall diſtances.*

166. In viewing objects at ſuch ſmall and different diſtances, that the breadth of the pupil bears a ſenſible proportion thereto, or objects that are within about arm's length of us; at every ſenſible change of diſtance, the eye muſt alſo change its conformation for procuring diſtinct viſion. As if the eye is adapted for ſeeing diſtinctly at 8 inches diſtance; it muſt gradually change its figure, to bring upon the retina the images of objects that are more remote. And thus, if we were accuſtomed to look attentively at objects, with one eye only; it is very likely, that the changes made in the eye upon theſe occaſions, would be ſenſible enough, after repeated trials, to enable us to judge pretty accurately of the different degrees of ſmall diſtances. But for want of experience, a perſon at firſt making the trial, will be ſubject to ſome ſmall miſtakes; as has been obſerved, if a perſon, having the uſe of both eyes, attempts to thread a needle with one eye ſhut. However, it does not follow from that experiment, that a perſon in ſuch a ſituation cannot diſtinguiſh the place of a near object within an inch or two; and after a little practice, I think, that with one eye only, we can ſcarce ever miſtake $\frac{1}{4}$ or at moſt $\frac{1}{2}$ inch, in the ſituation of a near object.

Latitude within which we can thus judge of diſtances.

167. The above method of eſtimating diſtances, will not ſerve us beyond a certain ſmall extent, not greater perhaps than about 20 or 30 inches: For if beyond that diſtance, a line be ſuppoſed infinitely extended, and in this line ſeveral points be conſidered as the focus's of pencils of rays proceeding to the pupil; the difference of the degrees of divergency of the extreme rays in theſe pencils, will be inſenſible, and bear no proportion to the different diſtances of their focus's. And

[a] 129. II. beſides in theſe caſes, there are other cauſes [a] that affect viſion, much more than

than the different inclinations of the rays. Hence it is, that when the diſtance of an image, ſeen through a lens with one eye only, is greater than about a foot or a foot and a half from the lens, we are ſo liable to err in our judgment of that diſtance.

168. *Schol.* As an object that is approached too nigh [a] will appear confuſed, and the nearer it is approached, the greater will be the confuſion: So in theſe caſes, by the different degrees of confuſedneſs, we may make an eſtimate of the different nearneſſes. Remote objects may alſo have the ſame apparent confuſedneſs, though from other quite different cauſes: But, I think, we can ſcarce ever miſtake a remote object to be within a finger's length of us; and eſpecially if we uſe both our eyes, ſuch a miſtake will be next to impoſſible.

[a] 89. II.

II. *Inclination of the optic axes, is another more certain means of diſtinguiſhing degrees of ſmall diſtances.*

169. In viewing an object attentively with both eyes, we always direct the axis of each towards it; and the nearer is the object, or the point of concurrence of the two axes, the more, conſequently, they will be inclined to each other; and the more remote the object is, the leſs will be the ſaid inclination: The ſenſations accompanying the different degrees, of theſe inclinations of our eyes, together with thoſe of their different conformations for ſeeing diſtinctly; ſeem to be ſufficient of themſelves, excluſive of other aid, to enable us to judge of ſmall diſtances, not exceeding a certain extent, pretty accurately.

It is very certain, that difference of brightneſs and ſhades of objects, are circumſtances, which in many caſes affect our ideas of diſtance. But notwithſtanding the painter's art, if one approaches within a certain diſtance of his picture, he will perceive very plainly, that the whole ſurface is ſmooth and flat. In the painted hall at Greenwich, for inſtance, the pilaſters viewed at ſome yards diſtance, appear fluted; but by approaching them nigher, at a certain diſtance, their ſurfaces appear ſmooth: Therefore one can perceive a ſmall difference in the diſtances of objects placed within this extent; and what is here to help him, but the ſenſations he has from the different motions of his eyes? To one that will make it, this experiment will enable him to determine that extent, within which he can judge to a given accuracy of different diſtances, by the above helps only. But the following experiment will be more deciſive.

Behind a ſcreen, place upright two or three objects a little ſideways of one another, and at little different diſtances from an horizontal ſlit made in the ſcreen. The objects may be cylindrical ſticks of different

thickneſs, and colours; and let them be ſo diſpoſed, as their ſizes, &c. may be moſt likely to deceive the ſpectator behind the ſcreen, in his notion of their diſtances. The ſlit in the ſcreen ſhould be ſo big, as to cauſe no manner of obſtruction to the light coming from the objects to the ſpectator; and it ſhould be alſo ſo limited, as to exclude all objects from between it and thoſe deſigned for the experiment; that is, neither the floor, or any covering above, or any objects to the right or left of the above-mentioned, ſhould be viſible. Things being thus diſpoſed, and repeating the experiment ſeveral times, by changing the places of the objects, without the privity of the ſpectator beyond the ſcreen; you may determine pretty accurately, within what degrees he will be able thus to judge of different diſtances. An experiment to this purpoſe, may be eaſily made with a ſmall looking-glaſs, inſtead of the above ſcreen.

The above farther confirmed, by the apparent places of images.

170. *Schol.* The ſudden and exact eſtimates, we make of the diſtances of images behind a ſpeculum, is a farther proof of what has been above advanced: For theſe images are generally attended with ſome circumſtances, that rather tend to miſlead than to help us, in our eſtimations of their diſtances. An image in a plain ſpeculum, is always darker than the object, and this might induce us to think it farther than it really is; but nevertheleſs, if it is not far, we never miſtake its diſtance, although it be detached from others, according to the conditions of the preceding experiment. The images of objects placed at [a] 111. I. ſome diſtance before a convex ſpeculum, [a] are always very much diminiſhed; and ſo they appear to a ſpectator: But notwithſtanding this diminution, if he is not above two yards from them, he will judge of their diſtances from the ſpeculum very accurately; or he will ſee them as near as can be eſtimated, in thoſe very places from whence the rays diverge. Let a plane ſpeculum be placed cloſe to the convex one; and the different diſtances of the images of the ſame objects in each, will be very remarkable; nor will it, from a near ſtation, be poſſible to fancy them ſo remote from the convex ſpeculum, as they are from the plane one.

If I advance within 4 or 5 feet of an image, projected before a concave ſpeculum, I can define its place very preciſely; and the image itſelf, though much ſmaller than the object, will appear very perfect, and continue ſtill at the ſame place, as I advance nigher: But on my retreating farther back than my firſt ſtation, I begin to be leſs certain of its place; and the miſtake lies, in ſuppoſing it farther from me, than it really is: My faculty of diſtinguiſhing diſtance in this caſe, not being ſufficient to overcome the prejudice ariſing from the greater faintneſs, and other imperfections of the image.

171.

171. From the preceding obſervations, I think, it is very manifeſt that with both our eyes we can diſtinguiſh, pretty accurately, the places of objects that are not above five or ſix foot diſtance from us. And indeed it ſeems very neceſſary, that we ſhould have ſomething within our ſelves, or ſome means that never forſake us, whereby we might unerringly judge of diſtances ſo very near us: Otherwiſe we might be frequently in danger of our lives, without perceiving it; as well as ſubject to perpetual miſtakes, concerning objects ſo near as within the reach of our arm.

Limits within which, by the motions only of our eyes, we can eſtimate diſtances to a tolerable exactneſs.

But as the diſtance becomes greater than $1\frac{1}{2}$ or 2 yards, we begin to be leſs certain in our eſtimations of it; and beyond 3 or 4 yards, the means hitherto conſidered, ſeem to be of little or no help for aſcertaining the places of objects. For beyond that extent, the differences of the optic angles on the account of different diſtances, are ſo very ſmall, as to become in a manner inſenſible.

III. *The length of the ground-plane, or the number of intervening parts perceived in it, is another mean, by which we perceive diſtance.*

172. When the floor or ground on which we ſtand, is uniformly extended before us; in a line produced directly from us on this plane, we can diſtinguiſh that ſuch ſucceſſive parts, as form ſenſible angles at the eye, are ſucceſſive or one behind the other; and the greater number of viſible parts the line contains, the greater conſequently is the viſible extent of the whole. Let AB be the height of the eye above the ſtrait line AC: Now if the viſual angles EBD, DBC are each of them about 2 *min.* the parts ED, DC will appear as two contiguous points, each being[a] a *minimum viſibile:* But if the ſaid angles are each ſenſibly greater than 2 *min.* then three objects placed at E, D, C, may be diſtinguiſhed as being ſituated one behind the other.

Fig. 27.

[a] 83. II.

173. An extent of ground lying before us, is itſelf properly an object, the viſible length or magnitude of which, is the viſible diſtance of an object placed at the farther end of it: The ſame obſervation is alſo juſt, with reſpect to the ſide of a ſtrait wall, or hedge, *&c.* In this caſe therefore diſtance is properly viſible, ſince the intervening chain that meaſures it, is itſelf viſible: And it ſeems very probable, that from the experience we have had in meaſuring ſuch a viſible extent of floor or ground lying before us, by walking it over or otherwiſe, it is that we firſt have the idea of diſtance. This idea is ſo fixed, that all objects have their viſible diſtance, although nothing is perceived to intervene betwixt them and the ſight. A near object directly facing us, is perceived to be within or without arm's length of us, or within the length of

Idea of viſible diſtance, how acquired.

of a ruler, containing a certain number of parts of a ſtated length: This judgment being made by the preceding methods of eſtimating diſtance. A crow flying in the air has its viſible diſtance, how widely ſoever this may differ from the real diſtance; that is, in all caſes, the viſible diſtance ſeems to be a line having a definite length.

Viſible diſtance a fixed idea, and much the ſame to all people.

174. We have the ideas of viſible diſtances ſo early, that we remember nothing of our having been ever without them: And theſe ideas are ſo firmly ſettled in our infancy, that our future experience does not afterwards ſenſibly alter them. Since I learned that the moon is many thouſand miles off, her apparent diſtance is to me, not a whit greater than it was before; nor did that new-acquired knowledge cauſe any alteration in her viſible magnitude. The viſible magnitude of the ſky, and viſible diſtances in general of given objects, I imagine are much the ſame to all common eyes; although for want of practice in meaſuring, and their being apt to confound the idea of viſible diſtance with preconceived notions of the real diſtance, people will differ in their eſtimate as to the number of real feet or yards, *&c.* contained in ſuch a viſible extent. Thus, if a given viſible extent be conceived as being divided into a few number of equal parts, which may be called degrees; I take it, that moſt people will judge much alike of the places of the dividing points, and that perhaps nearly as well in proportion, as they would judge of the places of points, that divide a ruler into a like number of equal parts: And if this be the caſe, it ſhews that viſible diſtance is much the ſame to different people, how widely ſoever they may differ in their notions of the real diſtance.

Viſion a faculty, and independent of the ſenſe called feeling.

175. *Schol.* It ſeems undoubtedly true, that all our ideas are acquired. But it ſeems as true, that ſeeing is learnt ſo perfectly in our infancy, that but little addition is made to it afterwards: Or, that viſible ideas, properly ſo called, or the appearances of objects during the act of viſion, are much the ſame that they always have been *, ſo far back as our memory reaches. Viſion therefore cannot, with any propriety, be reckoned an art, as ſome would ſeem to make it; ſince the appearances of objects are the ſame to the illiterate and the men of ſcience; and alſo continue always much the ſame, unleſs it may be in ſome few uncommon caſes, which had not earlier occurred to us. The judgments we form concerning objects are ſubject to great variations, according to the greater

* It ſeems as if our own ſtature is a kind of ſcale, to which we compare the magnitudes of objects: For it is certain, that in our infancy, the ſame things look bigger than they do afterwards; and diminiſh as we grow up, ſeemingly in proportion as we have enlarged our ſcale; and that having done growing, things have much the ſame apparent magnitudes ever after. But this affects not what is above ſuggeſted; for the proportions of apparent magnitudes of different objects, are not ſubject to theſe variations: However, it ſeems to create a doubt, not eaſy to be cleared, whether the ſame things do not appear larger to ſmall-ſized people than they do to others: But this queſtion affects not the preſent argument.

greater advances we make in knowledge: But it is not theſe rational ideas that are here conſidered. Viſion then is a faculty, which conveys an image to the mind, analogous to that upon the retina; nor does it ſeem to be always founded upon prænotions from tangible ideas, or notions ariſing from feeling. How elſe could diſtorted images, as thoſe in cylindrical ſpeculums; objects ſeen in microſcopes; diagrams drawn upon paper, the like of which we had never felt with our fingers; how elſe, I ſay, could theſe, and a thouſand other objects that are not tangible, be ſo like in their appearances to different people? Theſe inſtances ſeem to ſhew, that a perſon who never had had the uſe of his hands, would have the ſame viſible ideas with another; ſo far does it ſeem from being true, that viſible ideas have that near connexion or dependance upon tangible ideas, which ſome have imagined. And indeed, are not the ideas proceeding from each of our ſenſes, independent one of the other; however one idea may bring to memory, another idea peculiar to a different ſenſe?

There is a certain extent within which, viſible diſtance is adequate to the real.

176. If an indefinite line, extended from the foot of a ſpectator directly before him, be divided into equal parts, as yards, &c. the viſual angles ſubtended by theſe parts continually diminiſh, as the ſaid parts are at a greater diſtance. Yet there is a certain extent, within which, viſible diſtance differs not ſenſibly from real diſtance: That is, if a line of the ſaid extent be divided into a ſmall number of equal parts, the viſible or ſeeming length of the remoteſt part, will not be much leſs than that of the neareſt. This extent, in moſt caſes, I do not take to be much leſs or much more, than about ten yards; and if a line of that length, drawn upon an even floor, be divided into 10 equal parts, diſtinguiſhed by little ſtones or bits of paper, and a ſpectator ſtands at one end of the line; the firſt yard or part, will not appear much bigger than the remoteſt; although it is very certain, that to a perſon of a common ſtature, ſtanding upright at the end of an horizontal line, the two firſt yards of it ſubtend a larger angle, than the reſt of the line would, if it was infinitely extended.

The nearneſs that we can diſtinguiſh degrees of diſtances in given caſes, eſtimated.

177. An indefinite line being extended obliquely before the eye, and divided into equal parts, as above; the viſible extent of thoſe parts that are beyond the limit above-conſidered, continually diminiſh, and that faſter and faſter the more remote they are; ſo as beyond a certain limit, all objects have the ſame viſible diſtance from the eye, how great ſoever the differences may be of their real diſtances. There are ſeveral cauſes which, as they occur, tend to lengthen or ſhorten the apparent extent, of the ſame real extent. But in order to make ſome eſtimate of the nearneſs, that we can diſtinguiſh degrees of different diſtances; let us ſuppoſe this nearneſs at a medium, to be as the lengths of the leaſt

viſible

visible parts, taken at given distances in an horizontal line uniformly extended before a spectator of a given stature. Let AC be that indefinite horizontal line, AB the height of the eye, suppose $5\frac{1}{2}$ foot; and suppose the *minimum visibile*, or the angle subtended by the least visible part, to be 2 minutes. Now, if the $\angle$ E B e = 2 *min.* and the distance AE be 10, 20, or 50 yards; the least visible part E e will be accordingly about $1\frac{1}{6}$ inch, $4\frac{1}{2}$ inches, or 29 inches; and therefore in the respective cases, these lengths are the least differences of real distances, that can be perceived by the eye. If the $\angle$ GBC = 2 *min.* and AG be supposed 100 yards, the least visible part GC will be near 10 foot; and so beyond 100 yards, the visible falls very short of the real distance. If AG be $\frac{1}{4}$ or $\frac{1}{2}$ of a mile, GC will be accordingly 71 yards, or 347 yards; which last number is above $1\frac{1}{2}$ furlong. And therefore two objects, whereof one is at $\frac{1}{2}$ a mile, and the other at $\frac{3}{4}$ mile distance from a spectator, will seem very near together. When AG is $\frac{1}{4}$ mile, the $\angle$ AGB = HBG is less than 15′; and when AG is $\frac{1}{2}$ mile, the said $\angle$ is only about 7′: So that if the line AG be infinitely extended, all beyond G in the one case, would not contain above 8 visible points; and in the other, it would not contain 4 visible points. If the part AG be supposed 3151 yards, which is about $1\frac{3}{4}$ mile, all the line beyond G, if it be infinitely extended, will subtend an $\angle$ at B = 2′, and therefore will be a *minimum visibile*.

Fig. 27.

Visible extent of a given line, as the height of the eye.

178. *Schol.* 1, If in a plane passing through AB, a given line DF be seen very obliquely by an eye at B; the visual angle DBF, will be as AB, the height of the eye above the line DF produced to A, nearly.

Fig. 27.

Draw Fb parallel to AB: Now the triangles DFb, DAB being similar, and their bases DF, DA being invariable; the ratio of Fb to AB will be also invariable: But the $\angle$ DBF will be as Fb, nearly. Therefore, *&c. Q. E. D.*

Visible distancevariable, for various causes.

179. *Schol.* 2. It is manifest from observations, as well as from the reason of the thing, that a given extent appears longer, according as it contains a greater number of visible parts: And hence two remote and equal distances may appear very unequal, according to the different circumstances of the intervening parts, and as the spectator is differently elevated. Thus, by the preceding, a given part of an extended plane contains a greater number of visible parts, and therefore the whole made of the collective parts, appears longer, according to the greater elevation of the eye: But without regarding the differences upon this account, let the spectator have a given station, or stand upon even ground, and there will be many circumstances, which may vary the apparent or visible distances. Thus, a hedge having in it several grown trees, generally looks longer than a clipt hedge, or the same extent of

ground

ground in the open field; becauſe the ſeveral parts of the uneven hedge are more remarkable and perceptible, than the like ſpaces upon the even ſurface of the ground. For the ſame reaſon, a row of houſes, columns, or trees regularly planted, appear longer than a plain wall of the ſame extent: For, here we have not only more viſible and remarkable parts in the one caſe, but our præ-knowledge of the ſeveral intervening objects being equidiſtant, tends to protract the apparent length of the whole chain ſtill longer. A river at firſt, looks not ſo broad, as after we have had a ſide-view of the bridge acroſs it: And indeed, a given extent of water, does not appear ſo long as the ſame extent of land; it being more difficult to diſtinguiſh parts in the ſurface of the one, than it is in the ſurface of the other. In like manner, a village ſeen from an eminence, appears farther, as a greater number of fields, *&c.* are perceived to intervene: For the hedges being remarkable objects, and the preciſe breadths of the ſeveral fields unknown; their greater number affects the idea of viſible diſtance. And univerſally, a greater number of intervening objects, all other circumſtances being the ſame, render the whole apparent diſtance always greater; but not greater in proportion to that number.

A loſs of part of the intervening chain, leſſens the viſible diſtance.

180. Again, when part of the intervening chain is inviſible, or obſcurely ſeen; the apparent diſtances of objects that are beyond that part, will be accordingly leſs. Thus, a certain extent of rough uneven ground, appears ſhorter than the ſame extent of plain ground; the prominent parts hiding the cavities behind them, the whole apparent diſtance is ſo much the leſs, by the loſs of thoſe inviſible parts. For the ſame reaſon, the brow of an hill, ſeen over the top of another, frequently looks nearer, than it does after the vale between preſents itſelf to view. And the banks of a river at ſome diſtance, will ſeem contiguous, if none of the ſurface of the river is viſible. Theſe obſervations regard only the apparent diſtances of objects that are beyond a certain limit; [a] for there is a limit, within which the apparent diſtance will not be ſenſibly affected, by the interpoſition of a viſible chain.

[a] 171. II.

An aſcending plane appears longer than a level plane of the ſame extent.

181. An aſcent, as at AT or DT, ſeen from B, contains more viſible parts than an equal extent, AC or DC, upon a level; ſuppoſing each to be alike even and ſmooth ſurfaces. For the aſcending plane being leſs oblique to the eye at B, than the horizontal plane AC; each ſucceſſive part of the one, ſubtends greater viſual angles, than the correſponding parts of the other. And hence, an aſcent appears longer, than the ſame extent of level ground; and eſpecially, an aſcent DT at a conſiderable diſtance from the eye, appears much longer than the ſame extent of level DC, at a like diſtance. And, within a certain limit, the greater

Fig. 27.

is the acclivity, or the ∠CDT, the greater will DT appear. For the ſame reaſon, a deſcending plane appears ſhorter, than an horizontal plane of the ſame extent. However, the apparent lengths of thoſe correſponding parts, will not be exactly in the ratio's of the viſual angles; for we generally make ſome allowances, though ſeldom or never enough, on account of the different inclinations of thoſe planes with reſpect to the horizon.

Before we proceed any farther on the ſubject of apparent diſtance, it will not be amiſs to make a few obſervations on the appearances of planes in given poſitions.

An horizontal plane ſeems to aſcend.

182. An horizontal plane AC ſeems to aſcend, for ſome diſtance off, from the ſpectator. For the viſual rays cut a perpendicular, EI, to the horizon, in points that are higher and higher, or nearer to the horizontal line BH, according as they belong to points in the line AC, that are more remote from A; and the images of the more and more remote parts, having the like ſituations upon the retina, as the images of objects having greater and greater elevations, we form an idea of a gradual aſcent accordingly. And becauſe the greater is AB, or the higher is the point B, the [a]greater will be the viſual angles formed by the ſucceſſive parts of AC; ſo alſo, the greater will be the apparent aſcent of AC. The apparent aſcent of the ſea, to a perſon ſtanding on the ſhore is in ſome caſes ſo remarkable, that one might expect to ſee it come rolling towards him, and overflow its banks.

Fig. 27.

*178. II.

Phenomena of the cieling and ſides of a long gallery, &c.

183. It is for a like reaſon as the above, that a cieling, as ML, appears to deſcend gradually towards the horizontal line BH; and that faſter than the floor aſcends, in proportion as BM is greater than BA. In like manner the ſides of a long room, or two parallel rows of trees, as AC, ML, ſeem to a ſpectator ſtanding between them, to converge towards the farther end; and that ſide which is fartheſt, will ſeem to converge the moſt.

But as the viſual angles EBF, FBD, &c. ſubtended by the equal parts EF, FD, &c. continually diminiſh, the more remote thoſe parts are from AB; ſo the two parallel lines ML, AC, ſeem leſs and leſs inclined to the line BH, the farther they are extended; and inſtead of appearing ſtrait lines tending towards a certain remote point, they ſeem more like the legs of two hyperbola's converging towards the line BH. And hence, a long horizontal floor or plane, and the cieling over it, ſeem convex towards the eye; or to aſcend and deſcend reſpectively leſs and leſs, the farther they are extended.

184. Again, a tall upright pillar, or the wall of a high building, as AC, will ſeem to lean forward to an eye at B, towards the vertical line BH. But this phenomenon may be partly owing to another cauſe: The centers of the images of objects, that we directly look at, are always in the axis of the eye, or upon the ſame point of the retina; and we judge of the ſituations of objects in oblique directions, according to the ſituations of their images, with reſpect to that fixed point in the eye. In looking upwards, after inclining our heads all we can, we are apt to imagine that we have brought the axes of our eyes to point towards the zenith; but this not being the caſe, our eyes looking forwards many degrees from it; whatever is in the zenith, ſeems to be as many degrees behind us. And this accounts for the miſtakes, that people generally are apt to be under, concerning the altitudes of ſtars having very high elevations. Why an upright building ſeems to lean forward.

IV. *Different degrees of apparent diſtances are ſuggeſted, by the different appearances of known objects, or by the known magnitudes of their leaſt viſible parts.*

185. As objects become more and more remote, we continually loſe ſight of parts that are in themſelves ſtill bigger and bigger; and theſe loſſes perpetually vary the countenances of objects, ſo as, within a certain limit, to afford us means of diſtinguiſhing different degrees of diſtances. Thus, when I am at the neareſt diſtance of diſtinct viſion from a given object, its ſmalleſt viſible parts, and the little wholes made up of theſe, are then moſt diſtinct of all: But, as I retreat farther back, the ſmaller parts become firſt indiſtinct, then vaniſh; afterwards theſe are ſucceeded by others that are larger, which after the ſame manner dwindle away, and become abſorbed in greater ones. And as thus I am continually loſing ſight of more and more parts, till at length they are all abſorbed in one confuſed and ill-defined whole; ſo from the experience I have had in like caſes, thoſe different appearances of the object, which may be called different degrees of diſtinctneſs, ſuggeſt to me the reſpective degrees of real diſtances.

186. *Schol.* 1. Obſervations of the above kind, are the moſt univerſal means we have, and in ſome caſes the ſureſt, for diſcovering degrees of real diſtances: And the judgments we form of the real diſtances of remote objects, ſeem in ſome meaſure to affect the idea of viſible or apparent diſtance, properly ſo called. It is very probable, that generally, the greateſt apparent diſtance is not many ſcore yards; yet we cannot help fancying, that a building, none of whoſe parts are diſcernible, appears much farther than another, whoſe windows and doors are viſible;

ſible; and this other likewiſe appears farther than a houſe, having viſible parts which are known to be ſtill ſmaller; as the bricks in the wall, tiles on the roof, *&c.* In like manner, the different appearances of trees, animals, *&c.* as they are means which ſuggeſt different degrees of real diſtance; ſo they alſo cauſe ſome difference in the apparent diſtances.

187. *Schol.* 2. The occurrence of objects of unuſual magnitudes, and detached as it were from others, by their remarkable heights, *&c.* ſometimes miſlead us in our judgment of diſtances; the greater magnitude ſuggeſting the nearer diſtance. Thus, of two unequal ſteeples or towers, wide of one another, but at about the ſame diſtance from a ſpectator who is a ſtranger in the country, the larger tower may appear neareſt; and in conſequence of this miſtake, houſes or cattle that are near the larger, will appear ſmaller than others of the ſame ſize, near the other tower. In like manner, whenever we ſee a remarkable object afar off, whoſe real magnitude is much greater than common, or than we imagine it to be; we are apt from thence to imagine it nigher than it really is. But theſe are particular caſes, which ſeldom happen; and when they do, we are rarely left without ſome other helps, whereby theſe miſtakes might be corrected.

V. *All other things being the ſame, different colours and degrees of brightneſs of objects, cauſe a difference of apparent diſtances.*

188. As objects become more and more remote, the light proceeding from their leaſt viſible parts [a] is continually diminiſhed by the heterogeneity of the intervening medium; ſo they appear gradually more faint, languid, and obſcure: Alſo their colours not only loſe of their luſtre, but likewiſe degenerate from their native hue, and participate more of the blueiſh colour of the ſky, as the rays have paſſed through a greater body of air, or as the images upon the retina are tinged with a greater proportion of ſky-light. Theſe different appearances are of help to us in judging of degrees of real diſtances, of known objects; and ſo they affect the ideas of apparent diſtances; thoſe objects that are brighteſt, and whoſe colours are moſt vivid, appearing neareſt.

[a] 30, 31. I.

189. *Schol.* 1. By the above method of eſtimating diſtances, we are alſo ſubjected to frequent miſtakes; ſince objects at a given diſtance will appear farther, the darker and more obſcure they are: And alſo, the ſame objects, and at the ſame diſtances, will ſeem nearer or farther, as the weather is more or leſs ſerene. Thus, in foggy weather, all objects ſeem farther than ordinary: The diminution of light in that caſe, anſwering to that on account of greater diſtance, we are miſ-led accordingly.

dingly. A weſterly landſcape in a clear morning, with the ſun upon it, ſeems nearer than it does, when the ſun is higher up, or more weſterly: That aſpect of the ſun being the moſt advantageous for illuminating the objects, they appear brighter, and therefore nearer. To which may be added, that the air being leſs illuminated in the morning, than when the ſun is higher up, is another cauſe that favours the phenomenon. The diſpoſitions of the clouds, alſo affect the apparent brightneſs of objects; ſo that the different caſes that may affect the apparency of the diſtances of remote objects, are innumerable. And the miſtakes we fall into, on account of different circumſtances of brightneſs, are undeniable proofs, that the degrees of brightneſs of objects, have their ſhare in forming our ideas of remote diſtances.

190. *Schol.* 2. From the ſeveral preceding obſervations, it appears, that after joining together all the helps we can have, our eſtimations of diſtances, beyond a certain limit, are groſs and uncertain; and this limit alſo varies in different circumſtances. And the more certain eſtimates we always make of near diſtances, ſeems, as has been before obſerved, to prove that in theſe caſes, the ideas are principally formed from the motion of our eyes; otherwiſe the different colours and brightneſs of objects, and alſo the magnitudes of ſuch as we had not ſeen before, would lead us into perpetual miſtakes: And the leaſt viſible parts of unknown objects, can furniſh nothing to conjecture from.

The ſituations of objects frequently miſtook, and why.

191. *Schol.* 3. As the apparent ſituation, in reſpect of the optic axis, of a broad object, or of a row of ſeveral objects, is according to the apparent diſtances of the extremes from the eye; ſo by miſtaking the neareſt extreme or object to be the remoteſt, which may frequently happen, we alſo miſtake the ſituation, with reſpect to right and left, of the ſide next towards us; and the more is the real obliquity, the more liable we are to commit theſe miſtakes. Let A, B, C, D, be a row of lamps, trees, columns, the arches of a bridge, &c. Now if a ſpectator at O, happens to imagine the neareſt object A, to be the remoteſt, as at *a*; he will conſequently imagine all the reſt to be at *b*, *c*, *d*, in a line extended on his right hand, contrary to the ſituation of the line A D, which here is to his left †: And ſo whilſt he is under the above miſtake, if he ſees a body moving from A towards D, it will appear to him to take a quite different courſe, as from *a* towards *d*. The poſition of the line *a d* will be variable, according to the imaginary proportion of O*a* to O*d*. For the viſual angles AOB, BOC, COD will be the ſame, wherever in the lines OA, OB, OC, OD, the points *a*, *b*, *c*, *d*, be taken. Fig. 28.

192.

† The learned and worthy preſident of the *Royal Society*, MARTIN FOLKES Eſq; is the author of the above ſolution.

192. When the object is but of ſmall extent, in proportion to its diſtance from the ſpectator, it is ſo much the more difficult to determine its true poſition. And hence, beyond a certain diſtance, we are very liable to miſtake the poſition of a weather-cock, the fly or ſails of a windmill, *&c.* and conſequently to miſtake the courſe of their circular motion. Thus, if a ſpectator at O, ſituated nearly in the plane of the ſails, AE, CF, imagines the neareſt end A, of the ſail AE, to be the remoteſt; and the real motion of the ſails be according to the order of the letters ABCDE: Now when A is got to B, it will ſeem in D, as if it had moved from E to D; and ſo the ſail ſeems to go round according to the order of the letters EDCBA, contrary to the true courſe of its motion. This deception is ſo ſtrong, that it is frequently an even wager, that one miſtakes the true courſe of the above circular motion. We are alſo liable to the ſame miſtakes, and that for the ſame cauſe, as to the circular courſe of a branch or hoop of lighted candles, turned round at ſome diſtance.

Fig. 29.

Greateſt apparent diſtance: or, viſible extent of the horizon.

193. Before we diſmiſs this ſubject, it will not be amiſs to make an enquiry concerning the limit or extent at a medium, of the greateſt apparent diſtance. It is very certain, that no objects appear farther than the viſible horizon; and alſo, that the horizon itſelf appears fartheſt, when the whole intermediate tract of land is viſible. But as that viſible extent will be different, [a]according to the height of the ſpectator; [a]the evenneſs or roughneſs of the ground; [a]as the ground lies either level, aſcending, or deſcending: Let us ſuppoſe a ſpectator of a common ſtature, to ſtand in the midſt of an indefinite horizontal plane, and the viſible diſtance of the horizon will be [b] leſs than two miles; ſuppoſing the leaſt viſible angle to be 2 *min.* and in the preſent caſe, we may rather ſuppoſe that angle to be more inſtead of leſs. But as the appearance of the diſtance, upon which the above calculus is founded, is itſelf contracted; the apparent diſtance of the horizon muſt be ſtill leſſen'd; and, from [b]what has been already obſerved, it cannot ſenſibly exceed a quarter of a mile. Let AC be an indefinite horizontal plane, AB the height of the eye $5\frac{1}{4}$ foot, the leaſt viſible angle $= 2'$, AG 100 yards, CG 1 yard; and ſuppoſe GC appears from B, equal to a foot, at a near diſtance: Now the $\angle AGB = GBH$ is only $63'$; and therefore upon the preſent ſuppoſition, the greateſt diſtance beyond G, would be viſibly not above $31\frac{1}{2}$ feet. If we begin our calculation from a leſſer diſtance, the whole apparent diſtance will be ſtill leſs, ſo as to fall far ſhort of 100 yards, and even ſo as not to exceed half that length. This at firſt ſight, will ſeem ſurpriſing to moſt people; and, as[c] before obſerved, it is not unlikely, but that a pre-conceived notion of a greater diſtance, may in ſome meaſure affect the idea of apparent diſtance.

[a] 179, 180, 181. II.

[b] 177. II.

Fig. 27.

[c] 186. II.

But

But to throw aſide all doubt that might ariſe, from the ſuppoſition of the ſpectator's height, the extended plane, or the leaſt viſible angle being too great or too ſmall; let us make a calculus from the apparent magnitude of an object, ſeen in the horizon. Suppoſe the moon, for inſtance, when near the horizon, appears a yard in diameter, or equal to a round table juſt at hand, whoſe diameter is a yard: The viſual angle ſubtended by the moon is about $\frac{1}{2}$ degree; and therefore the apparent diſtance of the moon is the radius of a circle, whoſe circumference is twice 360 yards. For it is manifeſt, that twice 360 apparent moons, or yards, would go round the periphery of a circle of the above magnitude. Upon the above ſuppoſition, the apparent diſtance of the moon will be about 115 yards. In like manner, by having the viſual angle, and apparent diſtance of any object, we may find its viſible or apparent diſtance. But as the moon ſeldom looks near ſo big, as above ſuppoſed, I am apt to imagine upon the whole, that the greateſt apparent diſtance, is generally much leſs than an hundred * yards; and ſometimes not above fifty: But in ſome caſes, it may probably amount to a quarter, or may be half a mile; as when we ſtand upon an eminence, viewing a large vale or a gradual aſcent, on which are a great number of objects, whoſe different diſtances are very perceptible.

194. *Schol.* One main purpoſe of viſion being to diſcover, as near as may be, the true places of objects; we do not reſt ſatisfied with mere apparent diſtance, but from thence, and from ſuch other helps as occur, we always make ſome judgment of the real diſtances of terreſtrial objects. And hence it is, that moſt people are apt to think the ſimple apparent diſtance of a remote object, to be much more extenſive than it really is. It is very certain that the apparent diſtances of two ſteeples, one at a furlong, and the other at 10 miles diſtance, are much the ſame, if no objects are perceived to intervene betwixt them; and yet we are ſeldom without helps, by which we can diſtinguiſh that one is much more remote than the other. The principal aids for eſtimating real diſtances, are thoſe which have been already conſidered; and for very remote diſtances, the [a] two laſt are of moſt uſe. By theſe helps, thoſe who have had frequent experience, as country-men and people who often travel, will be able to gueſs at the real diſtance of a remote object, within, may be, $\frac{1}{3}$ or $\frac{1}{4}$ of the whole diſtance; from the obſervations they often had made, that certain objects, at ſuch and ſuch diſtances, have ſuch and ſuch appearances. Thus, ſeamen will not err many leagues, in their eſtimations of the diſtance of an object ten

Our eſtimations of real diſtances.

[a] 185, 188, II.

* It is ſcarce neceſſary, I think, to obſerve, that by a *yard* here is meant, the apparent extent of a ruler of that length, when it is pretty near us.

ten leagues off; and as the object becomes nigher, they will be more exact in proportion.

Of apparent Magnitudes.

When the apparent and real distances are alike, the apparent are also as the real magnitudes.

195. When objects are so near us, that their apparent distances are as their real distances; their apparent magnitudes are also as their real magnitudes. This, I think, is universally true, excepting a difference with respect to two objects, which in some cases proceeds from a difference of colour and brightness. Thus, a man at a yard from me, or at six yards from me, appears in both cases of the same magnitude. And the ideas we form of the magnitudes of different objects placed in the same room, are always pretty accurate.

Apparent distances being the same, the apparent magnitudes are as the visual angles.

Fig. 30.

196. Again, the apparent distances of objects, having nearly the same colour and brightness, being the same; their apparent magnitudes will be as the visual angles. Thus, the apparent distance OA, of two objects AB, AC being the same; their apparent magnitudes will be as the visual angles AOB, AOC. That is, in this case, the two objects being really at the same distance from the eye, their apparent magnitudes are as their real magnitudes. In like manner, if the distances OA, OD are so great as to appear equal, the apparent magnitudes of the two equal objects AC, DE, will be as the visual angles subtended by them, or as AC and AB. But if, as in the preceding, the objects AC, DE are so near, that their apparent distances, are as the real ones OA, OD; then will their apparent magnitudes be as the real ones, or as AC and DE.

Greater brightness suggests greater magnitude.

197. *Schol.* The apparent distance of an object being fixed in the mind, the degree of brightness affects a little the idea of apparent magnitude; the greater brightness, for instance, suggesting a greater magnitude. Thus, if two pieces of paper, of the same figure and bigness, one whereof is white and the other black, be stuck to a wainscot of a different colour from either; the white paper will seem to be bigger than the black. As the eye receives more light from an object of a given brightness, the bigger it is; so on the other hand, its receiving more light from a brighter object, seems to be the cause of that object's appearing bigger. The phenomena above, are not very sensible, but when two adjacent objects of opposite colours, are compared together.

Apparent magnitude, in the compound ratio of the visual angle and apparent distance.

198. It is very difficult to ascertain precisely, how far, or in what proportion, apparent distance affects the idea of apparent magnitude: But it seems probable, that the latter is always either exactly or very nearly, as the visual angle multiplied by the apparent distance. If that be the case, and the apparent distance was always as the true distance, then the apparent magnitude would be always as the true magnitude; or

or the ſame object would always appear of the ſame magnitude, at all diſtances; as the caſe really is, within a few yards of us, or ſo far as apparent diſtance is adequate to the real. But objects placed beyond a certain diſtance, generally appear leſs, the more remote they are: For two parallel rows of trees, *&c.* ſeem to converge towards the farther end. But it is certain, that the diminution of apparent magnitudes, is no ways proportionable to that of the viſual angles, or to the increaſe of real diſtance: A pane of glaſs appears nothing nigh ſo big, as the front of a houſe, or ſtreet, ſeen through it: And it is certain alſo, that in moſt caſes the apparent diſtances of remote objects, fall ſo much the ſhorter in proportion of the real diſtances, as the objects are more remote, which ſerves to account for the decreaſe that is obſervable in the apparent magnitudes. So that upon the whole, it ſeems likely, that *apparent magnitudes are either exactly, or very nearly, in the compound ratio of the viſual angles and apparent diſtances.*

199. *Schol.* 1. We are as frequently deceived in our notions of magnitude, as we are in thoſe of diſtance; and theſe deceptions are farther confirmations of the preceding theory. A fly ſkipping before an unattentive ſpectator, will ſometimes excite the idea of a crow flying afar off; but as ſoon as the miſtake in the diſtance is found, the large object or the crow, will dwindle into the little fly; and one may even perceive the diſtance and magnitude, as it were, contracting both at the ſame time. In like manner have I miſtook a crow, for a fly hopping betwixt me and the window.

200. *Schol.* 2. It is obſervable, that in foggy weather, and in the duſk, objects generally appear farther than they commonly do: So, for the reaſons above aſſigned, in theſe caſes they alſo appear proportionably larger; and ſometimes they appear of a very extraordinary ſize, according to the greater miſtake in the diſtance. A ſmall heap of ſtones, has been miſtook for the ruins of a large building, *&c.* And it has been obſerved, that a chamber appears leſs, when its walls are whitened; and that fields and mountains appear leſs, when covered with ſnow: The reaſons of which muſt be, the white walls and fields appearing ſo much nearer.

Why objects appear larger in foggy weather.

201. *Schol.* 3. In forming our ideas of magnitudes, it is not enough that we know, either from former experience of our own, or from the teſtimony of perſons on whom we can entirely rely, how far ſuch or ſuch an object is from us: But we form our idea, or calculus, if it may be ſo called, of the apparent diſtance, as it were *extempore*; from ſuch circumſtances as then preſent themſelves to the ſight, and by which we uſually judge of diſtances when we are left to our ſelves. And

hence it is, that we are not able to correct ſome miſtakes, as to apparent magnitudes. An horizontal walk, terminated by two parallel ſides, will not appear ſo narrow at the farther end, as a walk upon a deſcent of the ſame length and breadth: And the parallel ſides of an aſcending walk, will yet appear to converge leſs; and in ſome caſes, it may appear as wide at the farther end, as it does at bottom, and even wider. An inſtance of this latter ſort, I have frequently before me: The parallel ſides of a ſett of ſteps facing my window, and going up to a mount on the Tower wall, ſeem manifeſtly wider at top than they do at bottom. All the above different appearances, ariſe undoubtedly from the miſtaken judgments we make of diſtances; and theſe miſtakes alſo proceed from the reaſons that have been [a] before aſſigned.

[a] 181. II.

Apparent diſtances independent of the viſual angles.

202. *Schol.* 4. The diminution of apparent magnitudes, is ſo very ſmall, in compariſon of the greater increaſe of diſtance; that in the general, it ſeems that the viſual angles ſubtended by objects, have little or no ſhare in directing our judgments of their diſtances. And indeed, if the caſe was otherwiſe, it would be almoſt impoſſible for us ever to gueſs aright, either as to diſtance or magnitude. For inſtance, if we did not judge of diſtance independently of the viſual angle, how could we know a child from a grown perſon; or even a pin from a maypole: For the largeſt object, by being removed to a greater diſtance, may ſubtend a leſs angle than any aſſignable one.

Why objects at unuſual heights appear ſmaller than ordinary.

[a] 180. II.

[b] 198. II.

203. As objects generally [a] appear nigher, when the intervening chain is not perceived; ſo in theſe caſes, we have a reaſon why they ſhould alſo [b] appear proportionably ſmaller: As the caſe is, when we look from below at objects on the top of a ſteeple; or from above, at objects at the bottom. In looking upwards or downwards, the oblique and unuſual view we have of objects, adds ſtill to our deception; as from above, we look on the crowns of hats; tops of houſes, coaches; backs of cattle, *&c.* But after frequent experience in theſe caſes, things do not look ſo ſurpriſingly ſmall, as they did formerly.

A tree upon the ground, appears longer than it did ſtanding; and a ſteeple which is viſible all the way from the bottom upwards to the top, does yet not appear ſo high, as it really is; or not ſo long, as the ſame extent upon level ground. This phenomenon ſeems to be entirely owing to our want of experience, in meaſuring heights; and its being ſuch, accounts why even a globe on the ſpire of a ſteeple, appears much ſmaller, than a globe of the ſame diameter on the ground, and at the ſame diſtance from a ſpectator. When we are deprived of that help, which a view of the whole extent of the ſteeple affords us, as in looking

ing down, *&c.* it is a reaſon why objects ſhould appear ſtill ſmaller, as I believe, all other circumſtances being alike, they always do.

204. *Schol.* To prevent miſtakes, let it be here again obſerved, that, according to the [a]*defin.* by *diſtance* in this §, was every where meant, the extent of an imaginary line joining the eye and the object; and not the diſtance betwixt two objects, unleſs it was ſo declared. Alſo, by *apparent magnitude of an object*, was meant its apparent diameter, breadth, or diſtance betwixt two of its oppoſite ſides; and therefore all that has been ſaid upon that head, is directly applicable to the apparent diſtance between two objects; and that, whether the line joining the two objects be at right angles to the optic axis, or obliquely to it: For the apparent diſtance betwixt two ſides of an object, and betwixt two objects or points in them, are caſes that are exactly coincident.

[a] 161. II.

Of the viſible or apparent figures of objects.

205. Our faculty of diſtinguiſhing degrees of diſtances, is alſo of the laſt importance to us, in diſtinguiſhing things. For an object is not a mere coloured extenſion, contained within a certain out-line; but a figure, wherein are comprehended, beſides their colour and out-lines, the particular figures, *relievo's* or ſwellings, hollowings, *&c.* of every viſible part, and member of the whole; or a figure, wherein is conſidered the diſtances, from an imaginary plane, of every viſible point. Thus, a globe placed before me does not appear a circle, but a convex ſurface, ſwelling all the way from its outward or viſible verge, towards the vertex or point neareſt to me. The inſide of an empty bowl, is viſibly a ſpherical concave: In a face, the noſe is ſeen projecting before the reſt, and the eyes are plainly ſunk lower than the forehead, *&c.*

We have other helps for diſtinguiſhing the prominences of ſmall parts, beſides thoſe by which we diſtinguiſh diſtances in general; as their degrees of light and ſhades, and the proſpect we have round them: Both theſe, I think, contribute their ſhare to the viſibility of ſuch ſmall objects as the pile upon cloth, *&c.* And by the parallax on account of the diſtance betwixt our eyes, we can diſtinguiſh beſides the front, part of the two ſides of a near object, not thicker than the ſaid diſtance; and this gives a viſible *relievo* to ſuch objects, which helps greatly to raiſe or detach them from the plane, on which they lie: Thus, the noſe on a face, is the more remarkably raiſed, by our ſeeing each ſide of it at once. Theſe obſervations, I ſay, are of uſe to us in diſtinguiſhing the figures of ſmall and near objects; and when the breaks, prominencies and projections are more conſiderable, we do not want them. The diſtances betwixt the legs of a chair, are viſible many yards off; and the projection of a building is viſible ſtill farther. But as the diſtance is increaſed, different degrees of eminencies, cavities, *&c.*

diſappear one after another; ſo that at length the corners, projections, &c. of buildings become imperceptible; and both ſquare and round towers, at great diſtances, appear alike dark and flat ſurfaces, terminated by blunt and obſcure lines. It is for the ſame reaſon, that the moon appears a flat ſurface, and not globular; the difference betwixt the diſtances of the limb and middle part from us, being much too ſmall to be perceived.

Things that we are daily converſant with, we ſurvey in different poſitions; and from this familiarity, a tranſient glance only oftentimes ſuffices, to give us an idea of even the back part which is not ſeen: And we learn alſo to make allowances, on account of the different views we have of things. A chair appears a chair, and ſeemingly much the ſame object, in various poſitions. And when we look attentively, we have frequently great helps for diſcovering the figures of objects, that we are not acquainted with. Suppoſe a perſon, who had never ſeen one before, looks obliquely at a chair: The ſeat he ſees is parallel to the floor, and ſupported in the front by two legs perpendicular to it; one of the back rails he ſees from top to bottom, and that it is joined to one corner of the ſeat; he likewiſe ſees the whole upper part, and underneath, a little of the foot of the other rail; and by conſidering the uſes and ſimilarity of the viſible parts, he will go near to form a good idea of the whole figure. The figure of that part of the cavity of a bowl viewed obliquely, which is not ſeen, will be ſuggeſted by the convexity of the outer part. But there muſt be always ſomething viſible to help us; without this, prænotions will not do: The moon appears alike flat to an aſtronomer and to a ſhepherd.

§. 5. *Of the apparent diſtances, magnitudes, and figures of the cœleſtial objects.*

Of the ſky.

206. We ſeem to be incloſed by a kind of a vaulted roof, extending every way round us, and within which all viſible objects ſeem to be included: For all remote objects, detached from the earth, ſeem to float, or as being placed in this concave ſurface, which we call the *ſky*. We are certain nevertheleſs, that inſtead of being confined within a concave ſhell of a ſmall diameter, as the ſky ſeems to be; the medium round us is tranſparent, to a diſtance far greater than even our imagination can reach; for ſuch is the immenſity of the diſtance of the fixed ſtars. We are certain alſo, that light is a mediate cauſe of viſion; and therefore, that the above ſky-like appearance muſt be owing to the inflection of light by the air, or by ſome particles floating in it.

For,

For, if light was not inflected by the atmoſphere, it is pretty manifeſt, that the appearance which we call the ſky, would vaniſh; and in its room would ſucceed a dark void be-ſprinkled with ſtars. And as we are undoubtedly ſurrounded every way with an atmoſphere, ſo we receive light from every part of it; as indeed both are proved by the phenomenon itſelf.

Its apparent figure and magnitude.

207. Every pencil of light that enters the eye, excites the idea of colour, and of a viſible point: But the particles of air themſelves, are much too ſmall to be ſeen; and as we undoubtedly receive light, by the reflection of airy particles at innumerable different diſtances; it remains ſtill to be accounted for, why the ſky ſhould appear at a particular diſtance, or as having a definite magnitude and figure.

The air being tranſparent, the ſky cannot appear nearer than the remoteſt object in view; nor do I ſee any reaſon why it ſhould appear farther, as nothing occurs here to the ſight, whereby we uſually diſtinguiſh one object to be more remote than another. The ſky therefore ſeems to touch the remoteſt land-object in view: And light being reflected to us from all parts whatever, and that much in the ſame proportion as to ſort and quantity; ſeems to be the reaſon, why the ſerene ſky ſhould be a concave ſurface, of pretty much the ſame colour. And different parts of the atmoſphere being at different times charged with heterogeneous particles, ſuited for inflecting different ſorts of light, ſeems to be the reaſon why we ſee clouds, or parts in the ſky of different colours. But to define the magnitude and figure of the ſky, we muſt have recourſe to obſervations.

The apparent diſtance of the viſible horizon [a] has been already conſidered: And by comparing the apparent diſtance between two given ſtars, or the apparent diameter of the moon, at different altitudes, we may pretty nearly determine the apparent figure of the ſky. Or, by gueſſing at the places, whereabouts perpendiculars dropt from ſeveral points in the ſky, at different altitudes, would fall, and having the angles of elevations of theſe points; we may from thence alſo make an eſtimate, of both the apparent figure and apparent magnitude of the ſky; due allowances being made betwixt the apparent and r al diſtances of the places, where the perpendiculars are ſuppoſed to fall. As if a cloud 40 degrees high, be ſuppoſed to hang over a place 50 yards off; then ſuppoſing the apparent diſtance of that place to be 40 yards, the apparent diſtance of the cloud will be about 52 yards; or the hypothenuſe of a triangle, whoſe baſe is 40 yards, and angle at the baſe 40 degrees. And thus by finding the apparent diſtances of ſeveral points, we ſhall have the apparent figure and magnitude of the whole ſky.

[a] 193. II.

The

The two ſorts of obſervations above, are to be conſidered as being but groſs ones; for people will differ in their notions, both of the apparent magnitude of the moon, and alſo of the diſtance of that point, over which ſhe ſeems ſuſpended: However, we may be certain from them, that the figure of the ſky is not, at leaſt is not always a hemiſphere; but rather a kind of ſpheroid; the apparent diſtance of the horizon being generally much greater, than that of the zenith. And this is very agreeable to the theory of the apparent diſtances of terreſtrial objects.

Variations in the viſible magnitude and figure of the ſky, from different ſituations of the ſpectator.

208. To thoſe who have very extenſive proſpects, the viſible horizon appears ſomewhat farther, and the ſurface of the ſky appears ſomewhat flatter, than to thoſe whoſe proſpects are more confined. But theſe variations are not ſo conſiderable, as might be expected from the preceding theory; and that partly, becauſe the apparent diſtances of objects fall ſo very ſhort of their real diſtances. And alſo, becauſe the ſky is an object very familiar to us, and by our moving about, frequently ſeen under various circumſtances, as to extenſiveneſs of land-proſpects; we retain much the ſame idea of its figure and magnitude, ſo long as its colour and brightneſs is the ſame alſo; even though a wall is before us, beyond which nothing is viſible but the ſky itſelf.

Difference of ſerenity, a cauſe of difference in the apparent figure, and magnitude of the ſky.

209. The different brightneſs of the ſky, at different times, is a reaſon why its apparent figure and magnitude are neither of them immutable, but variable in different weather: A greater brightneſs, as in other caſes, ſuggeſting a nearer diſtance; as objects ſeen through a fog, generally ſeem farther than uſual; and nearer, when the air is clear and the ſun ſhines upon them. The atmoſphere is generally more loaded with vapours, the lower down; and ſo the ſky commonly appears duller that way, than it does higher up: And this alſo is another reaſon, why the ſky commonly appears fartheſt towards the horizon. The atmoſphere is alſo ſubject to greater changes, the lower down; and this accounts why the apparent diſtances of the lower parts of the ſky, ſhould be ſubject to greater variations, than thoſe of the parts which are more elevated.

Phenomenon of the horizontal moon.

210. From the preceding, the reaſon is manifeſt, why the moon when near the horizon, generally appears bigger than when ſhe is elevated higher up; and alſo, why ſometimes in the ſame ſituation, ſhe appears much bigger than ordinary. The ſun alſo frequently appears of an extraordinary bigneſs, when near the horizon; but the greatneſs of his ſplendour, prevents our obſerving ſo particularly the ſun's magnitude, as we do that of the moon. It is obſervable likewiſe, that the diſtance betwixt two ſtars, appears different according to their elevations.

By

By obſerving whether the apparent diſtance of two ſtars, in the neighbourhood of the moon, keeps the ſame proportion at different altitudes, with the apparent diameter of the moon at theſe altitudes; it may be determined, whether a greater faintneſs of the moon, abſtractedly conſidered, or without regarding the ſky, may of itſelf in ſome meaſure excite the idea of a greater magnitude. But as the moon in an eclipſe, appears no bigger than ſhe ordinarily does at the ſame altitude; it ſeems very probable, that the variations in her apparent magnitude, at different elevations, are moſtly, if not wholly, to be imputed to the apparent figure of the ſky.

How the aſtronomers expreſs the apparent magnitudes of the planets.

211. *Schol.* As the apparent magnitudes of remote objects, will be different, according to the judgment every one forms of their apparent diſtances; (thus, one deſcribes the moon as appearing about the bigneſs of the crown of his hat; another thinks ſhe looks bigger, and ſo on; but none think her ſo ſmall as a pea, though a pea at a ſmall diſtance will ſubtend a larger angle, than the moon:) So, to prevent miſtakes, aſtronomers expreſs the apparent magnitudes of the planets, by their viſual angles, as they find them by actual meaſuring with inſtruments; and not by comparing them to common meaſures, as inches, feet, *&c.* Thus, the moon's diameter is ſaid to be about half a degree; becauſe thereabouts is the viſual angle ſubtended by the moon, at the diſtance we are from her. The apparent diameter of the ſun, is about the ſame bigneſs with that of the moon; although in reality, the ſun's diameter is much bigger than the diſtance between the earth and moon. The angles ſubtended by the fixed ſtars are altogether imperceptible, or too ſmall to be diſtinguiſhed; although theſe bodies may, very likely, be each as big as the ſun; ſo the ſtars are ſaid to have no ſenſible diameters.

Why the ſtars appear of different magnitudes.

212. The ſtars appearing of different magnitudes, is probably owing entirely to their different ſplendor; thoſe ſtars appearing bigger, as they excel in brightneſs. And indeed, as the fixed ſtars have no definite figures, or ſenſible diameters; what is commonly expreſſed by degrees of magnitudes, can be properly conſidered as nothing elſe, than degrees of brightneſs. With reſpect to the naked eye, all the planets, excepting the ſun and moon, may be alſo claſſed among the fixed ſtars; the appearances of them all being much alike.

Phenom. of new moon.

213. The enlightned part of the moon's diſk, a few days from the change, ſeems to be a ſegment of a larger circle, than the remaining obſcure part, which is then alſo viſible; ſo that the *anſæ*, or horns, project ſenſibly farther from the center, than the limb of the ſaid obſcure part, as repreſented in the annexed figure. This phenomenon, I think, can be no ways eaſier or more naturally accounted for, than Fig. 31.

by ſuppoſing the exceſs of apparent magnitude of the enlightned part, to be owing to its greater brightneſs. For, here the apparent, as well as the real diſtance of both parts being the ſame, both appearing contiguous and in the ſky; the greater quantity of light received from the one part, excites the idea of a greater magnitude. I have looked at the new moon through concave glaſſes of different figures; * but could not perceive any ſenſible difference, as to the proportions of the diameters of the two afore-mentioned parts. And indeed it is difficult to conceive, how the aberrations of the rays coming from the ſame object, could cauſe ſo remarkable a difference in the apparent magnitudes of two contiguous parts.

Radiations of the ſtars.

214. The apparent figures of the ſtars, is a phenomenon that preſſes hard upon the hypotheſis of aberrations and falſe images of remote objects: For by that hypotheſis, the ſtars and planets, and all bright objects ſubtending inſenſible angles, ought to have definite and circular figures, gradually decreaſing in luſtre from the middle towards the circumference. But the ſtars, and alſo lamps or candles, ſeen at a diſtance in the night, inſtead of appearing whitiſh circles, ſeem only indefinite glares of light, ſhooting out, as it were ſpontaneouſly, all round as from a center into an indefinite number of ſlender and ſharp-pointed ſpokes, or *brachia*, of different lengths. The ſtar-like appearances of ſmall ſtones, bits of glaſs, *&c.* ſeen at ſome diſtance in the ſun-ſhine, as alſo the radiations from the edges of very bright objects, having ſenſible magnitudes, muſt all probably be owing to a like cauſe.

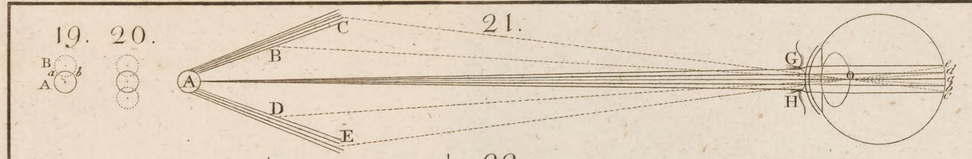
The hypotheſis for explaining the phenomenon of two beams ſhooting from a candle, will not do here; becauſe the beams are obſerved to ſhoot round every way alike: Nor can theſe phenomena be ſolved directly, by any of the foregoing principles. But it ſeems probable, that they are owing to ſomething in the conſtitution of the eye itſelf; and perhaps one of the following may be the cauſe of them.

If it be ſuppoſed, that an impreſſion made upon any point of the retina, is, by ſome vibrations of the fibres in that part, communicated and as it were ſpread further, like the undulation of water when a ſtone is thrown into it; but that, inſtead of ſpreading itſelf over one continued circular ſurface, it branches out into a ſtar-like image: And the light of a ſtar impinging but upon a ſmall point in the retina, and there being no impreſſions made round that point, at the ſame time, at leaſt but ſuch as are very faint and languid, compared to the impulſe of the ſtar's light; the undulating motion above ſuppoſed, has time to ſpread itſelf undiſturbed, and ſo to excite the ſame idea, as would be cauſed by the direct impulſe of light upon thoſe parts where the motion reaches.

* See 128. II,

Or,

19. 20.


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Or, if it be ſuppoſed, that ſome of the light proceeding from a viſible point into the pupil, inſtead of being collected with the reſt into a point on the retina, is, by an hetereogeneity in the humours of the eye, irregularly ſcattered round the center into a ſtar-like image. Either of theſe hypotheſes, will account for the above appearances of ſmall bright objects; and alſo ſhew why bright objects having ſenſible magnitudes, ſeem to branch out from the limb into pyramidal *ſpicula*, whilſt the whole ſpace within is all over bright: But they are hypotheſes only, and here offered as ſuch.

Why the ſtars appear leſs by twilight.

215. *Schol.* 1. The reaſon, why none but bright objects have theſe branches ſhooting from them, ſeems to be, that the ſky-light, which intrudes into the eye at the ſame time, overpowers and obliterates thoſe feeble impreſſions which are made by the few rays ſcattered from the true image: So that theſe impreſſions are never ſenſible, but when the reſt of the retina is very dark, in compariſon of the image of the radiating object. After the ſame manner may be accounted, why the ſtars do not ſparkle or radiate ſo much in the twilight or moon-ſhiny night, as they do when the ſky is leſs luminous; and as they ſparkle leſs, they appear leſs, being more like points: Alſo, why a ſtar near the moon, appears leſs than ordinary. The flame of a candle alſo from a remote diſtance, will have leſs apparent magnitude, as the medium between is more illuminated.

Why Venus never appears horned like the ☽.

216. *Schol.* 2. When the flame of a candle is at ſuch a diſtance, as to be a *minimum viſibile*, its pyramidical figure cannot then be diſcerned, and the candle has only a ſtar-like appearance, for the reaſons already aſſigned. It is for the ſame cauſe, that *Venus* never appears horned to the naked eye: For her whole diameter not ſubtending a ſenſible angle, it is impoſſible to diſtinguiſh her moon-like figure; and the whole diſk being as a point, Venus and all the reſt of the planets excepting the moon, have ſtar-like appearances.

Twinkling of the ſtars.

217. Thoſe who are accuſtomed to look through long teleſcopes, find that the air is generally affected with a kind of tremor or undulating motion, which is in different degrees in different weather. This tremor of the atmoſphere, ſome have thought to be the cauſe of the *ſcintillation* or *twinkling* of the fixed ſtars *. The ſtars do not appear as

* Another opinion concerning the ſcintillation of the ſtars, as it has been often printed, ſhould not be here paſſed unobſerved: *viz.* "The ſtars being but points, their twinkling is "ſaid to be cauſed by ſmall opaque particles "ſpontaneouſly floating in the atmoſphere." But although the ſtars are points, yet ſince they propagate light all manner of ways; it is pretty manifeſt, that an opaque body of a leſs breadth than that of the pupil, cannot totally hide one of them, in whatever part of the atmoſphere ſuch a body be ſuppoſed to float. And particles of that ſize in the air, are much too large to be admitted.

as if they were ſhaken out of their places, but only as if they were either nimbly covered and uncovered, or as if their light was impaired and renewed, by fits; and to an attentive eye, the brighter ſtars either do not twinkle at all, or very ſeldom in compariſon of the leſſer ones. The tremor of the air therefore, if it has any effect, does not ſeem of itſelf ſufficient to account for the phenomena: It is very likely that ſomething which paſſes in the eye itſelf, may here have the principal ſhare. We are continually ſhifting and cloſing our eyes, even when we are not conſcious of ſo doing; and a ſtar being but a point, the ſmalleſt motion of the eye is ſufficient to make us loſe it, or very ſmall opaque particles floating in the vitreous humour pretty near the retina, are capable of preventing the ſtars light from proceeding any farther. Perhaps, without any ſtoppage of the rays, the extream feebleneſs of their impreſſions, are not capable of keeping the viſive faculty conſtantly awake, but only as it were by fits. Theſe are only conjectures.

§. 6. *Of Viſion by images.*

The particular phenomena of viſion, in given caſes, by reflected and refracted light, are the chief ſubjects of the two enſuing ſections; but becauſe of their univerſality, it will not be amiſs to cloſe this with the following few obſervations.

218. Viſion of real objects ſeen directly, or naked viſion, and viſion by images, are both founded upon the ſame principles: That is, the like impreſſions, or the ſame kinds of images upon the retina, excite like ideas in both caſes. Whence,

219. Objects ſeen by reflected or refracted rays, are ſeen in the places of their laſt images, if theſe images are at moderate diſtances * before the eye; the ſeveral circumſtances by which we form the ideas of the apparent diſtances of objects ſeen by naked viſion, being here alſo taken into the account. And univerſally, every viſible point of any object appear ſomewhere in the directions of the axes of the pencils of rays proceeding from them to the eye, after their laſt reflection or refraction.

220. *Schol.* In viſion by images, we are generally deprived of many circumſtances by which we uſually judge of diſtances; and this makes it

* In naked viſion, the pencils of light always enter the pupil of the eye diverging, though in many caſes their divergencies are ſo ſmall, that they may be reckoned parallel: But in viſion by reflected or refracted light, the rays ſometimes fall on the eye converging, and yet the object, in ſome caſes, is ſeen tollerably diſtinct; but here we regard only ſuch caſes as are analogous to thoſe we meet with in naked viſion.

it difficult in moſt caſes, to determine the place of an image, if it be farther off than two or three yards [a], though we ſee it with both eyes; and with one eye alone, we cannot determine diſtance [b] beyond a foot or two. Theſe difficulties are again frequently increaſed, by ſome peculiarities appertaining to the images, which we are not accuſtomed to obſerve; and for which therefore we are at a loſs how to make the proper allowances. But when the diſtance of the image is within the above limits, we can determine its place, in moſt caſes, pretty accurately; and, as in naked viſion, the nearer we can determine the place from whence the rays diverge to the eye, the more diſtinct will the image appear.

[a] 171. II.

[b] 167. II.

221. The apparent magnitudes of objects ſeen by reflection or refraction, are, either accurately or very nearly [c], *as the rectangles under the viſual angles and apparent diſtances, of their laſt images:* Alſo in all caſes, the apparent place, poſition and figure of an object ſeen by reflected or refracted light, are as thoſe of its laſt image: For the rays * diverging to the eye, when the image is before it, from a ſucceſſion of phyſical points, after the ſame manner as if they came directly from a real object [d], excite an idea of that image, or of an object equal and ſimilar to it.

[c] 198 and 218. II.

[d] 218. II.

222. *Corol.* 1. A given ſpeculum or lens magnifies the moſt or leaſt, or diminiſhes the leaſt or moſt; when the rectangle under the viſual angle and apparent diſtance of the image, is the greateſt or leaſt poſſible.

223. *Corol.* 2. In viſion by images, we are liable to many deceptions, ſome of which are entertaining as well as ſurprizing. For not only the place of an image, but alſo very often its poſition, magnitude and even figure, ſome one or all of theſe, ſhall be quite different from the original object.

224. If in the place of an image, formed by rays converging either by reflection or refraction, be ſubſtituted a white plane, as a ſheet of paper; upon this white ſurface there will be formed a picture, exactly equal and ſimilar to the image †. And if the room be dark, ſo that the eye may be pretty free from extraneous light, this picture will be viſible to a ſpectator placed in almoſt any ſituation before it: For here the rays are reflected from every point of the white body all manner of ways, after the ſame manner as they are reflected at the ſurface of common objects. But if the white obſtacle that ſtopped them be taken away, the rays will proceed directly onwards; and therefore the image cannot be ſeen unleſs the eye be placed directly behind it: And even then the viſible field will, in moſt caſes, be much narrower than

* The caſes of viſion by parallel or converging rays, are not conſidered here.

† See 244. I.

than the picture; as the rays proceeding from parts of the image that are remote from the middle, do neceſſarily eſcape wide of the eye.

SECTION III.

Of viſion by light reflected from plane and ſpherical ſpeculums.

THE abſtract theories of images, formed both by the reflection and refraction of plane and ſpherical ſurfaces, having been already delivered in the firſt book; in this and the next ſection, what we have chiefly to conſider are the phenomena themſelves, of viſion by ſuch images.

§. 1. *Of viſion by* mirrors *or plane ſpeculums.*

225. *Phenom.* Objects ſeen by the reflection of plane ſpeculums, generally appear ſo perfect and natural, that if the ſpeculum itſelf is not perceived, one is liable to miſtake the images for the real objects of which they are types. A man appears alive, corporeal, and not a mere ſurface; and any ſeries of objects placed before the ſpeculum, exhibit alike ſeries on the other ſide, all appearing in their due places, agreeable to the [a] theory of images, and that of viſion in like caſes. The principal difference obſervable here is, that the images appear ſomewhat darker than the objects, on account of the loſs of light by reflection; and this, when the images are remote, may [b] affect their apparent diſtances; and more eſpecially if, as the caſe often is, part of the floor ſuſtaining the real objects [c] is not ſeen, or if the ſpeculum [d] is not vertical. But when the images are near, we find, as in viſion of real objects, that the above cauſes have no ſenſible effect.

[a] 37 to 49. I.

[b] 188. II.

[c] 180. II.

[d] 181. II.

Fig. 32. Let PQR be an object, whoſe image formed by the plane ſpeculum ST, is *pqr*. Now an eye placed before the ſpeculum ST, ſees every point of the object PQR, by the very ſame rays that originally diverged from thoſe points, without having undergone any other change, excepting that they are bent at the ſpeculum, as if they came directly from the correſponding points in the image *pqr*; and therefore it is, that the ſaid image appears equal, ſimilar and of the ſame colour with the real object. Thus, an eye placed at A, B, C, D or E; will accordingly

cordingly ſee the radiating points P, R, by the pencils of rays whoſe axes are S*a*, 5*b*; 1*a*, 6*b*; 2*a*, 7*b*; 3*a*, 8*b*; or 4*a*, T*b*, proceeding directly as from the reſpective points *p*, *r*. The rays that enter the pupil ſuppoſe at C, from any point P, have the very ſame divergencies as if they came from a diſtance[a] = P2 + 2C[a] = C*p*; and ſo the eye[a] 43. I. receiving the ſame impreſſions, as it would if the object P was placed in *p*, we form an idea accordingly.

226. *Schol.* If in any ſpeculum a perſon ſees his own image, or that of an object that is behind him: If the image appears erect, it will appear inverted as to right and left; and the reaſon is, becauſe the object and its image face each other, or look contrary ways. The caſe is not unlike when the object is between us and the ſpeculum, though we are apt to make it different, for want of conſidering that with reſpect to the image ſeen in the ſpeculum, it is the back of the object that is now towards us; and ſo we now call that the right ſide of the object, which we would call the left if we were on the other ſide of it. Thus, if a perſon ſees the image *pr* of PR; the image *p* appears Fig. 32. to the right wherever he ſtands; if he is behind the object, he will reckon the point P alſo to the right; but if he is between the object and the ſpeculum, he muſt turn his back to the ſpeculum to ſee PR, and then P is to his left. If PR is a printed page, the letters in the ſpeculum will be equally inverted as to right and left, whether the ſpectator be before or behind the paper. The phenomena are contrarily, when a perſon ſees an inverted image, or one that is turned upſide down; that is, if the object is behind him, the image will ſeem in its due ſituation as to right and left; but in that reſpect it will ſeem alſo inverted, if the object is between the ſpectator and the ſpeculum. But we ſeldom attend to the poſitions of images as to right and left, excepting in particular caſes.

Apparent poſition of images ſeen in ſpeculums.

The phenomena of viſion in plane ſpeculums agree, in moſt if not in all caſes, with the theories of the images; and what remains to be added under this head, is to explain the phenomena when two or more ſpeculums are combined together, or when one is placed in an inclined poſition.

PROP. XII.

227. *If two reflecting planes* BC, DC, *be placed at right angles to one another: A body* A *moving along the line* AB *perpendicular to the one ſpeculum, and conſequently parallel to the other, will have its images* a, b, *move the contrary ways; that is, while the body* A *is in motion,* Fig. 33.

the

the image b *goes onwards the ſame way, whilſt* a *moves directly towards* A; *and the motion of* A, b *and* a, *are all exactly at the ſame rate.*

It is manifeſt that the lines *a*B, *bc*, deſcribed by the images *a*, *b*, are the images of the line AB in the reſpective ſpeculums BC, DC; therefore *a*B = [a] AB, and *ab* is [a] perpendicular to BC; alſo *b*D = [a] AD, and *bc* is parallel to DC. Therefore &c. *Q. E. D.*

[a] 40. I.

Fig. 34. 228. *Schol.* If the line AB be inclined to the ſpeculums BC, CD; the lines *a*B, *b c* deſcribed by the images *a*, *b*, may be found by drawing the perpendicular A*a* to BC, and the perpendiculars A*b*, *bc* to DC; then taking *da* = *d*A, D*b* = DA, and C*c* = C*b*: Or, the lines *a*B, *bc*, may be found by making the ∠ *a*B*d* = ∠ AB*d*, and the ∠ *be*D = ∠ A*e*D.

PROP. XIII.

Fig. 35. 229. *If a plane ſpeculum,* RS, *be inclined to the horizon in an angle of* 45 *deg. with its face downwards: An upright object* CD *will have its image* cd *in an horizontal poſition; and the image* ab, *of an object* AB *lying horizontally, will be vertical.*

[a] 40. I. [b] Hypoth. For each object and its image forming [a] equal angles with the ſpeculum, and thoſe which the objects form being each [b] half a right angle; the ∠ [a] CR*c*, BS*b* are right angles. Therefore, &c. *Q. E. D.*

230. *Corol.* If RS be a plane ſpeculum, with its face upwards, AB will be the image of an object *ab*; and the image of an object *cd*, will be CD. And ſo a perſon ſtanding upright before ſuch a ſpeculum, will ſee his image extended horizontally, as it were on the floor, with his face upwards.

231. *Schol.* 1. A perſon looking in a plane ſpeculum inclined to the horizon in an angle of 45 *deg.* will ſee himſelf, as it were, in a flying poſture: For he will ſeem ſuſpended horizontally in the air, with his face downwards. And if the ſpeculum is ſufficiently long, he will ſeem to fly downwards or upwards, as he walks towards or from the ſpeculum.

Fig. 35. 232. *Schol.* 2. If the ſpeculum SR be inclined to the horizon in any other angle than 45 deg. the images *cd*, *ba*, of the vertical and horizontal objects CD, BA, will be accordingly inclined from the poſition they have now in the figure. And particularly, the objects CD, AB retaining their vertical and horizontal poſition; if the ∠ RSB be greater than 45 deg. the image *cd* will recline upwards, the point *d* being higher than *c*; and the image *ab* will recline backwards from the ſpeculum: And the contrary will happen, when the ∠ RSB is leſs than 45 deg.

233. *Schol.* 3. Upon a flat and ſmooth board FI, let there be made a grove or channel *abcde*, by means of thin ledges A, B, C, D, E glewed to the board FI; and let the edges of the boards A, B, C, &c. that are next the channel, form the ſame angles with the ſides of the rectangular board FI *. The board being thus prepared, and ledges fixed to the ſides of it, and another at *ae*; if the end FG be a little elevated, a marble or ivory ball placed at *a*, will roll with the ſame velocity, along the channel, till it gets to the bottom at *e*. Now if the board be placed at the bottom of a box, in which is a looking-glaſs ſo inclined that the board FI appears vertically, the end HI appearing uppermoſt; the ball will ſeem to roll upwards, till it gets to the top at *e*. Fig. 36. A ball ſeeming to roll upwards.

This little deceipt is ſurprizing enough till the cauſe is known, and to prevent its being immediately detected, the box ſhould be placed a little above the eye, and the aperture in the front, and the ſize of the glaſs ſhould be ſo limited, that, from a certain ſtation, nothing ſhall be viſible but the image of the board FI. In the ſide *ae* of the box, and below the end *e* of the channel, may be a narrow board, whoſe end *e* may be lifted up for throwing the ball back again to *a*; and then it will begin its journey as before. The end *e* may be lifted up by a ſtring guided by pullies to any particular place, and which might be concealed from the ſpectators various ways. Note, there ſhould be a proviſion made to prevent the ball from getting under the lifting board *ae*.

If the ſide FH of the board be placed to the front, the ball will ſeem to go almoſt directly up and down. Or, if HI be to the front, and made the uppermoſt ſide, it will be ſurprizing enough to ſee the ball roll downwards from *e*, ſo very gradually. The channel *abcde* may be made in a ſerpentine form; but then the board muſt have a greater declivity, and the motion of the ball will not be ſo uniform.

PROP. XIV.

234. *In any number of plane ſpeculums, all lying in the ſame plane, there can be ſeen from the ſame place, only one image of the ſame object.*

For in this caſe, there can be but [a] one image; and the object is ſeen by rays proceeding directly to the eye, as [b] from the image. Therefore &c. *Q. E. D.*

[a] 37. I.
[b] 41. I. and 219. II.

235.

* The breadth of the channel *abcde*, may be leſs than half the diameter of the ball; and the ball will run upon a leſs declivity, if the edges of the boards againſt which it bears, be champered or ſloped conſiderably.

235. *Schol.* If two ſpeculums be inclined backwards, ſo that perpendiculars to each, meet ſomewhere beyond them; it is manifeſt that rays diverging from any object cannot be reflected by both ſpeculums to an eye having any given poſition; and therefore they can exhibit only one image of the ſame object. See *fig.* 34. where *a* may be ſuppoſed the object, *e* the eye, BC, CD the two ſpeculums.

PROP. XV.

Fig. 37. 236. *If two plane ſpeculums* AB, AC, *be joined together ſo as to form any* ∠ BAC; *there will be a certain diſtance* AF, *within which a ſpectator will ſee his image in each ſpeculum, at the ſame time: And if to theſe be joined two other ſpeculums* BD, CE, *forming* ∠ˢ ABD, ACE; *a ſpectator placed any where within a certain diſtance* HG, *will ſee his image in each of the ſeveral ſpeculums. And univerſally, if ever ſo many plane ſpeculums are joined together in an arch of a circle, not too wide to be comprehended at one view; a ſmall object placed near the center of that circle, will be multiplied as many times as there are reflecting planes.*

Upon AB, AC, raiſe the perpendiculars BF, CF; now an object placed any where between F and A, will have its images in lines [a] parallel to FB, FC, till it arrives at A where both images will be united. Again upon BD and CE, raiſe the perpendiculars DG, BH, and EG, CH; and by the ſame way of reaſoning as before, a ſpectator placed any where between G and H, will ſee his image or part of it, alſo in BD and CE. And univerſally, if O be the center of an arch ED, any number of ſpeculums joined together within this arch, will each of them exhibit the image of an eye placed at or near O. Therefore &c. *Q. E. D.*

[a] 37. I.

237. *Schol.* As a ſpectator walks from F towards A, his images in the two ſpeculums AB, AC continually approach nearer together; and at a certain ſtation he will ſee two heads and but one body.

If the place of an object and of the eye be given, the phenomena may be eaſily [b] deduced: As if H be the place of an object, its image will be viſible in the ſpeculums AB, AC, at all diſtances; becauſe the angle of incidence HBF or HCF, is not leſs than the angle HFB or HFC. And in the ſpeculums BD, CE the image will be alſo viſible to a conſiderable diſtance beyond H; becauſe the angle of incidence HDG is not much leſs than the angle HGD. For, it is manifeſt that when the angle of incidence, is equal to the angle formed by the perpendicular to the reflecting plane at the point of incidence and the line AO, the reflected ray is [c] parallel to AO.

[b] 40 & 44. I.

[c] 11. I. & 27. I. *Euclid.*

PROP.

PROP. XVI.

238. *The images of any ſector* SRT, *contained between two reflecting planes* RS, RT, *inclined to one another in the given angle* SRT, *will be all concentric; and the number of theſe images will be ſuch, as will exactly compleat the whole circle.* Fig. 38.

The images of the ſector SRT, formed by the ſpeculums RS, RT, are[a] reſpectively the ſectors SRG, TRM: But it is manifeſt that the rays forming the image SRG fall on the ſpeculum RT, after the ſame manner as if SRG was a real object; and ſo the ſaid rays[a] form a ſecond image or ſector MRL. In like manner the rays forming the image TRM, do, by the reflection of RS, form the image GRH; and the images MRL, GRH, form the ſectors HRI, LRK, by the reflections of RS, RT reſpectively: Again, by the reflection of the rays forming either of the two laſt images, at the reſpective ſpeculums, the ſector IRK will be exhibited; and ſo on till the whole circle be compleated. But it is manifeſt, that none of the rays forming the image IRK, or which exhibit the ſpace IRK contained between the backs of the reflecting planes SR, TR produced, can be again reflected or form another image. Therefore &c. *Q. E. D.*

[a] 40. 1.

239. *Schol.* 1. When the ſector SRT is not an aliquot part of a circle, the laſt image, as IRK, will be leſs than SRT, if the two laſt perfect images HRI, LRK do not incroach one upon another; but when they do ſo incroach, they will both frequently appear only as one image, greater than SRT; but the ſaid image can never be twice as great as SRT. For it is manifeſt, that each ſpeculum forms the ſame number of images of the ſector SRT; and therefore the remaining ſector IRK, may be of any magnitude not ſo great as the double of the ſector SRT. Or, the two laſt images may perfectly join, ſo as to leave no ſpace between them.

240. *Schol.* 2. The images A, B, C, D, *a*, *b*, *c*, *d* of an object Q, placed between two ſpeculums RS, RT inclined to one another in any angle SRT, will be all in the periphery of the circle whoſe center is R, and radius $=$ RQ: There will be one image, and no more, in each image of the ſector SRT; and the arches contained between the images will be alternately $=$ 2QS and $=$ 2QT. This is ſufficiently manifeſt from the *Prop.* but it will not be amiſs to illuſtrate it farther. Upon the reflecting planes RS, RT, let fall alternately the perpendiculars Q*m*A, Q*na*; A*o*B, *apb*; B*q*C, *brc*; C*s*D, and *ctd*: Then take *m*A $=$ *m*Q, *na* $=$ *n*Q; *o*B $=$ *o*A, *pb* $=$ *pa*; *q*C $=$ *q*B, *rc* $=$ *rb*; *s*D $=$ *s*C, and *td* $=$ *tc*. Now A, B, C, D; *a*, *b*, *c*, *d*, are the images formed

by rays diverging from Q upon the ſpeculums RS, RT reſpectively,
[a] 37. I. after one, two, three, four reflections. For A, *a* are[a] firſt images of the object Q; and the rays proceeding from theſe images, fall upon the adverſe ſpeculums ſo as to form the ſecond images B, *b*; for the ſame reaſon theſe again form the third images C, *c*; and laſtly are formed the images D, *d*. But in this example, becauſe the ∠ SRT is an aliquot and alſo an even part of a circle, the two images D, *d* coincide, and make but one image. Hence it is ſufficiently manifeſt, that 1. All the images are in the periphery of the circle whoſe center is R, and radius is RQ; 2. there is one image and no more, in every image of the ſpace SRT; 3. the arch $Ab = Bc = 2QT$; $bC = aB = 2QS$; $Cd = Qa = 2QT$; and $Dc = QA = 2QS$. *Q. E. D.*

241. *Corol.* When the ſector SRT is $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{1}{20}$, &c. of a circle; the number of images of the ſaid ſector, and of an object Q placed between the two reflecting planes will be accordingly 2, 3, 4, 5, 9, 19, &c. But when the angle SRT becomes pretty acute, one or more of the laſt images will become inviſible.

Fig. 39. 242. *Schol.* 3. If two plane ſpeculums SR, TR, be placed parallel to one another; there will be a ſeries of images of the floor or ſpace between them, indefinitely extended both ways; there being no other limitation of their number, than what is cauſed on account of the decreaſe of light, by the continual reflections: Alſo an object Q placed between the ſaid ſpeculums will have a like ſeries of images, all ſituated in the indefinite line BQ*b*, perpendicular to the reflecting planes; and the diſtances between the ſaid images will be $= 2QT$ and $= 2QS$, alternately, all which is evident from the preceding *Schol.* A perſon ſtanding between two ſuch ſpeculums, will ſee in that he fronts, the images of both his fore and back part, repeated ſeveral times in a long row; but continually fainter the farther off.

PROP. XVII.

Fig. 40. 243. *Having the angle of inclination* SRT *of two plane ſpeculums* RS, RT, *and the ſituations of an object* Q, *and of an eye* O, *placed between them; to trace the whole progreſs of any ray diverging from* Q, *which after any number of reflections comes to the point* O.

To prevent the confuſion of too many lines, let us trace only thoſe rays which diverge from Q upon RS.

1. By drawing the perpendiculars QA, AB, BC, CD, upon the reſpective ſpeculums, and by taking $mA = mQ$, $oB = oA$, $qC = qB$,
[a] 240. II. and $sD = sC$; we have the places of the images A, B, C, D.

2. Draw

2. Draw a line from O to any given image (A, B, C, or D, &c.); and from the point where this line interſects the ſpeculum, draw a line to the next preceding image; and from the point where this line meets the ſpeculum, draw a line to the next preceding image, and ſo on, till you get to the object itſelf Q. This ray traced backwards from Q, will be that which exhibits the given image to the eye at O.

Thus, to find the ray that exhibits the image D; draw the line O*k*D, and from the point of interſection *k*, to the next preceding image C, drawn the line *ki*C; then draw the lines *ih*B, *hg*A, and laſtly *g*Q. I ſay Q*g*, *gh*, *hi*, *ik*, *k*O, is the ray that exhibits the image D to the eye at O.

For the ∠ O*k*T = [b] R*k*D = [c] R*k*C; and therefore the ray *ik* is reflected from *k* [d] to O. Again the ∠ *ki*S = [b] C*i*K = [c] K*i*B; and the ∠ *ih*R = [b] B*h*T = [c] T*h*A; alſo the ∠ *hg*R = [b] A*g*S = [c] S*g*Q. Therefore the ray diverging from Q to *g*, is reflected from *g* to *h*, from *h* to *i*, from *i* to *k*, and from *k* to O. *Q. E. D.*

[b] 15. I. *Eucl.*
[c] Conſt. and 4. I. *Eucl.*
[d] 11. I.

In like manner the image C, is ſeen by the ray Q*d*, *de*, *ef*, *f*O; the image B by the ray Q*b*, *bc*, *c*O; and the image A, by the ray Q*a*, *a*O.

After the ſame manner, we may trace the rays diverging from Q upon the ſpeculum RT, till they reach the eye at O.

244. *Schol.* 1. It is worth obſerving, that the pencil of rays which exhibits any image to the eye, is reflected orderly as from each of the preceding images; beginning with the firſt image, and from no other parts of the ſpeculums: And ſo the whole number of reflections, is always equal to the whole number of images; as the images A, B, C, or D, are ſeen reſpectively by rays having undergone 1, 2, 3, or 4 reflections.

245. *Schol.* 2. The diſtance OD, of any image D, from the eye at O, is equal to the whole ray proceeding from Q, till after all its reflections it arrives at O; that is OD = Q*g* + *gh* + *hi* + *ik* + *k*O.

It is manifeſt that Q*g* + *gh* = *h*A = *h*B, and *ih* + *h*B = *i*C; alſo *i*C + *ik* = *k*D. Therefore &c. *Q. E. D.*

In like manner OC = Q*d* + *de* + *ef* + *f*O; OB = Q*b* + *bc* + *c*O; and OA = Q*a* + *a*O.

246. *Schol.* 3. The laſt image, as D, of a ſmall object Q, will be inviſible, whenever the eye O is ſo ſituated that the ſtrait line OD paſſes through the center R; or ſo as not to touch either of the ſpeculums when they do not meet in R; and ſo in that caſe the next preceding image C, is to be reckoned the laſt image. Wherefore in ſuch caſes, if the arch ST be $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$, or $\frac{1}{20}$, &c. of a circle; the number of

visible images of a small object Q, will be accordingly 1, 2, 3, 4, 8, or 18, &c.

Fig. 39. 247. *Schol.* 4. In like manner as in *Art.* 243, may be traced the rays exhibiting any image of an object Q to an eye at O, placed between two parallel speculums SR, TR. Thus the image A, is seen by the rays Q*r*, *r*O; the image B, by the rays Q*s*, *st*, *t*O; the image *a*, by Q*v*, *v*O; and *b*, by the rays Q*x*, *xy*, *y*O. But the images on one side, will be visible to a much greater distance, if the eye be placed behind one of the speculums, looking through a hole in it, or over its edge, at the other speculum.

Fig. 41. 248. *Schol.* 5. By placing two plane speculums AB, CD, at the end of a box parallel to one another; and looking over or by the edge of one of them, the images of the bottom of the box will [a] appear continued to a considerable distance. Or if two speculums AC, BD be placed upright in a box, but inclined to one another; the trapesium CB [b] would be repeated on each side, till the images met; the images of the lines AB, CD being the chords of two concentric circles, whose center is the point where AC, BD produced, would meet.

[a] 242. II.

[b] 238. II.

A shew-box by plane speculums.

By joining the above four speculums together, as is expressed in the figure, the bottom of the box will be multiplied surprizingly; there being here no other limitation to the number of images, but what is owing to their continual loss of light. The top of the box may be covered with thin canvas, which will let sufficient light within, where may be placed some toy, as two people playing cards, or soldiers standing century, &c. If these be set in motion, by wiers coming from their feet through the bottom or side of the box; it will afford a spectacle still more entertaining. Or, three plane speculums joined together, will have a like effect: For the polygon formed by two of the reflecting planes, will be repeated by the other, &c. The bottom of the box may be covered with artificial moss, shining pebles, flowers, trees, &c. but whatever they are, the upright figures between the speculums, should be slender and not too many in number; otherwise they will too much obstruct the reflected rays from coming to the eye.

Fig. 42. 249. *Schol.* 6. If the sides of a little room, whose floor or base is a polygon as expressed in the figure, be all lined or covered with looking-glasses; the room being enlightened either through the top, or by a lustre of candles suspended in the middle; objects in the room will be surprizingly multiplied, and that for the same reasons as were given in the preceding. The shows lately exhibited, called *luminous amphitheatres* are of this sort. The room should not be very large; for

for the ſmaller it is, the oftner will the images be repeated before they grow too faint to be perceived.

The preceding propoſitions are, I think, ſufficient for explaining any phenomena, that may ariſe from the combinations of plane ſpeculums; and ſome of theſe phenomena are ſo very entertaining, that I believe thoſe who have a reliſh for theſe ſorts of amuſements, will not think I have dwelt too long upon them. But before we cloſe this head, it will not be amiſs to explain the theory of looking-glaſſes, conſidered as they are mediums terminated by a refracting and a reflecting ſide.

PROP. XVIII.

250. *Having the focus of incidence, of rays diverging upon a plane looking-glaſs, whoſe thickneſs is given; to find the focus of the emergent rays, after their ſecond refraction at the fore ſide of the glaſs.*

Let CF be a plane looking-glaſs whoſe back, reflecting or quick-ſilvered ſide is EF, and thickneſs is AB; and let Q be the focus of incident rays. Having through the point Q, drawn the indefinite line NP perpendicular to the ſides of the glaſs; in this line take the point G ſuch that AQ : AG :: n : * m; [a] then will G be the focus of the rays after their refraction at the firſt ſurface of the glaſs CD. 2. Take BH = BG, and H [b] will be the focus of the rays after their reflection at the ſurface EF. 3. Find the point R, by ſaying m : n :: AH : AR; and [a] R will be the focus required. That is, an incident ray, as Qa will be firſt refracted into ab, ſo as to belong to the focus G; then at b it will be reflected into bc, proceeding from the focus H; and laſtly at c, it will be refracted into ck, ſo as to belong to the focus R. *Q. E. I.* Fig. 43.

[a] 50. I.

[b] 37. I.

251. *Schol.* 1. Let Ar = AQ; that is, let r be the focus of rays reflected by the ſurface CD, before they enter the glaſs: And it will be $Rr = \frac{2n}{m}AB$; that is, in a looking-glaſs, $Rr = \frac{4}{3}$ its thickneſs.

For AH = [c] AG + 2AB; Ar (= AQ) = $\frac{n}{m}$AG; and AR (= [c] $\frac{n}{m}$AH) = $\frac{n}{m}$AG + $\frac{2n}{m}$AB; whence $Rr = \frac{2n}{m}AB = \frac{4}{3}AB$. *Q. E. D.*

[c] Conſt.

252. *Schol.* 2. Some of the rays diverging upon the looking-glaſs ED, may be ſeveral times reflected betwixt its ſurfaces, before they totally emerge; whilſt ſome others of the pencil emerge each time at the ſurface CD. The emergent rays will have different focuſes s, t, v, &c. all lying orderly one behind the other in the perpendicular BP; and the

* See Art. 35. I.

the diſtances R*s*, *st*, *tv*, &c. between the focus's *s*, *t*, *v*, will be each $= Rr = \frac{2n}{m}AB$.

Thus, of a ſlender pencil of rays ſuppoſed to flow along the phyſical line Q*a*, ſome will be immediately reflected at *a* into *ap*, belonging to the focus *r*; but the greateſt quantity will enter the glaſs, along the line *ab*, and being reflected at *b*, along *bc*, will emerge into *ck* belonging to the focus R. Again, ſome of the rays proceeding along *bc*, will be reflected into *cd*, *de*, as if they proceeded directly from I, K reſpectively; and then ſome of them will emerge along *el*, proceeding directly from *s*. In like manner, ſome of the rays may be reflected back at *e* into *ef*, then into *fg*, and at *g*, ſome will be refracted into *gm*; whilſt ſome will be returned back along *gh*, *hi*, *in*, &c. the focus's of the reflected rays *ef*, *fg*, *gh*, *hi*, will be reſpectively L, M, N, P; and of the refracted rays *gm*, *in*, will be *t*, *v*; ſo that GI, IL, LN, *&c.* and HK, KM, MP, *&c.* are each $= 2AB$; and R*s*, *st*, *tv*, &c. are each $= Rr$.

By making the angles of incidence and reflection equal; it is manifeſt that $AI = {}^{d}AH = {}^{d}AG + 2AB$; $BK = {}^{d}BI = {}^{d}BH + 2AB$ or $= BG + GI$; and therefore $GI = 2AB = KH$. In like manner it is proved that IL, KM, LN, MP, *&c.* are each $= 2AB$. Again $As = {}^{c}\frac{n}{m}AK = \frac{n}{m}AH + \frac{n}{m}HK = {}^{d}AR + \frac{n}{m}2AB$; and therefore $Rs = \frac{2n}{m}AB = {}^{c}Rr$. In like manner it may be proved that $st = tv = Rr$. Therefore, &c. *Q. E. D.*

[d] 250. II.

[c] 251. II.

Light conſiderably obſtructed by glaſs.

253. That ſome of the rays of light, are reflected ſeveral times betwixt the ſurfaces of a looking-glaſs, and that ſome of them are tranſmitted each time they come to the fore ſurface; appears very plainly from the flame of a candle ſeen obliquely in it, exhibiting a ſeries of ſeveral images: In this caſe, all the images beyond R, will continually decreaſe in brightneſs, till at laſt the rays become too faint to exhibit a viſible image. The flame of a candle will exhibit more viſible images than an opaque object, becauſe of its great brightneſs: And the ſeveral images are more diſtinguiſhable, the more obliquely they are viewed; becauſe then the angles formed by the ſpaces *r*R, R*s*, *&c.* become ſo much the larger *. In other caſes, the images will partly cover one the other; and the more ſo, the larger is the object, or the thinner is the glaſs. From the above phenomenon we have a proof, that light ſuffers a conſiderable diminution in its paſſage through glaſs; that is,

a

* It frequently happens that a looking-glaſs exhibits ſeveral images, at much greater diſtances one from another than according to the above theory; but theſe are owing to the irregularities of the ſurfaces of the glaſs.

a great part of the light which comes to the ſurface of glaſs, is prevented from going the way it ſhould take, according to the theory of refraction.

254. *Schol.* 3. The diſtance beyond the fore ſurface CD of a *looking-glaſs*, of the image of an object ſticking cloſe to that ſurface, is $= \frac{4}{3}$ the thickneſs of the glaſs. For in this caſe, the points Q, A, *r*, do coincide; and ſo $RA = Rr = {}^{a}\frac{4}{3}AB$.

[a] 251. II.

Hence we have a ready way of eſtimating the thickneſs of a looking-glaſs, by obſerving the diſtance betwixt a wier or a ſlender object laid cloſe to the glaſs, and its image, when both are ſeen pretty nearly in the ſame viſual line: For the thickneſs of the glaſs in this caſe will be $= \frac{3}{4}$ of that diſtance, nearly; and the ſaid diſtance may be eſtimated, by comparing it with the diſtance betwixt two parallel lines ſeen at the ſame time.

How to eſtimate the thickneſs of a looking-glaſs ſet in a frame.

255. *Schol.* 4. In the annexed figure, $Qabc$O is the courſe that a ray would take in coming from Q to an eye at O; if a ſlender pencil of rays impinging obliquely on the looking-glaſs CF, had its *foci* after refraction at the ſurface CD in the points G, R, found as in the preceding figure. But becauſe of the [a] aberrations of refractions, at the ſurface CD; the ray Qa will be refracted into *ad*, whoſe focus is ſomewhere in the line AG, [a] farther from Q than G. Again the ray *ad* will be reflected into *de*, making the $\angle Bdh = {}^{b}Aed = {}^{c}Aag = {}^{b}Dep$; and laſtly the ray at *e* will emerge into *eo*, whoſe focus is *x* lying ſomewhere between R and *r*; and the greater is the obliquity of the incident ray, the greater will be the aberration R*x*. For, when the point of incidence *a*, is neareſt of all to A, then $Ax = {}^{d}AR$: And it will always be $Aa : ae :: {}^{e}Ar : rx$: For the $\angle Aag$ being $= Dep$, the $\angle Deo = {}^{f}QaA = {}^{g}Aar$; and ſo *ex* is parallel to *ar*. Suppoſe the line *ag* to be produced, till it meets the perpendicular AG produced in a point *y*; then it will be $yA : AB :: {}^{e}Aa : \frac{1}{2}ae$. But as the point *a* recedes from A, the ratio of *y*A to AB, is continually increaſing; and therefore the ratio of *ae* to A*a* is conſtantly decreaſing; and ſo the point *x* is conſtantly approaching nearer to *r*. But *x* can never coincide with *r*, till A*a* becomes infinite; for till then the ratio of A*a* to *ae*, and conſequently of A*r* to *rx*, will be finite. Therefore the point *x* is always betwixt R and *r*; and as the point *a* is nearer to or farther from A, the point *x* will be nearer to R or *r*. *Q. E. D.*

Fig. 44.

[a] 56. I.

[b] 29. I. *Eucl.*

[c] 11. I.

[d] 50. I.

[e] 2. VI. *Euc.*

[f] 13. and 16. I.

[g] Conſtruct.

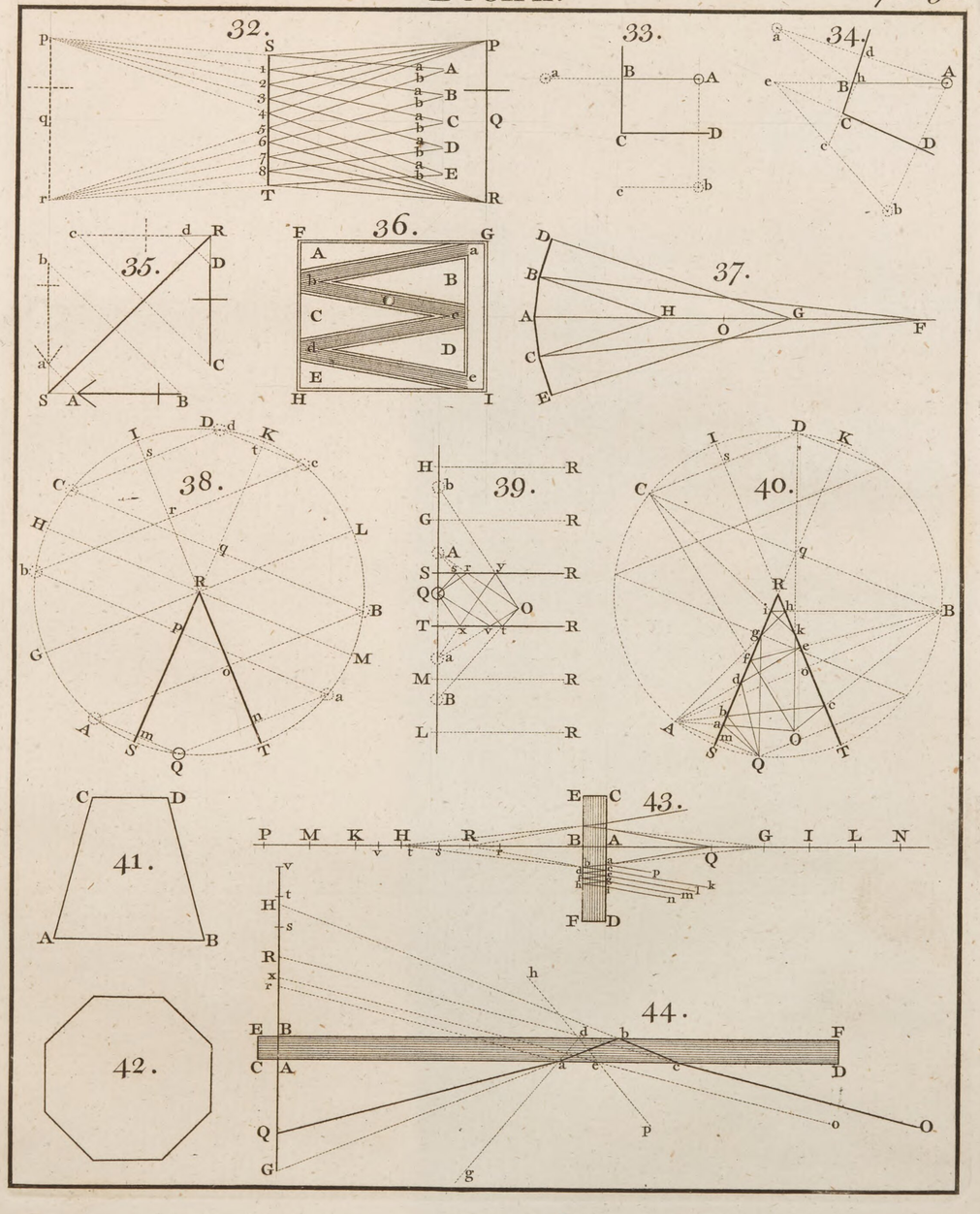
In like manner the diſtances between the focus's R, *s*, *t*, *v*, &c. which here correſpond to thoſe in fig. 43, will, as *a* recedes from A, be alſo leſſened, ſo as to be always about $= rx$. But the focus of a pencil whoſe axis is $Qadeo$, will be in the line *oex*, nearer to *e* than *x*.

For a ray diverging from Q upon a point of the ſpeculum which is
[a] 56. I. nearer to A than *a*, will have its focus after emerſion, [a] betwixt *x* and R; and a ray falling on the other ſide of *a*, will emerge in a line proceeding in a point [a] betwixt *x* and *r*; and ſo theſe two rays will interſect the line *ex*, in a point between *x* and *e*. So that a line connecting the ſeveral ſucceſſive images made by the reflections within the glaſs, or thoſe correſponding to R, *s*, *t*, *v*, &c. will be inclined from BR towards *d*, and alſo a little curved. When the ∠ AQ*a* is 60 deg. *rx* will be only about $= \frac{2}{3}$AB.

Some obſervations whereby to judge of the goodneſs of plane ſpeculums.

256. All ſpeculums ſhould be exactly figured, and well poliſhed: The goodneſs of figure of a plane ſpeculum is eaſily known, by obſerving if images, ſeen in all poſitions, eſpecially in very oblique ones, and from all parts of the ſpeculum, appear exactly equal and ſimilar to the objects; that is, if the images, eſpecially of the remoteſt objects in the room, appear natural without having any part of them diſtorted, the ſpeculum has a good figure: The ſtrait edges of the rails of wainſcot, are good objects for this experiment. A plane muſt be exceeding bad, that will diſtort a face looking into it; becauſe the rays being returned almoſt directly back to the eye, ſmall aberrations cannot be ſenſible: But let two perſons look at each other's image as obliquely as they can, and they will ſooner perceive if the figure of the ſpeculum is faulty.

It is deemed a very difficult thing to grind a true plane; and the difficulty of making a good looking-glaſs is ſtill greater, becauſe the two ſides ſhould be exactly parallel, as well as flat. If the images of a candle, ſeen very obliquely, as in *fig.* 44, and in different obliquities and from all parts of the glaſs, do not always keep pretty nearly, at equal diſtances one from another, it is a proof that the ſides of the glaſs are neither flat nor parallel. Another method, analogous to the preceding, of trying the goodneſs of a looking-glaſs, is thus: Stick a pin or ſlender wier by the point, in the upright jaum of a window frame, or in a bar of the ſaſh, ſo that the pin may be nearly horizontal and in the plane of the window; then hold the looking-glaſs, and turn it about ſo, as to ſee the images of the pin very obliquely, and from all parts of the glaſs: In this caſe, there will be ſeldom viſible but two images, correſponding reſpectively to R, *r*, *fig.* 43, 44; and if theſe images keep always exactly ſtrait, parallel, and at regular diſtances one from another, according to the theory in the preceding, the glaſs is well figured; and the phenomena, if the glaſs is good, will be alſo a confirmation of that theory. The phenomena of the different diſtances of the images from each other, will be the more conſpicuous, if two pins be ſtuck parallel to one another, and at a ſmall diſtance aſunder:


32.
33.
34.
35.
36.
37.
38.
39.
40.
41.
42.
43.
44.

By this method alſo, the correctneſs of figure of a metalline plane ſpeculum, may be alſo examined. By making ſuch trials, one ſhall obſerve a prodigious difference in the goodneſs of looking-glaſſes; and it is very rare to meet with a common glaſs, as will bear the teſt in all its parts *.

In all ſpeculums, *cæteris paribus*, the better they are poliſhed, the brighter will be the images; that is, the more light the eye will receive from given objects: And this affords us a method of examining the goodneſs of ſpeculums as to their poliſh, whenever we have an opportunity of comparing ſeveral of the ſame ſort, and in the ſame light, together. The darker is the colour of a glaſs ſpeculum, it is generally a ſign of a higher poliſh: For the glaſs itſelf can be no otherwiſe ſeen, than by the reflections of ſcabrous particles, not lying even and ſmooth with the reſt of the ſurface. But different glaſſes, though equally well poliſhed, will not always appear equally dark; yet generally the rule above takes place, and the darkeſt is to be preferred.

§. 2. *Of viſion by the reflection of ſpherical ſpeculums.*

PROP. XIX.

257. *To explain the phenomena of viſion by light reflected at a convex ſpeculum.*

In a convex ſpeculum, the images of objects are always ſeen erect, a little convexed towards the eye, and diminiſhed, yet pretty nigh to the ſpeculum: And the greater is the convexity of the ſpeculum, or the ſmaller is its radius, the nearer, the ſmaller, and the more convex will be the images of given objects.

Theſe phenomena agree exactly with the theories of [a] images, and of [b] viſion. In the annexed figure, BAC is a convex ſpeculum, whoſe center is O; and the principal *foci* in the axes OP, OQ, OR, are E, F, G; the curved line *pqr*, is the geometrical image of the ſtrait object PQR; and *svt* is the image of the object SVT. The lines ſpreading from BA towards PR, repreſent rays that originally diverged from P, and are now reflected from the focus *p*; and the lines proceeding from the part CA of the ſpeculum, repreſent rays belonging to the phyſical point R, and now diverging from the focus *r*. In like manner, as rays diverge upon the ſpeculum from every phyſical point in the object PQR;

[a] 111 to 116. I.
[b] 169 to 171. II.
Fig. 45.

* I have a ſmall looking-glaſs by me, ground in a peculiar manner by a ſkilful hand, that anſwers much better than ordinary; though by examining it very critically, one may diſcover that even this is not quite perfect.

PQR; ſo after reflection, they diverge from every correſpondent point in the image *p q r*. And as it is manifeſt, that ſome rays belonging to *p* and *r*, will after reflexion from ſome part or other of the ſpeculum, be mixed throughout the whole ſpace BCKH; ſo wherever the eye is placed within that ſpace, it will ſee the whole image *p q r*: Thus, if K*a* be the breadth of the pupil; an eye placed at K*a*, will ſee the point *r*, by the pencil of rays whoſe baſe is K*a*, and vertex is *r*; and the point *p*, by the pencil of rays K*p a*. And as this is agreeable to the manner of naked viſion, ſo *p q r* appears as a real object, of the ſame colour with PQR; becauſe it is ſeen by rays propagated from PR, without having undergone any other change, than that of being made to diverge more, which is the cauſe of the diminution and convexity of the image.

If the ſpeculum is a pretty large ſegment, it will exhibit the images of objects that are pretty wide aſunder; ſo that part of the ceiling, floor, and two ſides of the room, may be ſeen at the ſame time; the whole making a kind of * picture, very agreeable in its effect: And the nearer the eye is to the ſpeculum, the [c] larger will be the field of viſible images. If while the objects and ſpeculum remain fixed, a ſpectator moves towards or from the ſpeculum; the apparent places and magnitudes of the images, will remain invariable: But if an object moves towards or from the ſpeculum, its image will alſo appear to move the contrary way; and as it approaches nigher or recedes farther, it will accordingly appear more and more enlarged or diminiſhed. All which is entirely agreeable to the [d] theories of apparent diſtances and magnitudes of near objects; and a notable confirmation of that theory.

[c] 122. I.

[d] 169 and 195. II.

258. *Schol.* 1. If a convex ſpeculum be placed againſt a window having an extenſive proſpect, or facing the end of a ſtreet; the great multiplicity of objects that are ſeen, one behind the other, and the diminution of their images; will, ſometimes after poring into the ſpeculum, make us fancy that theſe images are a great way off, and perhaps farther than the objects are: But if we look more attentively, this miſtake will ſoon be corrected; and the apparent places of the images, will not differ ſenſibly from their real places, or thoſe places from whence the rays diverge to the eye.

259. *Schol.* 2. When the eye ſees the image very obliquely, the rays in that caſe diverging from focus's [e] that are nearer to the ſpeculum than *p q r*; the place and figure of the apparent image will be different from thoſe of *p q r*: But the image will not be ſenſibly the leſs diſtinct; becauſe

[e] 105. I.

* The appearance here is a kind of mean betwixt that of the objects themſelves, and that of a good picture of them on a flat ſurface; and upon this account therefore, and alſo for grouping the objects, a convex ſpeculum may be very uſeful to a landſkip painter.

becauſe, though the rays that enter the eye have great aberrations from the axes of the pencils, yet as they make but a ſlender pencil, they have but ſmall aberrations from one another, and therefore cauſe no confuſion.

Thus, to an eye at *b c*, the apparent image of the object PQR is *s t v*, lying in a different poſition from the geometrical image *p q r*, and betwixt it and the ſpeculum. And as the pencil of light coming to the eye from P, falls more obliquely on the ſpeculum, than that which comes from R, ſo *s p* is greater than *v r*; and for a like cauſe the curvity of *s t v*, is different from that of *p q r*; and it will ſtill vary according to the different ſituations of the eye and the object. Fig. 46.

260. *Schol.* 3. As the rays diverge much more after reflection by a convex ſpeculum, than they did before; ſo this is another reaſon, beſides that on account of the uſual loſs of light by reflection, why images ſeen in ſuch ſpeculums, appear much darker than the objects. Thus, the eye at *b c*, receives no more light from P, than what is contained in the ſlender pencil P*d*; the two extream rays of the pencil P*d*, being reflected reſpectively to *b* and *c*, the edge of the pupil. In like manner, the two extream rays in R*g*, are reflected reſpectively to *b* and *c*. But the pencil R*g* is a little broader than P*d*, becauſe it falls leſs oblique on the ſpeculum; and therefore the point *v* of the image, will be a little brighter than the point *s*. Why images in convex ſpeculums appear very dark. Fig. 46.

PROP. XX.

To explain the phenomena of viſion in ſpherical-concave ſpeculums.

This propoſition admits of three caſes, according as the object is between the ſpeculum and its principal focus; between its center and that focus; or farther off than the center. In the annexed figures, BAC is a concave ſpeculum, whoſe center is O, and principal foci are in the arch EFG: PQR is a rectilineal object, placed at right angles to the axis AOQ, of the ſpeculum; and *p q r* is its geometrical image. The ſtrait lines iſſuing from the ſpeculum, repreſent rays which diverged upon it from the reſpective points P, Q, R; and which now diverge from, or converge to, the reſpective points *p*, *q*, *r*; the aberrations of the more remote rays not being here conſidered. The rays iſſuing immediately from the object to the ſpeculum, are not here repreſented, to prevent the confuſion of too many lines. Fig. 47, 49, 50, 51.

261. CASE 1. *When the object is between the ſpeculum and its principal focus.*

In this caſe, the images of objects are always[a] erect, and[b] magnified; and the nearer is the object to the principal focus F, the greater will be Fig. 47. [a] 112. I. [b] 111. I.

be its image. If the object is a ſtrait line or a plane, placed at right angles to the principal axis AO, its geometrical image will be always [c] concave; and the nearer the point Q is to F, the greater will be the concavity of the image *pqr*. In the figure, the rays diverging from the part BA of the ſpeculum, are the rays belonging to P; and thoſe diverging from the part CA, are thoſe which came from R. If the rays belonging to P and R, had been alſo drawn from the other halves of the ſpeculum, they would have interſected throughout the whole ſpace BCKH; and therefore to an eye placed any where within that ſpace, the images of the points P, R, and conſequently the images of every other point of the object PQR, will be viſible. As if K*a* be the place and breadth of the pupil; the eye will ſee the image of the point P, by the pencil of light *ab*AK; and the image of the point R, by the pencil *ad*CK. In like manner, the eye will ſee every point of the image *pqr*, by pencils of rays, reflected from the ſpeculum between *b* and C.

[c] 116. I.

Phenom. In the caſe above, the image always appears erect, and larger than the object; and, if while the object and ſpeculum retain the ſame poſition, the eye moves a little either way in the axis AO; the apparent place, figure and magnitude of the viſible image, will continue much the ſame. Alſo, if the geometrical image is pretty near the ſpeculum, ſo that the rays in the pencils entring the eye, have ſenſible divergencies; the viſible image will appear pretty near in the ſame place, and of the ſame figure and magnitude with the true image. But when the geometrical image is remote from the ſpeculum, its apparent place will be in a manner indeterminate; and variable according to different circumſtances of light, and particularly as the eyes happen to have a different conformation for ſeeing at different diſtances. But the viſible image never appears very remote; and in ſome caſes, it will ſeem to approach the ſpeculum, when the geometrical image recedes from it; and ſometimes it will appear ſtationary, when the object is in motion.

Thus, let PR repreſent the length of the face of a perſon viewing his own image in the ſpeculum BAC, from different ſtations in the axis AF. When PR is near to the ſpeculum, its viſible image on the other ſide will be pretty nearly in the ſame place, and of the ſame figure and magnitude with the geometrical image *pqr*, as has been obſerved above; the rays in this caſe having a ſenſible divergency, and the geometric image *pqr* being not very concave. As PR recedes from A, its image for a little while will alſo appear to recede from A, but not ſo faſt as the geometric image *pqr*, till at length the viſible image

will become ſtationary; or it may even ſeem to approach towards the ſpeculum, more eſpecially if PR is an object whoſe image is viewed by a ſpectator placed near the center O. For, as the object advances towards F, the divergencies of the reflected rays become leſs and leſs ſenſible, whilſt at the ſame time the images of the ſmaller parts of the object, appear, pretty nearly, equally bright and diſtinct; alſo as our ſpectator recedes from A, his image fills a larger part of the ſpeculum; this part * being a ſegment, whoſe breadth is the meaſure of about half the angle POR. So from all theſe conſiderations, we ſee why, as the ſpectator retreats from A towards F; his apparent image at firſt retreats from the ſpeculum pretty quick; then ſlower and ſlower; at length becomes ſtationary; and at laſt may even ſeem to advance forwards †. If the ſpectator be at O, the viſual angle of an object PR will be conſtantly increaſing, as Q retreats from A towards F; that angle in this caſe being always $= \angle$ POR.

262. Again, When the apparent diſtance of the image is leſs than the geometrical diſtance; its apparent magnitude [d] will be leſs than the geometrical magnitude. And while the apparent diſtance increaſes but ſlowly, or remains the ſame; the apparent magnitude will increaſe but ſlowly, or it may remain for a little while invariable; for the viſual angle ‡ decreaſes, though ſlowly, whilſt the ſpectator ſees his image reflected from a larger part of the ſpeculum; and for this laſt cauſe, the apparent magnitude ſcarce ever ſeems to decreaſe by retreating from the ſpeculum.

Apparent magnitude of the image.

[d] 198. II.

For the above reaſons, the image of a flat and blank ſurface, will not appear ſo concave as the geometrical image: For not being able to determine the true diſtances of the different parts of this image, we ſometimes fancy it to be pretty near flat, like the object. But when the geometrical image is not very remote, our prejudice will be overcome; and the image will appear more concave, as we view it more attentively. In ſome caſes, we have farther aſſiſtance for judging more accurately of the place of the image; and then the phenomenon is ‖ ſurprizing.

263.

* If P be the eye, and PQ = QR; the image of the point R will be ſeen by a ray reflected from a point near to A, and that of P, by a ray OP after reflection.

† As the rays have but a little way in coming from the object to the eye, and therefore as many as come ſtrike as vigorouſly upon it, as they uſually do when we ſee near objects; this may be thought a cauſe why the image ſhould not appear very remote: The pencils of light are alſo denſer, than when we ſee an object at the ſame diſtance by naked viſion; and the *impetus's* upon the retina being probably chiefly in proportion to the quantities of light in each pencil, this may be yet another and more powerful cauſe of the above phenomena.

‡ If P be the eye, and PQ = QR; the viſual $\angle$ of the image will be $= APp = PAO + POA$; and the $\angle APp$ is leaſt when Q is in F.

‖ See Prop. XXI.

Phenomena of a hollow cone or ſtick held in the axis of the ſpeculum.

263. *Schol.* 1. If a hollow cone be held with its baſe towards the ſpeculum, and at ſuch a diſtance that its vertex may be near the principal focus F; its image will appear projected into a prodigious length, according to the theory of the geometrical images of different parts of the cone. The reaſon of which, is, our having in this caſe, a ſeries of theſe images, lying orderly one behind the other, we are enabled thereby to judge of the places from whence the rays diverge to the eye, more truly; and the whole viſible length is protracted [a], becauſe of its really containing a great number of viſible parts. It is for the ſame reaſon, that if a ſtrait ruler or ſtick be held before a concave ſpeculum, ſo as to lye pretty near in the axis, as AF, and ſo that the fartheſt end from the ſpeculum is pretty near to F; the image of this end, will appear much farther, than will the image of the ſtick if it be held croſs-wiſe at the ſame diſtance from F. The image of the ſtick will alſo appear conical; the remoteſt end of the image appearing the biggeſt. And for the ſame reaſon, the image of the above cone, will be nearly cillindrical, or much more acute than the cone itſelf. In like manner, and for the ſame reaſon; the noſe, and ſide of a face, will be protracted much more, and the hollows ſunk deeper, than in the proportion that the length and breadth of the face are magnified.

[a] 172. II.

Fig. 48.

264. *Schol.* 2. The pencils of light coming to the eye from an object PQR, placed before a concave ſpeculum, diverge from [a] a greater diſtance than the geometrical image *p q r*, as from *s t v*. The image *s t v* will be alſo bigger, and have a different figure than *p q r*; and if the eye is out of the principal axis, as in the annexed figure, the poſitions of *p q r* and *s t v*, will be alſo different. But the phenomena will be as in *Art.* 261, and that for the reaſons there aſſigned; excepting that when the eye ſees the image obliquely, as from *b c*, its apparent poſition will be nearly ſimilar to the real poſition of *s t v*.

[a] 105. I.

265. Case 2. *When the object is between the principal focus, and the center of the ſpeculum.*

We have before brought a ſpectator viewing his own image, as far from the ſpeculum, as its principal focus; and for the reaſons [a] that have been given, his image ſeen from that focus will appear erect, magnified, and pretty near the ſpeculum. As he retreats from hence towards the center, the geometrical image of his face [b] will be behind him, [b] advancing ſtill towards him, and at the center [b] the image and object will both unite. During this retreat, the image appears ſtill erect, becauſe the rays belonging to each viſible point, do not interſect before they reach the eye: The image becomes leſs and leſs diſtinct,

[a] 261. II.

[b] 97. I.

and at the ſame time its viſual angle is increaſing, ſo that at the center or rather before, the whole will be loſt in confuſion.

In this caſe, the picture is made before the retina; and yet the indiſtinctneſs is not like that of remote objects, but on the contrary, like that of objects viewed too near. For the ſmall parts of the object, inſtead of becoming inviſible, for want of ſubtending angles ſufficiently large, do appear larger than common; the increaſe of magnitude being manifeſt, by the image filling a larger part of the ſpeculum. But yet in the preſent caſe, I think, the image never appears before the ſpeculum, but rather as it were ſticking to it: And that perhaps may be owing to a prejudice ariſing from our frequent looking into plane ſpeculums, where the images are always on the other ſide: Or, having here no aſſiſtance from the different conformation of our eyes for viſion at different diſtances, we naturally look directly at the ſpeculum, and nothing being perceived betwixt us and it, we cannot fancy the image nearer than the ſpeculum we look at; nor can we fancy it much farther, becauſe of the manifeſt increaſe of its apparent magnitude ‡. So that here, the apparent places of the image continue pretty near the ſpeculum; and therefore its apparent magnitudes, are nearly as the viſual angles.

266. To illuſtrate this caſe farther: Let an object PQR be fixed between the principal focus F and the center O, whilſt the eye looks for its image in the ſpeculum, from different parts of the axis A*q*. If the pupil of the eye be *bc*; the images of the points P, Q, R, will be ſeen by the pencils of rays * *bd*A*c*, *befc*, *b*A*gc*, whoſe reſpective focus's are *p*, *q*, *r*: Now if the conſtitution of the eye be ſuch, that the rays contained in the above pencils are brought to unite before they reach the retina, the repreſentation of PR will be † confuſed; as remote objects appear to ſhort-ſighted perſons. But although the rays come to the eye converging, which is a caſe that never happens in naked viſion; yet as they are orderly reflected according as they come from different parts of the object, they convey an idea of that object, though an indiſtinct one, both as to its ſhape and colour. And in the preſent Fig. 49.

‡ See *Art.* 261. II. and its *note* †.

* Some have thought the confuſion in the preſent caſe, to be owing to the mixture of the pencils upon the ſpeculum: But although the largeneſs of the ſpaces upon the ſpeculum taken up by the pencils entring the pupil, are proportionable to the degree of convergencies of the extream rays; yet, from the obſervations lower down in the above paragraph, it is manifeſt enough, that this is not the immediate cauſe of the indiſtinctneſs. The pencils belonging to the adjacent viſible points of an object, are mixt upon a plane ſpeculum; and the nearer the eye is to the ſpeculum, looking at the image of an object that is behind him, the larger will be the ſections made at the ſpeculum of the ſeveral pencils: But yet no indiſtinctneſs ariſes upon that account.

† The confuſion occaſioned by the convergencies of the rays, may be remedied by a concave lens of a proper concavity; and the nearer the eye is to the image, the more concave the lens muſt be. If the eye be at *q*, the confuſion will be irremediable by any lens held cloſe to it; but it may be remedied by a concave lens held at a proper diſtance before the eye, which diſtance muſt be the greater, the leſs concave is the lens.

preſent caſe, an aged perſon might ſee diſtinct enough, and perhaps diſtincter too, than he is ever able by his naked ſight: For whatever be the diſpoſition of the rays coming from every viſible point of an object, before they enter the eye, as diverging or converging; viſion will be ſtill diſtinct, if the ſeveral pencils are not mixed upon the retina; and the apparent places and magnitudes will be according to ſome general laws of viſion in other caſes; that is, as ſome of the ordinary helps are wanting, others will preponderate.

As the eye recedes farther back from the ſpeculum, the confuſion of the image neceſſarily increaſes: As when the pupil *xy* is at the center O, it receives from the points P, Q, R, the pencils B*xyh*, *dxyg*, *kxy*C, whoſe extream rays converge about twice as much as thoſe of the former pencils. And before the eye is got quite ſo far as *q*, the whole image will be loſt in confuſion. Again, all the way from A to *q*, the image neceſſarily appears erect; and alſo its viſual angle is conſtantly increaſing.

Fig. 49. 267. *Schol.* 1. As the eye recedes from A towards *q*, the field of viſion in the given ſpeculum, of an indefinite object PR, is continually diminiſhing; ſo that beyond K, none of the rays belonging to the points P, R, can enter the pupil; and at *q*, the ſaid field will be diminiſhed to nothing. After the eye has paſt *q*, the field of viſion of the indefinite object PR, will again begin to open, and the farther back the eye is from *q*, the wider will be the ſaid field. The cauſe of all which is manifeſt, from an inſpection of the figure: *See alſo* 121. I.

After the eye has paſt *q*, the image will appear inverted, and for ſome time continue indiſtinct, as in the caſe of objects viewed too near, till the eye is got ſo far, that the rays entring it have a proper divergency for diſtinct viſion. But as the image itſelf is ſo much the rarer, in proportion as it is magnified, and the pencils of light entring the pupil continually decreaſe in denſity, as the eye retreats farther back; ſo beyond a certain diſtance, it will be difficult to find the image, unleſs it be that of a bright object, as a candle; and when it is found, for the above reaſons, the ſpectator will be apt to fancy it farther from him, than it really is; and conforming his eyes accordingly, this alſo will cauſe an indiſtinctneſs.

268. *Schol.* 2. If the object be a ſtrait ſtick or a plane at right angles to the axis of the ſpeculum, its image, to a ſpectator at a proper diſtance behind it, inſtead of appearing concave, will be convex, and that pretty much too. This phenomenon, as at firſt ſight it may ſeem to involve ſome difficulty, deſerves to be farther explained.

Let,

Let, as before, *pqr* be the geometrical image of the object PQR, after reflection by the ſpeculum BAC; and let *bc* be the breadth and place of the pupil. Now the extream rays *dtc*, *gtb* of the pencil belonging to Q, being not remote from the axis AQ*a*, will diverge to the eye from a point *t* [a] that is very near to *q*. But the neareſt ray *ksc* to the axis POD of the pencil belonging to P, that can enter the pupil, being very oblique to that axis, will interſect it at a point *s*, [a] much nearer to the ſpeculum than *p*; and the reſt of the rays belonging to this pencil will interſect the axis a little farther from *p*, than *s*; the extream ray C*yb* interſecting the ray *ksc* in *y*. In like manner, the neareſt ray *hvb*, to the axis ROH of the pencil belonging to R, interſects that axis at the point *v*; and the extream ray B*xc* interſects the other extream ray *hxvb*, at the point *x*. And ſo the apparent image is a convex curve *ystvx*, agreeable to the theory. Fig. 50.

[a] 105. I.

269. CASE 3. *When the object is farther from the ſpeculum than its center.*

In this caſe the geometrical image is always [a] inverted, a little [b] concave if the object is a plane, and [c] between the center and principal focus of the ſpeculum. The farther the object is from the center, the [d] leſs will be its image, and the [c] farther from the ſaid center, or the nearer to the focus; and the converſe. If the object be in the center, its image will coincide with it.

[a] 112. I.
[b] 116. I.
[c] 97. I.
[d] 111. I.

Phenom. Let PR, repreſent a ſpectator viewing his own image, at a conſiderable diſtance beyond the center O of the ſpeculum: In this caſe, his image will appear inverted, ſmall, faint and indiſtinct: It is faint, becauſe but few rays are contained in the pencils that enter the pupil; and this faintneſs, [e] cauſes the apparent diſtance to be ſo much the greater, ſo that the image generally appears beyond the ſpeculum; and this miſtake adds to the indiſtinctneſs. As our ſpectator advances forward, his image will appear gradually larger and brighter, and alſo to approach nearer to him; but if he does not ſee it betwixt him and the ſpeculum, it will continue ſtill indiſtinct. Fig. 51.

[e] 188. II.

Phenom. of a ſtick held before the ſpeculum.

270. In the caſe above, if the obſerver holds out a ſtick with its end towards the ſpeculum, he will immediately perceive its image come out to meet him; the length of the image now aſſiſting him to judge more accurately of its diſtance: The image of the ſtick will appear ſhorter than the ſtick itſelf; and if the object be a cillinder or a ruler with parallel ſides, the image will appear taper; that end being ſmalleſt which is held in the hand, or which appears fartheſt off: Both which phenomena, of the ſhortening and tapering of the image, are exactly agreeable to the theory of the geometrical image. If the ſtick be

ſtretched onwards nearer to the ſpeculum, its image will be ſeen to advance farther out; and when the end of the ſtick is in O, its image will be ſeen there alſo. If the ſpectator advances farther, he will ſee the ſtick and its image paſs by one another. After this, ſtretching out his arm without the ſtick, he will ſee its image come out, and he may graſp, as it were, his own fiſt: And if he now minds the image of his face, he will ſee it ſuſpended in the air, at the proper place, betwixt him and the ſpeculum.

When the inverted image appears diſtincteſt.

271. As ſoon as the image is ſeen in its true place, it will become a determinate and fixed object, whereas before it ſhifted its place as it were caſually; and then, and not till then, it will appear perfectly diſtinct; and ſmall ſpecks or pits in the ſkin, though diminiſhed, will yet be well defined. This phenomenon is ſo ſtrong and lively, that I have known people ſtartled at their own image, the firſt time of their obſerving it.

After making a few trials, a perſon will find the true place of his image pretty readily, if he is not too far off or too near it; and it will continue diſtinct, as he moves to or fro, for ſome diſtance. When he has got within a few inches of it, the image will begin to be indiſtinct; and this indiſtinctneſs will increaſe, till he has got to the center O, where the image will be loſt in confuſion. If he advances beyond the center, he'll begin to have ſome glimmering of an erect image, which will grow diſtincter as he advances forwarder, as has been already explained.

272. If PQR be an object remaining fixed, while the eye is in motion along the axis AOQ; the image will appear inverted all the while that the eye is behind *pqr*; and when it is at ſuch a diſtance, that the image appears at the place *pqr*, it then alſo appears diſtinct. If the eye is far behind the image, it will appear faint for want of ſufficient light; and if it be too near it, it will be indiſtinct becauſe of the two great divergencies of the rays: If the eye gets before the image, it will appear erect and confuſed, which confuſion will leſſen as the eye gets nearer the ſpeculum, becauſe the convergencies of the rays will be leſſened.

If the eye be fixed, while the object PQR moves towards the center of the ſpeculum; as the real magnitude of the image will be thereby all the way increaſing, ſo alſo will its apparent magnitude. If the eye is pretty near the center O, the above motion of the object will cauſe an indiſtinctneſs, which will increaſe as the image approaches nearer the eye. If the eye had been placed between the image and the ſpeculum, the indiſtinctneſs of the image would have leſſened, as

the object advanced nearer towards the center. And in both cases, the contrary would happen, if the object retreated farther back.

273. *Schol.* 1. In the two last cases, if the eye be out of the principal axis, the figure and position of the image formed by the pencils entring the pupil, will be different from those of *p q r*; as has been before[a] exemplified in other cases. But as generally here, vision is indistinct, and the apparent image necessarily differs in some circumstances from that formed by the concourse of the rays entring the pupil, it would be needless to dwell upon this any farther.

[a] 259 and 264. II.

When the eye is in the axis *Qq*, at a small distance behind *p r*; the apparent image will be less concave than *pqr*, and may be even convex; because the pencils entring the pupil from the extream parts, diverge from focus's that are[b] nearer the speculum than the points *p*, *r*, as in *Art.* 268. Fig. 51.

[b] 105. I.

274. *Schol.* 2. When the eye is in or near the principal axis, and between the image *p q r*, of a plane object PQR, and the speculum; the image generally appears concave or hollow. In this case the object is somewhat less indistinct about the middle, than towards the extreams; because the rays belonging to the middle of the object converge less than the others, at their entrance into the pupil. And this seems to be the reason why the middle appears farther, or at a properer distance for distinct vision; the greater indistinctness in this case, suggesting a nearer distance of the object, as when we approach one too nigh, though the causes are quite opposite. See 265. II. Fig. 49, 50, 51.

275. *Schol.* 3. A small image *q* of an object Q, seen with both eyes in a concave speculum, will appear double, as at *m* and *n*; if the optic axes *avF*, *bvF* are not directed to *q*, but to some point F beyond it. This phenomenon frequently happens, as we are apt to look towards or beyond the speculum, for the image; and the[a] cause of it is, the points *r*, *s*, where the visual rays *Qmqvr*, *Qnqvs* meet the retina, falling on different sides of the points *a*, *b* in the optic axes. In like manner, if *p q r*, fig. 49, be the image of an upright and slender object, as a wier, &c. this image to a spectator beyond it, will also frequently appear double, and that for the same cause. Fig. 52.

[a] 59. II.

Again, when the image *q* is behind the spectator; there will appear a double image, if the speculum be broad enough, to throw rays diverging upon it from Q, into both eyes. For in this case also, the points *r*, *s*, where the visual rays *Bvr*, *Cvs*, meet the retina as they are tending towards *q*, are on different sides of the optic axes; and therefore vision is double: The case here is the same, as when the object seen, is farther than the point we look at; which is the reverse of the preceding. Fig. 53.

Fig. 51, 45. 276. *Schol.* 4. The inverted image of a perſon ſtanding a little way behind it, appears to him equally bright and diſtinct, with his image in a convex ſpeculum of the ſame radius with the concave one, and when he is at ſuch a diſtance that his image is of the ſame ſize in each. For, it is manifeſt that the denſities and divergencies of the rays in the ſeveral correſponding pencils, at their entrance into the eye, are the ſame in both caſes.

I have been the more particular in endeavouring to illuſtrate the phenomena of viſion in ſpherical ſpeculums, not only becauſe ſeveral of theſe phenomena are curious and entertaining, but becauſe they alſo greatly illuſtrate and confirm the theory of viſion in general. In the concave ſpeculum, we have a quick tranſition, by a ſmall motion of the object, through all the caſes that can happen, as to the different degrees of the divergencies and convergencies of the rays at their entrance into the eye: And the ſeveral phenomena do all concur to prove, that our viſible Ideas are always conformable to certain general laws, notwithſtanding any prejudices we may have in oppoſition to them; ſo far are we from being left to the uncertain guidance of mere fancy. The opportunity we have of viewing the images here with both eyes, is an advantage we ſeldom have when we look through lenſes; and ſo the phenomena are generally more ſtriking, conſtant and regular in the former, than they are in the parallel caſes in the latter. Before we diſmiſs this ſubject, it will not be amiſs to ſubjoin the following obſervations.

Fig. 54. 277. The image of an object PR, placed in the principal focus of a concave ſpeculum, will appear under the ſame viſual angle to an eye D, wherever in the axis the point D is taken. For, neglecting the aberrations, the viſual rays pD, rD, [a] will be always parallel to the reſpective axes OPB, ORC; and ſo the $\angle^s pDr$ will be always $= \angle POR$ ſubtended by the object from the center. But the apparent magnitude of the image will be greater or leſs, as it appears farther or nearer.

[a] 85. I.

Fig. 55. Alſo, if the eye be placed at the principal focus F of the ſpeculum; the viſual angle ſubtended by the image of an object PQR, will be invariably the ſame, wherever in the axes the object be placed. For the rays PB, RC will be reflected to F, wherever in the indefinite lines BP, CR the points P, R be taken; and ſo the viſual angle will be always $=$ BFC. In this caſe the image will appear always erect; but its diſtinctneſs, apparent place and magnitude, will be variable, according to the different places of PQR.

Fig. 45, 47, 49, 50, 51. 278. *In all caſes, the object and its geometrical image, form each the ſame angle at the vertex of the ſpeculum*; that is, the $\angle PAR = \angle pAr$.

At the points Q, *q*, draw perpendiculars to the axis QO, till they meet the axes or lines P O*p*, R O*r*, produced if need be; and, to avoid having new references, let theſe points of interſections be called reſpectively P, *p*; R, *r*. Now it will be, QO : *q*O[a] :: QP : *qp*; but becauſe QF : AF (=OF)[b] :: OF : F*q*; it will alſo be AQ (QF ± AF) : OF :: A*q* (AF ± F*q*) : F*q*. But OF : F*q*[c] :: QO : *q*O; and therefore QO : *q*O :: AQ : A*q* :: QP : *qp*. But becauſe the ∠ PQA = [d] *p q* A, the ∠ QAP = [e] *q* A*p*. After the ſame manner is proved that the ∠ QAR = *q*A*r*; and therefore the whole ∠ PAR = *p*A*r*. *Q. E. D.*

[a] 4. VI. *Euc.*
[b] 94. I.
[c] [94]. I.
[d] Conſtr.
[e] 6. VI. *Euc.*

279. *Corol.* Hence the ∠ *p*A*r* is always increaſing, whilſt the object is approaching nearer towards the ſpeculum; and in a convex ſpeculum, the magnitude of the image is alſo increaſing, whilſt the object is advancing forwards. When the object is between a concave ſpeculum and its principal focus, the image is leaſt of all when the object is cloſe to the ſpeculum; and the magnitude of the image continually increaſes, as the object advances towards the focus; after this, the inverted image will conſtantly decreaſe, as it approaches nigher to that principal focus, or as the object retreats farther from that focus. But the apparent magnitude of the image, will not be always as its true magnitude; becauſe the apparent diſtance is always finite, and frequently much leſs than the true diſtance. It may be obſerved farther, that the image in a convex ſpeculum is always between the ſpeculum and its principal focus: But in a concave ſpeculum, the diſtance betwixt it and its principal focus, is the only part of the axis, produced both ways to infinity, that the image is excluded from.

280. In all caſes, if the eye be in or near the principal axis, and the breadth of the pupil be neglected, the viſual angle of the apparent image, is equal to that angle which the real image ſubtends at the center of the pupil. When the geometrical image is before the eye, it affects us then as if it was a real object, and any miſtake we may be under as to its true diſtance, cauſes no difference in the viſual angle. When the image is behind the eye; ſuppoſe the point *b* or *x* to be the center of the pupil; then is the viſual angle *db*A or B*xk*, formed by the rays reflected towards *p* and *r*, equal to the ∠ *pbr* or *pxr*. But in this caſe, if the breadth of the pupil be conſidered, the above angle will be ſomewhat greater, and that becauſe the rays enter it converging. Thus, if *bc* be the breadth of the pupil, the viſual angle of the whole image will be *dog* = *por*, formed by the interſections of the rays *db*, *gc* tending towards *p*, *r*. And ſince, when the image is behind the eye, the [a] apparent diſtance remains much the ſame, the apparent magnitude will increaſe as the eye recedes from the ſpeculum towards *q*.

Fig. 49.

[a] 265. II.

281.

281. The limits of the diminiſhing powers of convex ſpeculums, and of the magnifying powers of concave ſpeculums, are reciprocally as their focal lengths. For if two equal objects PR, be placed each at the greateſt diſtances before two convex ſpeculums, or in the focus's of two concave ſpeculums; the image *pr* in the one caſe, will be directly as OF; and in the other, it will be reciprocally as OF. This is manifeſt, becauſe OQ is ſuppoſed to be infinite in the one caſe; and O*q* in the other; whence *pr* in the one caſe is as OF; and *pr* in the other is as the ∠ *p*O*r*. But yet theſe powers, as to their effects, are contained within bounds not very extenſive.

Fig. 45, 47.

An object ſeen in a convex ſpeculum, unleſs it be the ſun or moon, muſt be at a finite diſtance, and the more convex is the ſpeculum, the nearer the object muſt be to it, to be ſufficiently diſtinct. If the diſtances of the objects from the focus's of two convex ſpeculums, be reſpectively as their focal lengths; the diſtances of the object and its image from the center of each, will be alſo in the ſame proportion; that is, QF : OF :: OQ : O*q*; and OQ : O*q* :: QP : *qp*. And therefore in this caſe, the diminiſhing powers of the two ſpeculums are equal.

Fig. 45.

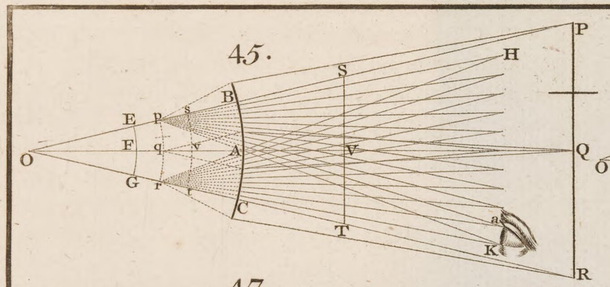
An object in a concave ſpeculum appears diſtincteſt, when it is at ſuch a diſtance from the focus, that the geometrical image is at a moderate diſtance off, not exceeding a foot or two from the ſpeculum; and generally the more concave is the ſpeculum, the nearer the image, *cæteris paribus*, will appear to it. And if the apparent diſtances of the images ſeen in two ſpeculums, be reciprocally as their viſual angles; the apparent magnitudes of the images, will be as the objects. That is, if the objects before each ſpeculum be equal, their images will appear alſo equal.

282. Hence it is, that one does not uſually perceive any great difference in the bigneſs of his face, when ſeen in two ſpeculums of different concavities. And the effects of concave ſpeculums in * magnifying a large object, as a face, are not ſo great as at firſt ſight one might imagine. The image generally appears biggeſt when the ſpectator is near the principal focus, and then the viſual angle of the image is about double of what it would have been, if the ſpeculum had been plane, the ſpectator ſtanding at equal diſtances from each ſpeculum: And admitting the image appears three times the focal length of the ſpeculum beyond it; the whole apparent breadth of the face, will be four times greater than in a common looking-glaſs. But the image ſeldom appears ſo far, and therefore is ſeldom magnified ſo much, as four times in breadth. But the apparent magnitude of the whole face, being as the

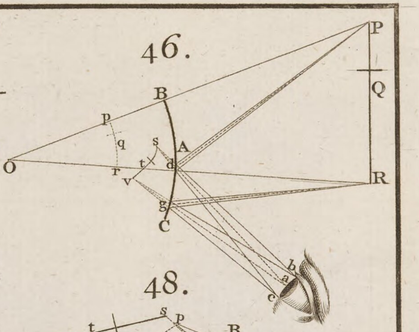
Fig. 47.

* Their effects in magnifying ſmall objects, will be conſidered in the next book.

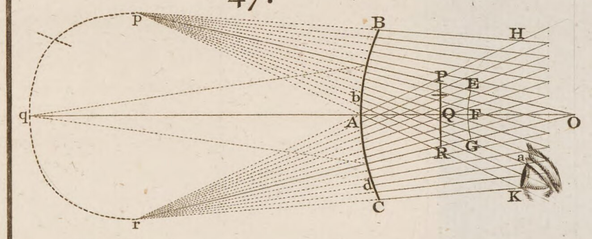
45.



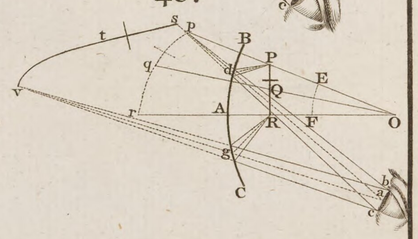
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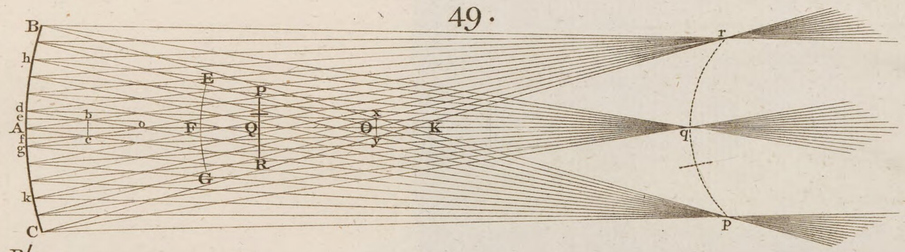
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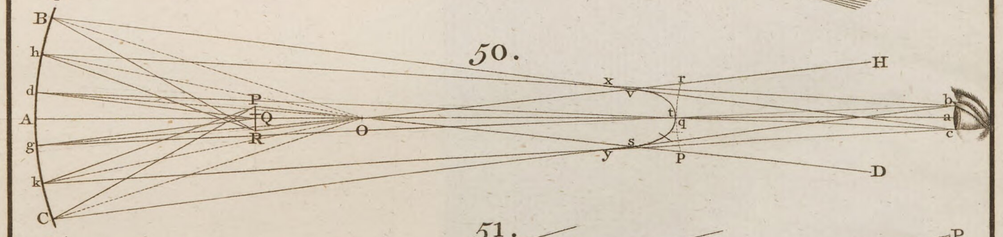
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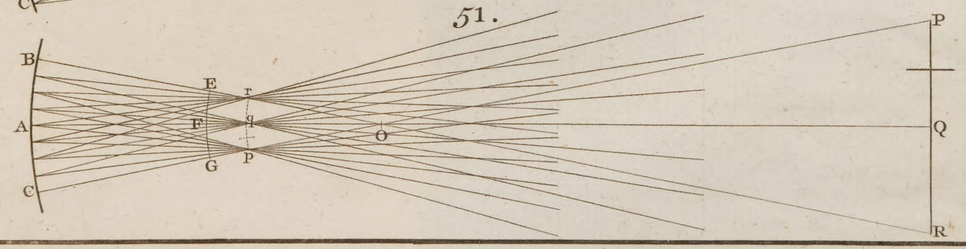
49.



50.



51.



the ſquare of that of its breadth, is the cauſe of our ſurprize; the whole face being really magnified nine times, when it appears only three times broader than uſual.

PROP. XXI.

283. *To explain the phenomena of pictures ſeen in concave ſpeculums.*

Phenom. If a picture drawn according to the ſtrict rules of perſpective, and wherein are repreſented objects having different real diſtances, be placed before a concave ſpeculum, a little nearer to it than its principal focus; the image of the picture will appear extremely natural, the whole ſcene appearing very like the real one from whence it was taken: For not only the objects are greatly magnified, ſo as to approach nearer to the natural ſize, but they have alſo different apparent diſtances; inſomuch that a view of the inſide of a church, appears very like a real church: And landſkip pictures, &c. appear very like as the real objects would do, to a ſpectator at the place from whence the view was taken. Theſe curious and very entertaining phenomena, may, I think, be accounted for in the following manner.

In the annexed figure, the curve *p q r* is the geometrical image of the ſtrait object PQR; or it is that curve in which are contained the foci of all the pencils of light that diverged from PR, and whoſe axes paſs through the center O of the ſpeculum BAC, after their reflection by that ſpeculum. Now the geometrical image of a circle facing the ſpeculum, and whoſe center is at Q, and diameter is PR; will be a hollow figure, formed by the rotation of the curve *p q r* round the axis O*q*. If this hollow figure be ſuppoſed a real thing, whoſe inſide ſurface is variouſly diſtinguiſhed into parts by different colours, and a picture of it be drawn in perſpective upon the circle PR, the point of ſight being at O: A ſpectator placed at O, would be affected much alike by the rays coming to him from the picture, after reflection by the ſpeculum, as he would by rays coming to him directly from the hollow figure, the ſpeculum and the picture being removed; for the hollow figure, and the geometrical image of the picture upon the circle, are, in this caſe, both coincident. Fig. 56.

Again, the geometrical image of a rectangular parallelogram AB, fig. 57, placed where PQR is fig. 56, will be alſo a hollow figure; but more like a pyramid with four ſides, than to the figure deſcribed by the rotation of the curve *p q r*. For the geometrical images of each of the ſides of the parallelogram, lye all in planes paſſing through the center of the ſpeculum, and theſe ſides reſpectively: Theſe images, it is true, will be curvilineal; but theſe curves lying in the planes above

mentioned,

mentioned, the ſtrait lines joining their extreams, will be the ſides of a parallelogram, like unto the original. And in like manner, a leſſer parallelogram *ab* within the former, will have the images of its ſides like thoſe of the former, but at a greater diſtance. And ſo likewiſe the ſides of the ſeveral parallelograms *cd*, *ef*, &c. will have their images in a ſeries one behind the other; the middlemoſt being fartheſt of all: So that the geometrical image of the whole figure, does ſomewhat reſemble the *fruſtum* of a hollow pyramid with four ſides, and which becauſe of the greater apparent diſtance of the ſmaller or middle parts, appears nearly like a hollow priſm, a ſection whereof is *pstr*.

Now if *ps* or *rt* be the length, and *st* or *pr* the breadth of the inſide of a church, a perſpective view of which from O, is drawn upon the plane PR; the geometrical image in this caſe will not be very unlike the church itſelf. For the picture upon the plane PR, is a figure properly conſiſting of ſeveral parallelograms, diminiſhing towards the middle, after the manner of thoſe above deſcribed; and if the picture is not too large in proportion to the ſize of the ſpeculum, the curvities above noted of the ſides of the ſeveral included parallelograms, will not be very conſiderable, and theſe are ſtill leſſened in the appearance, by the viſible concurrence of the images of certain lines, inclining to one another in the picture. But as moſt of the pencils of light entring the eye, diverge from points that are at great diſtances, their different divergencies are not alone ſufficient for determining the true places of their foci: And the apparent image of a blank ſurface placed at PR, will not appear near ſo concave as the geometrical image. We muſt therefore have recourſe beſides to ſome other cauſes; and indeed we have here many concurring together.

The contiguous parts of the picture of the floor, for inſtance, eſpecially if it be checquered or otherwiſe diſtinguiſhed, form a long ſeries of viſible images contiguous likewiſe to one another: The images of the remoter parts appear alſo at the ſame time fainter, and ſmaller; they appear fainter, becauſe they are drawn ſo in the picture, which faintneſs contributes much to enlarge the apparent diſtance; and they appear ſmaller, becauſe the pictures of the remoter parts of the real floor, diminiſh faſter in proportion, than the apparent diſtances of the images of theſe pictures increaſe: So that from all theſe cauſes conſpiring together, the eye receives much the ſame impreſſion as if it looked at a real floor; and ſo in both caſes the appearance is much the ſame; a long extended ſurface, a little diminiſhed in breadth, and that gradually, towards the farther end. In like manner, the walls appear erect and extended on each ſide, and the roof above facing the pavement; and all gradually inclining, after the ſame manner as a large

large room appears to the naked ſight when viewed from one end. Beſides the above helps, the window-lights in the ſides, the ſhades of upright objects thrown on the pavement, &c. in the picture, do alſo contribute their ſhare; and all theſe conſpiring together, do ſufficiently out-weigh the imperfections of the geometrical image of the whole: So that inſtead of a diſtorted picture, we ſee in a manner a real church; the great magnitude of the whole, its viſible concavity, and proportionable lengths and diſtances of the parts, contributing very powerfully to excite in us that idea.

In like manner, landſkip pictures, different views of buildings, &c. are alſo ſurprizingly improved by the concave ſpeculum; moſt or all of the cauſes above aſſigned, conſpiring together to make the images appear natural, as in naked viſion. Theſe phenomena appear rather more perfect to both eyes, than to one alone; and the appearance will be the ſame, to one who does not know what the picture on the cloth or paper repreſents; both which * ſhew that the ſaid phenomena are not founded upon mere prejudices, but upon the general laws of viſion in like caſes.

The pictures that receive the greateſt improvements by a concave ſpeculum.

284. Thoſe pictures appear to the beſt advantage in a concave ſpeculum, which ſtand moſtly in need of improvements; that is, thoſe which repreſent objects having conſiderable differences of apparent diſtances, from the point whence the view was taken: For the geometrical images of different parts of the picture, will be really at different diſtances. Alſo moſt pictures in general that are drawn leſs than the life, as landſkips and all ſorts of views of remote objects, are generally improved by the ſpeculum becauſe they are magnified. But portraits, and pictures of objects that lye nearly in a plane at right angles to the optic

* I have known a young child, who had never ſeen any thing like what was repreſented, ſhew great marks of joy and ſurprize, upon looking at the image of a print in a concave ſpeculum. And if it could be ſaid with propriety that nature is any where improved upon, I think, this is the place: For, if the print or picture is finely done, the opportunity we have of viewing its image without extraneous light, intruding at the ſame time into the eye, is an advantage we cannot have when we look at remote objects, and has a very pleaſant effect. For the eye being leſs tired, ſees the parts ſo much the diſtincter, and examines them more at leiſure; and I do not know but that in ſome caſes, the image of a fine picture, wherein perſpective has been accurately kept, may convey as good an idea of the ſcene repreſented, as if we were carried to the very ſpot from whence the artiſt took his view. How uſeful and entertaining are good prints when thus improved? And what ſtrong inſtances have we here, of the excellencies and defects of painting, both at the ſame time: How lively and natural is the image of a good picture drawn in perſpective; and how faint is the reſemblance, or how little there is of likeneſs, in the picture itſelf. But though pictures for this purpoſe, ſhould be drawn according to the ſtrict rules of perſpective; it does not follow from hence, that thoſe rules are to be cloſely kept in pictures that are to be viewed only by the naked ſight.

optic axes, receive no great improvement by ſuch ſpeculums. The glaſs ſpeculums commonly ſold in the ſhops, are ſegments of ſpheres of proper ſizes enough for this purpoſe; but the pictures to be viewed in them, ſhould not be ſo broad as they are; and they generally appear beſt, when they are not much above half that breadth: For when the picture is pretty wide, the images of the extream parts generally appear too near, and ſomewhat diſtorted; which is perfectly agreeable to theory. And hence, a given picture generally appears beſt, the larger is the ſpeculum, or the longer is its focal length. The picture, as it ſtands in its place, ſhould alſo be flat; otherwiſe it is manifeſt that its image cannot appear natural. Objects that are intended to ſtand upright, as perſons talking, &c. ſhould not be drawn in a part of the picture that is intended to appear level, as in the midſt of the pavement of a church, &c. For in this caſe, as the geometrical image will be in a manner thrown flat upon the floor; ſo the apparent image, inſtead of being erect, will have an uncouth poſture, and generally be protracted into an unnatural length. To avoid theſe inconveniences, all erect figures or objects ſhould be placed near the ſides in the picture; and then the defects of the geometrical images are the eaſier overcome, by the other aids that are preſent. Theſe few hints may ſuffice as to this head.

The propereſt poſition of a picture before the ſpeculum.

285. The picture ſhould be placed nigher to or farther from the principal focus, according as the image ſhould appear more or leſs concave. That is, if the objects repreſented, have conſiderable differences of apparent diſtances from the point of ſight; the image in the ſpeculum will appear moſt natural, when the picture is placed a * little within the principal focus, as in *fig.* 56. If the picture be moved much nearer to the ſpeculum, the apparent image will be flatter or leſs concave, and ſmaller, which is exactly agreeable to theory. And ſo, as the difference of the apparent diſtances of the neareſt and remoteſt objects, is greater or leſs; by moving the picture nearer to or farther from the principal focus of the ſpeculum, we ſhall obtain that poſition wherein it will exhibit the moſt natural image.

Again, the image is beſt, and appears beſt, when that point in the picture which repreſents the remoteſt part of the exhibited ſcene, is in or near the principal axis of the ſpeculum, and the eye is placed behind a hole made round the ſaid point. But as ſuch a hole would damage

* If the picture be placed in the focus or without it, the image to a common eye, will not be ſo diſtinct, and the indiſtinctneſs will increaſe, as the picture is moved farther out; ſo that before the picture has got half-way between the focus and the center, the parts are generally loſt in confuſion.

damage the picture; if the horizontal line is near the bottom of it, place the picture higher againſt the upper part of the ſpeculum, ſo that the image may be ſeen underneath it. In this caſe, if the ſpeculum is vertical, the ground plane will appear aſcending; the reaſon of which is manifeſt from an inſpection of *fig.* 56. To remedy this inconvenience, let the ſpeculum be inclined a little backwards, and alſo the picture, but not quite ſo much; for the picture ought always to be nearly at right angles to the axis paſſing through the principal point in it. But after all, if the breadth and depth of the picture be reſpectively above $\frac{2}{3}$ and $\frac{1}{3}$ of the breadth of the ſpeculum, ſome part of the image, in moſt caſes, will appear unnatural: For inſtance, the geometrical image of the roof of a church, in a picture placed and viewed as above, will be ſhorter than that of the pavement; and ſo will alſo the apparent image. If the picture be placed lower, and the image viewed over it, the phenomena will be the reverſe, but not ſo natural as before.

In all caſes, the horizontal line in the picture, ſhould have an horizontal ſituation; and when the ſaid line is near the top, it will be beſt to place the picture againſt the lower part of the ſpeculum, and look over it; and ſo incline the ſpeculum and picture a little forwards, for the reaſons above given. Again, when the picture repreſents an oblique view, as of a long range of buildings, &c. it ſhould be then placed a little ſideways, and a little obliquely, and alſo its image viewed on that ſide next the middle of the ſpeculum. From theſe hints, it will be eaſy by trials to find the propereſt poſitions of the pictures, in the ſeveral caſes that offer. And it is alſo to be obſerved, that the picture ſhould be no more enlightened, than what is juſt neceſſary: For the faintneſs of the image, contributes alſo its ſhare, to enlarge its apparent diſtance.

A box for holding the ſpeculum and the pictures.

286. A proper apparatus for ſupporting the ſpeculum, and the pictures, in their due poſition, will be very advantageous; as the image will remain ſteady, and may be viewed at leiſure without trouble or fatigue to the ſpectator: This may be effected by two different methods; both which I ſhall deſcribe, regarding at the ſame time, as much as may be, the principles explained in the preceding.

Fig. 58.

In the annexed figure DEGHIKLM, repreſents a vertical ſection of a box through the middle, the fore part of which ML is open. The glaſs * ſpeculum BAC, whoſe principal axis is AF, and focus F, is repreſented with its wooden frame, and placed in the narrow chamber GHIK fitted for it; the top part of the ſpace HK is left open for admitting the ſpeculum in and out, and is ſhut cloſe occaſionally by the

* Speculums for this purpoſe are commonly made of glaſs, as metal ones would be very cumberſome and expenſive. One that I have is a ſegment of about 26½ degrees.

narrow board HG. The board GK has a round hole *a b* cut in the middle of it, of the bigneſs of the ſpeculum: This board ſerves to ſtrengthen the box, otherwiſe the ſpeculum might have been confined by narrow ſlips of boards. For ſupporting the prints or pictures; on each ſide of the box is a dove-tail ſlider, carrying each a wooden bar, as *r s*, which turn ſtiffly upon a pin *t* going through each of the ſaid ſliders. If the prints are ſtuck on paſteboards, a ſlit in each of the above bars will be ſufficient to hold them. But as this method of ſtiffening the prints is inconvenient; a thin board whoſe width does not exceed half that of the ſpeculum, and whoſe ends ſlide in grooves made in the bars *r s*, will anſwer better. The board ſhould ſlide ſo ſtiff as to ſtand at any height, and this will be eaſily effected by a ſpring fixed at each end of it. The grooves ſhould be alſo deep enough to hold the board a little ſlanting; that is, when one of the bars *r s*, is a little forwarder than the other. The board may remain fixed while the prints are ſhifted; and the prints may be kept flat to it by a narrow frame turning upon hinges, or by ſliding ledges binding their ends, &c. The front ML is cloſed by a pair of folding doors, having at their edges about the middle a deep notch, making a tranſverſe ſlit or opening long and deep enough, for looking into the box with both eyes at once. This ſlit is cloſed occaſionally by a ſliding board; and which, by means of the grooves on each door, ſerves to faſten them together. In the top ED is a ſquare hole, which is ſhut occaſionally by the looſe board *e d*. The pictures are enlightened through the ſlit or † opening GE, which extends the whole width of the box, and may be made wider or narrower by two ſliders drawing to each ſide, and with which the ſlit may be ſhut quite cloſe. The prints may be put in and taken out, either through the front ML, or the top ED, or through the ſlit GE, or through a ſlit made for the purpoſe in one of the ſides of the box. If the print or picture has the point of ſight near the bottom of it, as is commonly the caſe, place it oppoſite to the upper half of the ſpeculum; then ‡ incline the box backwards, till the image appears vertical, which may be eaſily done by placing a book under the front L, giving the bottom an inclination, as I*i*: This being done move the ſliders to or fro, and if needful incline the bars *r s*, till the image appears to the beſt advantage. If the print requires to be placed at bottom, the

† This opening may be cloſed with thin canvas; but this is needleſs unleſs it be for concealing the print within, as it is eaſy to proportion the degree of light upon the print by a different poſition of the box with reſpect to the window.

‡ If the chamber HGIK be made about three times as wide, the proper inclination may be given the ſpeculum without tilting the box. But this would be attended with more trouble, beſides enlarging the box, and alſo endangering the ſpeculum, from its unneceſſarily having much room to play in.

the box muſt be inclined the contrary way, ſo that its bottom may have the poſition L *l*.

The box above deſcribed is deſignedly contrived with a view to other optical purpoſes, as will be ſeen in the next ſection; and I think it ſerves each of theſe purpoſes as conveniently, as if it was deſigned for one only. But by the above method it is troubleſome to ſhift the prints, and to keep them flat; and alſo the image is inverted as to right and left. Both theſe inconveniencies might at once be remedied, by a plane ſpeculum ſo placed as to reflect the rays diverging upon it from a print placed at the bottom of the box, upon the concave ſpeculum; and ſo as to be reflected from thence again to the eye. But in this caſe ſome part of the image will be much diſtorted, as the rays come to the concave ſpeculum very obliquely: This may in a great meaſure be remedied in the following manner.

Another method of placing the ſpeculum and the pictures.

287, The out-lines, in the annexed figure, repreſent a vertical ſection of the * ſame box, but at right angles to the former; that is, this ſection is parallel to the front ML in fig. 58. The concave ſpeculum BAC is placed near the top of the box, where it is ſupported with its face downwards, by the ledges *g*, *h*. The ſpeculum is placed a little inclining, that the rays coming to it from a picture at *a b*, may be the better reflected towards O; the principal axis being AF, and focus F. The ſpeculum is alſo placed higher than is neceſſary for a ſingle print, for the ſake of receiving a book; the print when furtheſt from it, being at about *a b*. *r s* is a plane ſpeculum; and *a c*, *c r*, *r* O; *b d*, *d s*, *s* O, are the principal rays in the pencils diverging from the points *a*, *b*, that come to the eye at O, after their reflection, by both ſpeculums. The ſpeculum *r s* need not be above 2 inches long, and 1 $\frac{1}{2}$ inch wide; and its frame or a little board for holding it, may turn upon a ſtiff hinge, one ſide of which may ſlip into a groove in the ſide of the box; and the proper inclination of *r s* is ſoon found by trial: The hole at O may be about 3 inches wide, and 1 inch deep. The prints are here enlightened through the front ML of the box, part of which is ſufficient for the purpoſe, or the whole might be ſcreened with thin canvas. Fig. 59.

The ledges *g*, *h*, may have a vertical motion, which will ſerve both for giving the due inclination to the ſpeculum, and for adjuſting the diſtance betwixt it and the print at bottom: Or, the print may be raiſed to

* The top part of the box is now uſeleſs; and it might be reduced ſtill ſmaller by cutting the concave ſpeculum into a parallelogramic form. But as the box in the ſhape it is now in, not only ſerves for both the above methods, but alſo for other optical purpoſes; it ſeemed needleſs to alter it.

to the proper height by placing one or more books under it, the ſpeculum remaining fixed. The plane ſpeculum might be placed near the oppoſite ſide of the box to O; in that caſe, the picture muſt be placed near the ſide O; the plane ſpeculum muſt be wider, and the ſpeculum BAC muſt have a contrary inclination: But I think the other method is preferable. If the ſpeculum BAC be more inclined, the print *a b* may be brought nearer to the middle of the box; and the contrary.

Both the methods above deſcribed of placing the ſpeculum and the prints, may be uſed very readily in the ſame box; but I think the laſt method of placing the prints at the bottom, is upon the whole the moſt convenient. Thoſe who are fond of ſurprizing unſkilful ſpectators, might, by either method eaſily conceal the whole contrivance, and ſhift the pictures unobſerved *.

PROP. XXII.

To find the focal length of a ſpherical ſpeculum, by experiment.

There are different methods of ſolving this propoſition, which are as followeth.

By the chord and verſed ſine.

Fig. 60.

288. Let O be the center, OA the principal axis, and F the principal focus of the ſpeculum BAC. If the ſpeculum is concave, lay the edge of a ſtrait ruler B*v* C over the middle of the ſpeculum, and meaſure exactly the length of the chord BC, and of the verſed ſine A*v*: But if the ſpeculum is convex, the ruler muſt be laid as at *a b*, ſo that the diſtances *a* B, *b* C, betwixt its edge and the verge of the ſpeculum, are equal. And in both caſes, it will be;

*v*A (= B*a* or C*b*): *v*B (= *v*C or $\frac{1}{2}$*a b*) :: *v*B : *v*O + AO; to which adding *v*A, the fourth part of that ſum will be = AF the focal length

* Dr. *Graveſande* deſcribes a box for concealing a concave ſpeculum, in order to ſurprize a ſpectator ſtanding at a proper diſtance, and looking through a large hole in the front of it, by exhibiting his inverted image ſuſpended near that hole. Our box will alſo ſerve for the ſame purpoſe. But the doctor takes no notice of the phenomena of pictures ſeen in ſuch ſpeculums; nor have I met with any account of theſe phenomena any where, which made me the more particular in endeavouring to explain them. By the beſt information I could procure, the phenomena above-mentioned were not obſerved here till about the year 1730; and it might be about 6 or 7 years later before they were commonly known. Mr. Felix Clay at *Aldgate*, I am informed was the firſt here who made a box for the above purpoſe, the hint of which, he told me, he took from a ſmall kind of ſhow-box or toy brought from *Holland*; and this was in the year 1741. Soon after theſe things became known to the printſellers, and boxes were copied from Mr. *Clay*'s; theſe were different, tho' upon the ſame principles from that above deſcribed. Mr. *Clay* I believe was alſo the firſt who placed the ſpeculum at top, as in fig. 59. But theſe boxes were not calculated for ſerving any other purpoſe than that one; and large ſpeculums being expenſive, theſe ſhew-boxes have of late been laid aſide; to give room for another ſort with a convex lens, which ſhall be deſcribed in the next ſection.

length required. But this method will not admit of great exactneſs, becauſe a ſmall error in the length of A*v*, will produce a great one in AF.

By expoſing the ſpeculum to the ſun.

289. Place the ſpeculum ſo that its principal axis may point very near towards the center of the ſun, or towards the moon when it ſhines bright: If the ſpeculum is concave, find the burning point, or receive the image upon a white piece of paper; in either caſe, the diſtance between the focus ſo found, and the vertex of the ſpeculum is the focal length required. Or, cover the ſpeculum with a ſheet of opaque paper, in which make two or more holes, as A, D; and by obſerving where the beams of light reflected from theſe holes unite, the focus may be found rather more accurately, than when the whole ſpeculum is expoſed.

If the ſpeculum is convex, it muſt be either covered, as above, leaving two holes as A, D; or leaving the ſpeculum naked, ſtick on it two round opaque patches, A, D; then holding a white paper as GH perpendicular to the principal axis of the ſpeculum, obſerve where the ſhades of the patches A, D fall upon it, as at G, H; meaſure exactly the diſtance AG, and the diſtances betwixt the centers of the ſhades G, H, and betwixt the centers of the patches A, D; and it will be, GH — AD (= *e*H) : AD :: AG (= D*e*) : AF. For D*e* is here parallel to FAG, and GH to DA, and ſo the $\triangle^s$ D*e*H, FAD are ſimilar. In this caſe, it would be beſt to deſcribe circles at G, H beforehand, whoſe diameters and diſtance betwixt their centers, ſhall be in ſome given ratio to the diameters and diſtance betwixt the centers of A, D: Suppoſe this ratio to be as 2 to 1, then AG = AF; if it be as 3 to 1; AG will = 2AF; for FG : FA :: GH : AD.

The focus of a concave ſpeculum may be found with rather more eaſe and exactneſs, by the following method: Becauſe here it will be troubleſome to hold the ſpeculum in its due poſition, unleſs it be furniſhed with an apparatus for the purpoſe; and the burning focus, or place of the diſtinct image of the ſun or moon, admits of ſome latitude.

Focus of a concave ſpeculum found by the image of a candle. Fig. 61.

290. At the end of a long table, place the ſpeculum in a vertical poſition, or ſo that its principal axis AOQ may be nearly parallel to the table; then place a candle towards the other end of the table, ſo that its flame Q may be oppoſite to the vertex of the ſpeculum. This being done, take a clean piece of white paper, and having fixed it at the end of a ſtick, place this ſtick in the ſocket of another candleſtick, ſo that the paper may be ſupported at about the ſame height with the candle; then move the paper or the candle to and fro, till the image *q* of

of the candle appears moſt diſtinct upon the paper: If the candle and the paper are not nearly in the principal axis AqQ, they muſt be moved a little, as occaſion requires. And having meaſured exactly the diſtances Aq, AQ; becauſe,

[a] 278. II. [b] [94]. I. [c] 94. I.

$AQ : Aq :: {}^{a}OQ : Oq (:: {}^{b}OF : Fq) :: {}^{c}QF : OF$; it will be,

$AQ + Aq : Aq :: Qq (QO + Oq) : Oq :: AQ (QF + OF) : AF = OF.$

And ſo either qO or AF gives the focus required. The ſame may be found without calculation, by moving the candle till its image hangs over it; or by obſerving where the end of a ſtick and its image touch one another; in both caſes, the point of coincidence is the center of the ſpeculum.

Fig. 60. The focal length of a convex ſpeculum may be alſo found by the light of a candle, thus: The ſpeculum, candle and paper being placed upright, as in the preceding; let the ſpeculum be covered with paper, having in it two round holes A, d; and let the candle and one of the upright ſides of the paper, be nearly in the principal axis AQ, the candle being placed hindmoſt, ſo that none of its direct light falls on that ſide of the paper which faces the ſpeculum. Then having found the places of the white circles g, h, upon the paper, and having meaſured the diſtances Ad, Ag, gh; the point of interſection q, of the lines gA, hd produced, may be found as in the preceding; and calling the flame of the candle Q, it will be in this caſe; $AQ - Aq : Aq :: Qq : Oq$.

A more exact method of finding the focal length of a concave ſpeculum.

Fig. 61. 291. If the greateſt nicety be required: The ſpeculum being placed in a vertical poſition, as in the preceding, find its principal axis, and alſo its center O, which may be done near enough by moving a candle till its image coincides with it. Before a convex lens a of about one or two inches focal length, and to a proper frame for ſupporting it, fix a thin opaque plate b with its edge upright, and a little ſide-ways of the axis of the lens, and alſo at ſuch a diſtance that the ſaid edge appears diſtinct through the lens. The edge of the plate being placed near to the center O, and the axis of the lens nearly in the principal axis of the ſpeculum; move the lens a little to or fro along this axis, till the edge of the plate and its image from the ſpeculum, are both ſeen diſtinctly through the lens, at the ſame time, and alſo very near to one another. Now the diſtance betwixt the ſaid edge and the vertex of the ſpeculum, is exactly twice the focal length required. Note, during this experiment, the ſpeculum muſt be in the dark; that is, it muſt be either ſcreened in a corner of the room by a tube or otherwiſe; or day-light muſt be excluded from the room altogether. In this laſt caſe,

the plate muſt be illuminated by a candle placed a little ſide-ways before it, with a ſcreen to prevent its light falling directly upon the ſpeculum.

The plate above might be perforated near the edge into ſeveral holes of different ſizes; ſo that the holes and their images might be ſeen at the ſame time: Or the objects might be ſmall wiers fixed to the frame of the lens.

Goodneſs of ſpeculums, how to be examined.

292. *Schol.* 1. A ſpeculum free from ſpecks, fiſſures, *&c.* is *cæteris paribus* preferable to another not free from theſe blemiſhes: The goodneſs of the poliſh may be eſtimated from the luſtre of the ſpeculum, but with more certainty from the brightneſs of the images. But the principal thing to be ſought after is the goodneſs of the figure; and this may be judged of in the general, by the likeneſs of the images. But the laſt preceding method for finding the center of a ſpeculum, affords alſo a pretty critical one for examining its figure; and that by obſerving the degree of diſtinctneſs of the ſmall holes in the thin opaque plate, with the ruggedneſſes, and duſts, *&c.* ſticking to them, compared with theſe objects ſeen directly through the lens; both being ſo brought as to be ſeen at about the ſame diſtance from the middle of the lens. Let a ſmall round hole in the thin plate be placed near the center of the ſpeculum, and place the candle in the axis behind it: If the ſection of the rays received upon a white paper, placed a little before or behind their focus, does not appear circular, but oval, ſquariſh, *&c.* it is a ſign that the ſpeculum has not the ſame curvature throughout its ſurface. If the light received upon the paper, when placed before the focus, is brighter towards the circumference, and darker about the middle, than when the paper is placed at the ſame diſtance beyond the focus; it is a ſign that the ſpeculum is flatter about the middle, than towards the circumference: And if the phænomena be the contrary, the ſpeculum is flatteſt towards the circumference. Inſtead of receiving the rays upon the paper, an eye-glaſs might be uſed, moving it a little to and fro along the axis, the perforated plate and the candle remaining fixed *. By ſuch trials as theſe, which may be alſo a little varied, a perſon who has had experience will be able to judge pretty nearly what figure the ſpeculum is of: And it will be a farther help, if trials be made by different zones of the ſpeculum, the reſt of it being covered. If the ſpeculum is for a teleſcope, it muſt be

* *John Haaley*, Eſq; who by his ſuperior ſkill and unwearied pains was the firſt who brought reflecting teleſcopes to any degree of perfection, is the author of the above method of examining the figure of a concave ſpeculum.

be farther examined by the effect, when joined with its proper appurtenances.

293. *Schol.* 2. If it be required to find a point in the axis of the ſpeculum, at which an object being placed, its diameter ſhall be to that of its image in any given ratio, as that of a to b; a being always greater than b, if the ſpeculum is convex. Make $b : a :: AF : FQ$; and the point Q will be that ſought.

Fig. 45. to 51.

[a] 290. II. For $FQ : AF ::$ [a] $OQ : Oq ::$ [b] $a : b$. Therefore, *&c. Q. E. D.*

[b] 111. I. 294. *Schol.* 3. Both convex and concave glaſs ſpeculums, quick-ſilvered on one ſide, will have double images, as has been ſhewed of plane looking-glaſſes; and the places of each of theſe images may be found by ſome of the rules in the preceding book. But as theſe glaſſes are uſually ſo thin, that theſe double images are ſcarce ever perceived, it would be needleſs to exemplify the manner of finding their places here. However, without regarding the reflection by the firſt ſurface, it will not be amiſs to conclude this ſection, with ſome obſervations concerning the effects of glaſs lenſes, when uſed as quick-ſilvered ſpeculums.

Fig. 62. Let the plano-convex lens AB be quick-ſilvered on the plane ſide, and parallel rays will have a focus Q before the lens. Let O be the center of the ſpherical ſide AB: Now a ray DB parallel to the axis will be refracted towards f, ſo that $AO : Af ::$ [c] $m - n : m$; and at b the ray will be reflected towards q, ſo that $aq = af$. Again, at c the ray will be refracted at its emerſion out of the glaſs towards Q, ſo that $fq : Of ::$ [d] $Aq : AQ$. If the thickneſs of the glaſs lens be neglected, its focal length in this caſe as a ſpeculum, will be to its focal length as a lens, as 1 to 2. But becauſe of the thickneſs aA, the focal length AQ as a ſpeculum, will be ſhorter and aQ longer than in the above proportion.

[c] 126. I.

[d] 144. I.

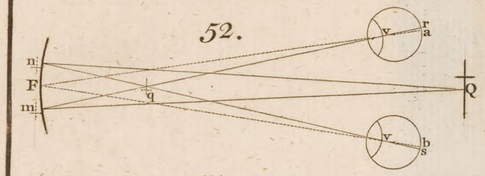
It is manifeſt, that if the convex ſide of the lens be quick-ſilvered, it will have a real focus, and nearer to it than the former. Alſo all *double convex lenſes*, quick-ſilvered on either ſide, will have a focus before them: And if the radii of the ſides are equal, the focal length in that caſe will be $= \frac{1}{4}$ the focal length of the lens, nearly †.

A *plano concave lens* will have its focus behind it, which ever ſide is quick-ſilvered; and conſequently ſo will likewiſe all *double concaves*, and *concavo-meniſcus's*, when the concave ſide is quick-ſilvered. A *concave meniſcus*, the radii of whoſe ſides are as $m - n$ to m, when the rays are incident upon the concave ſide, has no focus: This is [e] manifeſt, becauſe an incident ray parallel to the axis, will be refracted perpendicularly

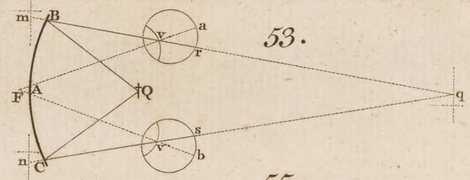
[e] 126. I.

† Suppoſing the glaſs lens, fig. 62. to have the ſide ab of equal convexity with AB, and neglecting the thickneſs, the center of this ſide will be in Q; then the focal diſtance from A of the ray reflected at b will be $= \frac{1}{4}$ A Q (by the rule 94. I;) and the focus of the emergent ray will be $= \frac{1}{4}$ AQ, by 144 I.

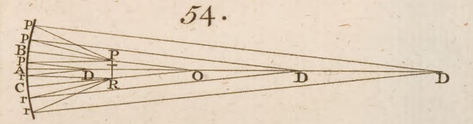
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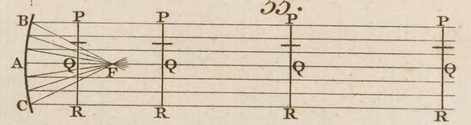
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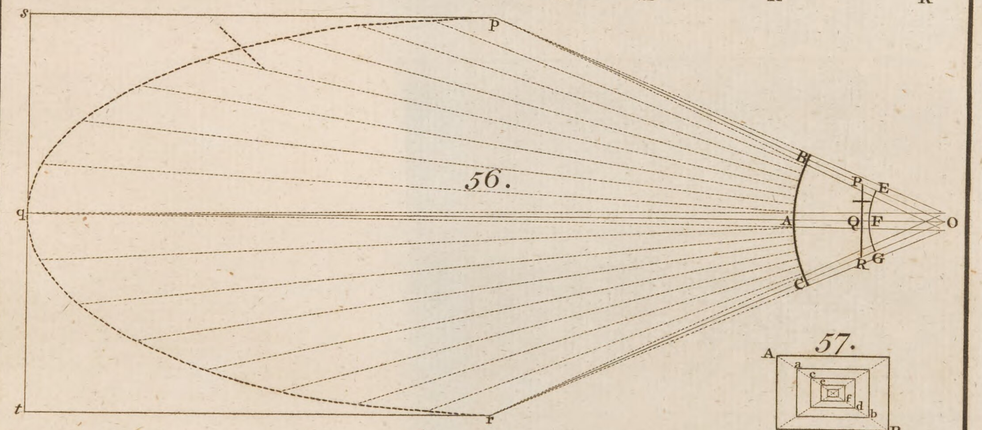
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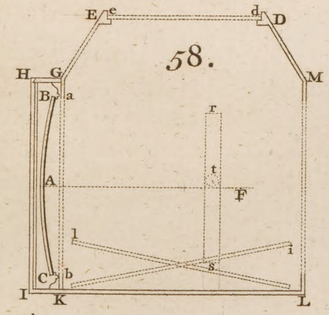


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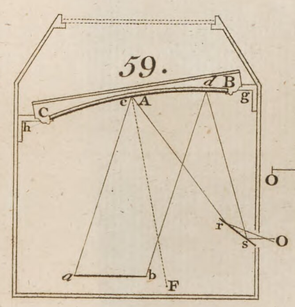


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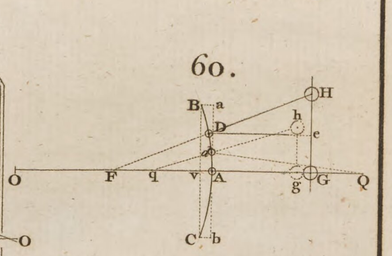
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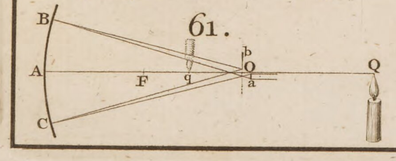
59.



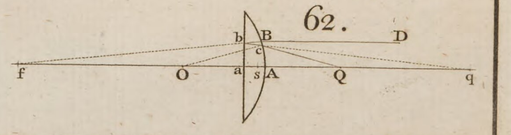
60.

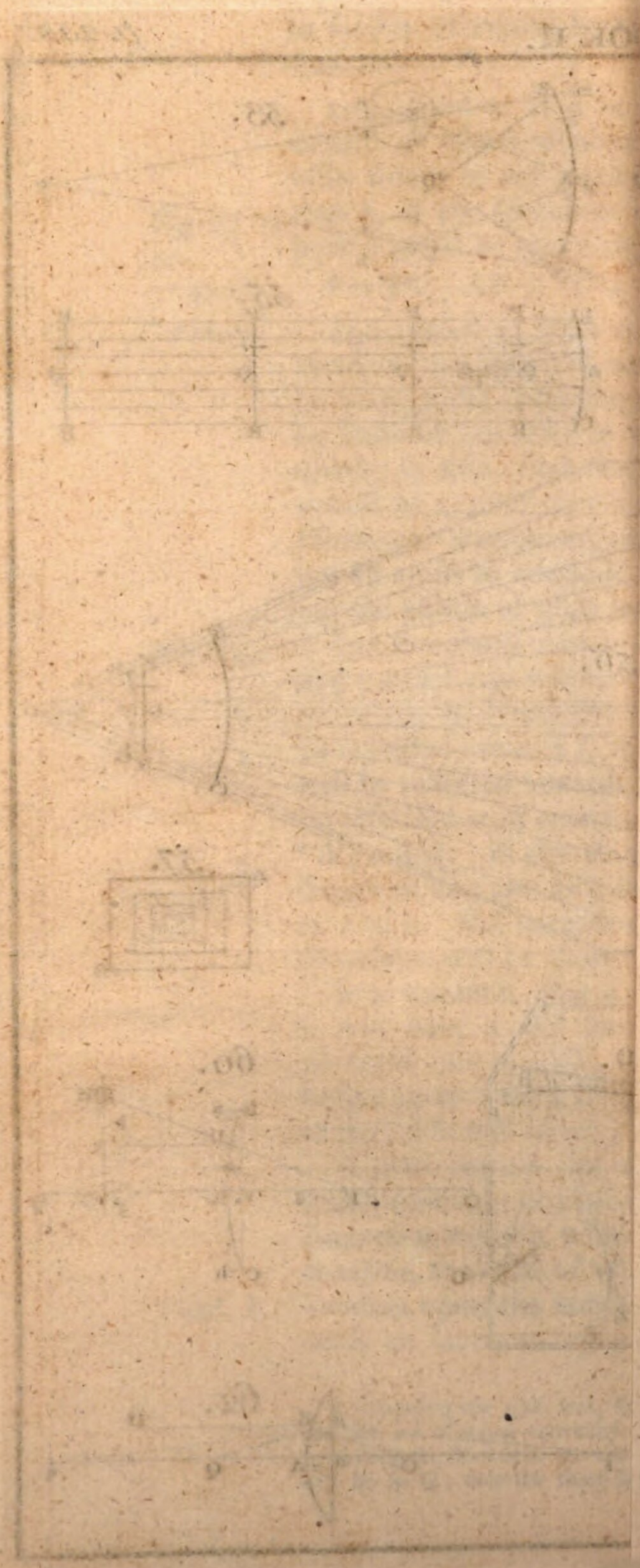


61.



62.





cularly to the reflecting ſide. And when the ſaid proportion of the radii of the ſides is nearer to an equality, or more unequal, the lens will accordingly have either a focus on the concave or convex ſide.

But becauſe of the great aberrations by the reflections and double refractions, lenſes are very unfit for ſpeculums, unleſs it be for illuminating objects, for which purpoſes thoſe that have their focus's before them, may anſwer indifferently well.

SECTION IV.

Of viſion through given mediums: of pictures ſeen through convex lenſes; and a deſcription of different ſorts of camera-obſcura's.

§. 1. *Of viſion through mediums, terminated by plane ſides; and ſome farther obſervations relating to the refraction of light.*

PROP. XXIII.

295. *An object ſeen through a medium, terminated by plane and parallel ſides, appears pretty nearly in the ſame plane, and of the ſame magnitude, and exactly of the ſame figure, as it would do to the naked eye in the ſame poſition, the medium being removed.*

For the geometrical image *p q r*, is [a] erect, equal, and parallel to the object PQR; and the rays here undergoing but ſmall refractions, the proper foci of the pencils of light that enter the eye will be very nearly in *p q r*. The image alſo is very near to the object; for the diſtance Q*q* between them is to the thickneſs of the medium, only in the ratio of $m - n$ [a] to m. And although the pencils of light CO, DO, at their entrance into the eye diverge, as if they came from the focus's *p*, *r*, and conſequently form a greater angle than if they proceeded directly from P, R; yet if the image appears in its true place, it will appear [b] equal to the object. Therefore, *&c. Q. E. D.* Fig. 63. [a] 79. I. [b] 195. II.

296. *Schol.* 1. We ſeldom ſee objects through mediums that are very thick: The thickneſs, for inſtance, of a coach-glaſs in the caſe before us, being very inconſiderable. But an object, by being rendered more obſcure by the medium, may appear [c] farther than otherwiſe it would; [c] 188. II.

[d] 198. II. and in that caſe it will alſo appear [d] larger: So that the phenomena of viſion through ſuch mediums vary, as the circumſtances are different. It may be farther obſerved, that the nearer the object and the eye are to the medium, the greater, *cæteris paribus*, will be the effect; becauſe the diſtance Qq, between the object and its image, is invariable.

Viſion of objects in a denſe medium.

297. *Schol.* 2. Objects placed in a denſe medium, whoſe refracting ſurface is plane, appear erect, and nearer to that ſurface than their true place; but in moſt caſes, they appear ſomewhat diſtorted. We have frequent opportunities of obſerving theſe phenomena, by looking into ſmooth and clear water; and the very near agreement of the phenomena with the [e] theories of the images in the preſent caſe, is another notable proof of the great uſe we make of the different conformations of our eyes, and the inclinations of their axes, according to the different places from whence the rays diverge, in our eſtimations of the ſaid places, where thoſe places are not very remote. For what elſe have we here to direct us? The greater obſcurity of the objects, and our knowing perhaps from experience that the water is deeper than it appears, are cauſes that here tend to deceive us; and yet we find that they have little or no effect. The phenomena are alſo the ſame to perſons who have no ſkill in optics.

[e] 62. to 66. I.

PROP. XXIV.

298. *To explain the phenomena of viſion through a medium, whoſe refracting ſides are planes inclined to one another.*

Phenom. An object ſeen through ſuch a medium, appears in a new direction, and generally in a different poſition, ſomewhat indiſtinct, diſtorted, and coloured; and the greater is the refracting angle of the medium, the greater will be theſe alterations.

Fig. 64. Let PQ be a rectilinear object, ſeen by the eye at O, through the triangular priſm RST. Now the geometrical image of the object is pq; but becauſe of the great obliquities of the rays to the refracting ſides of the priſm, the apparent image will not be pq; and here, in order to define the apparent image, it will be neceſſary to find the proper foci of the extream rays, at leaſt, in each pencil. The dotted lines repreſent [a] See 57. I. the [a] method of finding the proper foci of the rays Qc, Qe, after their refractions at each ſide of the priſm; thoſe rays being ſuppoſed homogeneal, and their ſines of incidence and refraction as the numbers 3, 2. Thus g, h, are the proper foci of the rays cd, ef; and m, n, the proper focus's of the rays dO, fO; and ſo upon the above ſuppoſition, the point b, where the rays dm, fn interſect, would be the place of the viſible image

image of the point Q. In like manner was alſo found the point *a*, the place of the viſible image of P. But as the pencils of light conſiſt of heterogeneal rays, which impinge the ſides of the priſm very obliquely; they will be here ſeparated, eſpecially at their emerſion out of it, into their different kinds: The image will therefore neceſſarily appear [b] coloured; it will be indiſtinct, becauſe the rays in the ſeveral pencils have different foci, as they differ in refrangibility, and have different inclinations to the refracting ſurfaces. The rays entering the eye being produced backwards, will form a kind of a compound image, which will be diſſimiliar to the object, and which may alſo appear in a different place and poſition from *a b*; and becauſe of the indiſtinctneſs, the apparent place and poſition may be different at different times of looking, the eye, priſm, and the object retaining their former places and poſitions. Therefore, *&c. Q. E. D.*

[b] 283. I.

299. *Schol.* In like manner, if a medium has ſeveral plane ſides HK, E, D, A, B, C, inclined to one another, ſo as to form the triangular priſms BFK, CHK, DGH, EKH, and the light proceeding from a ſmall object Q, be tranſmitted through each of the ſaid priſms to the eye, as at O; an image will be ſeen through each priſm, and ſo there will be as many viſible images in the whole, as there are priſms, together with one more ſeen through the ſides A, HK, if theſe are parallel. And hence a glaſs having plane ſides cut in this manner, is called a MULTIPLYING-GLASS. Of multiplying glaſſes. Fig. 65.

The image *a* ſeen through the parallel ſides A, HK, will be diſtinct, equal, and ſimilar to the object Q, and alſo pretty near it, the diſtance betwixt their centers being $= \frac{1}{3}$ thickneſs of the glaſs AHK. The images ſeen through the common ſide HK, and each of the ſides B, C, D, E, will be reſpectively *b*, *c*, *d*, *e*. And theſe will be all of them ſomewhat indiſtinct, diſtorted and coloured; and the more ſo, the more remote they are from the middle image *a*, becauſe the priſms through which they are ſeen have larger refracting angles.

The diſtance OQ being given, the greater is OA, the leſs will be the angles Q O *b*, *b* O *c*, *&c.* but the more indiſtinct and coloured will be the extream images; theſe images appearing beſt, when the incident and emergent rays form equal angles with the reſpective refracting ſides: And that will happen, if the flat ſide be towards O, when the glaſs is much nearer to O than to Q. When OA is given, the greater is AQ, the greater alſo will be the angles Q O *b*, Q O *c*, *&c.* and the more remote from the middle of the glaſs, will be the incidences of the pencils coming to the eye. And hence the diſtance OQ being given, the diſtance OA, at which an object may be ſeen through each ſide of a given glaſs,

glaſs, is limited; that is, OA muſt not exceed a certain given diſtance, or be leſs than another aſſigned diſtance. If the glaſs be turned round the perpendicular line QO to the ſides A, HK; the image *a* will appear immoveable, whilſt all the reſt ſeem to turn round with the glaſs.

LEMMA.

300. If a ray of light be tranſmitted through ſeveral contiguous mediums terminated by parallel planes, out of a given medium into the ſame again, as out of air, through water and glaſs, into air again; the emergent ray will be always parallel to the incident ray. The truth of this * is manifeſted by experiments, and the reaſon of it may be illuſtrated as followeth:

Fig. 66. Let AB be an incident ray, W, G two mediums of different denſities, as water and glaſs, and terminated by the parallel planes BP, KO, LF; and let the ray be refracted at B into B*b*, at *b* into *bf*, and at *f* into *f* D: I ſay, *f* D is parallel to AB. Draw the perpendicular B*n* to the refracting ſurfaces; and having from the center B, with the radius B*b*, deſcribed the arch *n b d*, produce AB till it interſects this arch in *d*, and through the point *d* draw *d m* perpendicular to B*n*. Now *m d* is the ſine of incidence, and *w b* the ſine of refraction of the ray AB into the medium W. Let *g a* be the ſine of refraction from air immediately into the medium G, the ſine of incidence being *m d*; through the point *a*, draw the line B*a e*, and B*a e* would have been the refracted ray, if the refractive power of the medium W had been equal to that of G; that is, if the incident ray IK be parallel to AB, B*e* will be parallel to the refracted ray KL.

Now it is reaſonable to ſuppoſe, and it ſeems very evident, that the refraction between the mediums W, G, muſt be equivalent to the exceſs of the refraction of G above that of W, when rays come with a like obliquity out of the ſame medium into each; that is, the ſines of incidence and refraction out of W into G, are as *w b* to *g a*. And upon this ſuppoſition, the refracted ray *bf*, will be parallel to B*e*. Draw *b i* perpendicular to the refracting ſurfaces, and from the center *b* with the radius *bf*, deſcribe the arch *i f h*; produce B*b* till it interſects this arch in *h*, and draw *hk* parallel to *of*. Now becauſe *w b* : *g a* :: [a] *k h* : *of* and the ∠ *o b h* = [b] ∠ *w* B *h*, the ∠ *o bf* is = *w* B *e*; therefore *bf* is parallel to B*e*; but B*e* is parallel to KL, and conſequently *f* D is parallel to LM; but LM is [c] parallel to IK, and IK is [b] parallel to AB, and ſo *f* D * is parallel to AB. *Q. E. I.*

[a] Hyp.
[b] Conſt.
[c] 67. I.

In

* See *Newton*'s Lect. Optic. par. I. ſect. 2. N°. 33.

In like manner, a ray EF immerging firſt into the denſer medium G, will emerge at *x* out of W, in the line *x* H parallel to EF. Produce EF to *t*; and let the ſines of incidence and refraction from air to glaſs, and from air to water, be reſpectively as the perpendiculars or ſines *t*, *r*, *s*. Now F *r* is the refracted ray; and it would have been F *s y*, if the refraction of G had been equal to that of W. At *r* the ray is refracted from F *r v* into *r x*; and for the ſame reaſon that *b f* is parallel to B *e*, *r x* is alſo parallel to F *y*, and *x* H is parallel to EF. *Q. E. D.* If EF be parallel to D *f* and to ON; F *r* will be parallel to *f b*, and *r x* will be parallel to *b* B and to OP.

301. *Corol.* Hence, if a ray paſſes through ever ſo many refracting mediums terminated by parallel planes, it will have the ſame inclination to the ſurfaces of the laſt medium, as if it had ſuffered but one refraction in going immediately out of the firſt medium into the laſt. Thus *b f* is parallel to B *e* and KL, and *r x* to F *y* and OP. And the caſe will be the ſame, though the number of different mediums be increaſed to infinity, and the intervals of their parallel ſurfaces be as much diminiſhed; ſo that the ray may be continually refracted into a curve line.

PROP. XXV.

302. *The proportion of the ſine of incidence to the ſine of refraction of one and the ſame ſort of rays out of one medium into another*, is, *as the rectangle under the ſines of incidence out of the firſt medium into a third medium, and out of that third medium into the ſecond; to the rectangle under the ſines of refraction out of the firſt medium into the third, and out of the third medium into the ſecond.*

Let as before, A B *b f* D be the courſe of a ray from a third medium through the two refracting mediums W, G into the ſame third medium again; and let S B *n*, Z *b i*, V *f* T, be perpendicular to the refracting ſurfaces at the points B, *b*, *f*. Alſo, let the proportion of the ſine of incidence to the ſine of refraction out of the third medium, as air for inſtance, into the medium W, be as F to *f*, i. e. S. ∠ ABS (= *w* B *d*) : S. ∠ *w* B *b* :: F : *f*; and out of air into the medium G, as G to *g*, i. e. S. ∠ T *f* D : S. ∠ V *f b* :: G : *g*; and let the ſine of incidence to the ſine of refraction out of the medium W into the medium G, be as *x* to *y*, i. e. S. ∠ B *b* Z : S. ∠ *o b f* :: *x* : *y*. I ſay *x* : *y* :: *f* × G : F × *g*. Fig. 66.

Becauſe S. ∠ ABS : S. ∠ *w* B *b* :: [a] F : *f* :: F × G : *f* × G; and [a] Hyp.
S. ∠ V *f b* : S. ∠ T *f* D = [b] ∠ ABS :: *g* : G :: *g* × F : G × F; it will be [b] 300. II.
S. ∠ *w* B *b* : S. ∠ *b f* V (= *o b f*) :: *f* × G : *g* × F :: *x* : *y* :: *w b* : *g a*. *Q. E. D.*

[a] Hyp. Otherwiſe thus. Becauſe $wb : md :: {}^{a}f : F$; and
$md : ga :: {}^{a}G : g$; therefore
$wb : ga :: f \times G : g \times F :: x : y$. *Q. E. D.*

And thus the refractions between any two mediums, and a third medium being known, the refractions between them both may be readily found. As if the ſine of incidence of any ray out of water into air, be to its ſine of refraction as 3 to 4; and the ſine of incidence of the ſame ray out of air into glaſs, be to its ſine of refraction as 17 to 11; the ſine of incidence of that ray out of water into glaſs, will be as 3×17 to 4×11, or as 51 to 44: Or if the ſines of incidence and refraction between air and glaſs be as 3 and 2, the ſines between water and glaſs will be as 9 and 8. For $3 \times 3 : 4 \times 2 :: 9 : 8$.

303. *Schol.* If a medium terminated by parallel planes, be contiguous to another medium whoſe refracting ſides are inclined planes; and a ray of light be tranſmitted through both the mediums, and another homogeneal ray whoſe incidence is parallel to the former, be tranſmitted through that medium only whoſe ſides are inclined; the two emergent rays will be parallel to one another, and inclined to the incident ray.

Let W be the medium, whoſe refracting ſides are parallel planes; and let the inclined ſides of the medium G be K*r*, LR. Now if the incident homogeneal rays be AB, IK, parallel to one another; [b] 300 II. becauſe the rays *bf*, KL incident upon the plane ſide LR [b] are parallel; [c] 16. I. the emergent rays will be alſo [c] parallel, but inclined to LM, *f*D, becauſe LR is inclined to LF. If the emergent rays be conceived to flow back again, ſo as now to become the incident rays, [d] 13. I. [d] they will proceed along the ſame tract they did before, and ſo the emergent ravs BA, KI will be parallel. In both caſes, therefore *&c. Q. E. D.*

It is almoſt needleſs to obſerve, that the caſe will be different if Fig. 64. the three refracting ſides are all inclined to one another. Let the priſm RST be conceived to conſiſt of two mediums RS*v*, TS*v*, of different denſities; it is manifeſt that the emergent rays will be inclined to thoſe in the figure, whether thoſe within are perpendicular to S*v*, as *cd*; or are inclined to it either way, as *ef*, *rs*.

§. 2. *Of viſion through lenſes; and of finding their focal lengths by experiments.*

The effects of concave and convex lenſes, are, in the general, analogous to thoſe of convex and concave ſpeculums reſpectively; but as ſpeculums and lenſes have ſome diſtinct properties, they deſerve to be explained apart.

PROP.

PROP. XXVI.

304. *An object ſeen through a concave lens, always appears erect, as it doth alſo thro' a convex lens, if the geometrical image is not betwixt the lens and the eye: If the focal lengths of the lenſes are but ſmall; the image through a concave lens appears diminiſhed, and nearer to the lens than the object; and through the convex lens, it appears magnified; and if the rays come to the eye diverging, the image will appear farther than the object.*

That the image, in the above caſes, does neceſſarily appear erect, is ſelf-evident: And as the eye can never come to the center A of the lens, but muſt be at ſome diſtance behind it, ſuppoſe at G; the object ſeen by the naked eye, and its image through the lens always form different angles, theſe angles being as PGR, to *p*G*r* or *i*G*o*. And conſequently, in the firſt caſe, the object appears diminiſhed, and in the ſecond magnified; and when the focal length AF is but ſmall, and the object, as in *fig.* 68. is between F and the convex lens; the image through the concave lens appears nearer to it, and through the convex lens farther from it, than the object. The reaſons of all which phenomena are very manifeſt. *Q. E. D.* Fig. 67, 68, 69.

305. *Schol.* 1. When we look through lenſes having long focus's, the image appears pretty nearly at the ſame diſtance off, that the object does to the naked eye. For in this caſe, the differences of the divergencies of the refracted rays and of the unrefracted ones, are quite imperceptible; and ſo, there being nothing here to direct us otherwiſe, we naturally look for the object thro' the lens, in the ſame place the naked eye ſaw it juſt before. Again, this deception as to the place of the image, is farther favoured by the difference of the viſual angles; the viſual angle being diminiſhed by the concave lens, and inlarged by the convex one, aſſiſt the imagination, in both caſes, to bring the image nearer to the object. In theſe caſes, the apparent magnitudes of the object to the naked eye, and through the lenſes, are as the viſual angles PGR and *p*G*r*. But in looking through lenſes having ſhort focuſes, if the geometrical image by the convex lens is erect, and at a moderate diſtance off, the true places of the images are perceived, at leaſt very nearly; and then the apparent magnitudes of the object and its image, are nearly as their [b] true magnitudes; that is, as PR and *pr*. And I think it holds univerſally, as well as can be judged by experiments, and as has before been taken notice of, that *The apparent magnitudes of objects and their images, are as the rectangles under their apparent diſtances and viſual angles reſpectively.*

An eſtimate of the proportion in which objects appear magnified or diminiſhed by lenſes.

[b] 198, 218, & 221. II.

Lenſes having long focus's do not ſenſibly alter the apparent magnitudes of objects.

306. *Corol.* 1. Hence, lenſes having long focuſes, have ſcarce any ſenſible effects in magnifying or diminiſhing objects; and the nearer they are brought to the eye, the leſs will be their effect; becauſe at the center of the lens, the object and its image ſubtend the ſame angle. Nevertheleſs ſuch lenſes may be uſeful ſpectacles, as their cauſing a different divergency of the rays of light may contribute towards a more diſtinct viſion.

Objects appear darker through concave, and brighter thro' convex lenſes.

Fig. 67, 68.

307. *Corol.* 2. An image ſeen through a concave lens appears darker, and through a convex lens brighter, if it be beyond the lens, than the object; neglecting in this laſt caſe the conſideration of the loſs of light by the lens. If *a b* be the breadth and place of the pupil; it can receive no more light in the firſt caſe from the point P, than what is contained in the ſlender pencil P*s t*; but in the latter caſe, the pencil P*s t* is much broader than the former. In the one caſe the rays diverging more, and in the other leſs; the pencils entering the eye in the firſt caſe are rarer or contain fewer rays, and in the latter they are denſer or contain more rays, than if theſe pencils had come directly from the object.

A concave lens placed cloſe to the eye cauſes indiſtinctneſs.

308. *Schol.* 2. An object ſeen through a concave lens placed cloſe to a common eye, always appears indiſtinct; and this indiſtinctneſs will be the greater, the ſhorter is the focal length of the lens. But if the eye recedes back to a certain diſtance from the lens, the object will become diſtinct.

When the focal length of the lens is ſo long, that the image appears as far as the object; the indiſtinctneſs is owing to that miſtake which cauſes a wrong conformation of the eye. But by retreating farther from the lens, the proportion of the true and apparent diſtances of the image become nearer to an equality; and ſo the indiſtinctneſs vaniſhes by degrees.

Fig. 67.

Again, when the focal length of the lens is ſhort, the object appears indiſtinct, becauſe the rays diverge too much. Thus, if *q a* be the neareſt diſtance of diſtinct viſion, and *a b* the ſmalleſt breadth that the pupil of the eye can be contracted to; the reaſon is manifeſt, why viſion will be diſtinct when the eye is at *a*, and that it muſt be confuſed, if the eye approaches nearer to the lens than the ſaid diſtance A*a*.

An object ſeen through a concave lens may be indiſtinct through the want of light.

309. *Schol.* 3. If the eye continues to recede farther from the lens than the point *a*, the object will continue diſtinct, if it be ſufficiently bright. For the image being near a concave lens having a ſhort focus, does alſo always appear near it, and not far from its true place; the lens itſelf, in this caſe, being as it were an object, to which the eye conforms according to the different diſtances. But as the eye recedes

farther from the lens, the image becomes continually darker; and it may become ſo dark, as to be indiſtinct upon that account.

An object may be indiſtinct through a convex lens by being placed too near it.

310. *Schol.* 4. If a convex lens, having a ſhort focus, is not placed at ſuch a diſtance from an object, as to cauſe its erect image to be as far from the lens, as the neareſt diſtance of diſtinct viſion; the object will be confuſed to an eye placed cloſe to the lens. As if qa be the neareſt diſtance of diſtinct viſion; the image pqr will be neceſſarily indiſtinct to an eye placed nearer the lens than the point a. But if the eye retreats to a, the ſaid image will become diſtinct, and ſo it will continue as the eye recedes farther back, but diminiſhed in its brightneſs. But if Aq is not leſs than the neareſt diſtance of diſtinct viſion, the object will appear the diſtincter the nearer the eye is to the lens, becauſe then it will receive the more light. Fig. 68.

A flat object ſeen through a lens appears ſpherical.

311. *Schol.* 5. The geometrical image pqr, of a ſtrait or plane object PQR placed perpendicular to the principal axis of the lens, is [c] concave towards a concave lens, and [c] convex towards a convex lens. This curvity of the image, appears very ſenſible to both eyes looking through a broad lens having a ſhort focus; but to one eye it is not ſo apparent; and ſometimes the image will appear bent the contrary way, as wxy. Fig. 67, 68.

[c] 239. I.

In all caſes, the middle of a broad image appears diſtincteſt; and our not being ſo well able to diſtinguiſh different degrees of diſtance with one eye alone, the different diſtinctneſs of the middle and extream parts, ſeems to be the medium that cauſes our miſtake as to the true figure of the image. In the firſt caſe, the image being diminiſhed and obſcure, ſeems to be what leads us to fanſy that the rays from the extream parts, which are the moſt obſcure, diverge from a greater diſtance than thoſe towards the middle. And in the latter caſe, the greatneſs of the image ſeems to be what cauſes our falling into a contrary miſtake; that is, the indiſtinctneſs of the extream parts ſeems to be now * imputed to their being too near, as they were before to their being too far, for diſtinct viſion. But in both caſes, by looking more attentively, theſe miſtakes are frequently corrected.

Viſion thro' a convex lens when the object is beyond the principal focus.

312. If the eye be between the lens and the image pqr of an object PQR, that object will appear erect, magnified, and confuſed; and the farther the eye is from the lens, or the nearer it is to the image, the greater will be the confuſion.

* It is true we are not ſenſible or conſcious of theſe reaſonings, but this is no more than what often happens to us in other caſes. Ideas excited by our ſenſations are frequently productive of others that were originally acquired from reaſoning; and being thus once acquired, they follow the others ſo quick as not to allow us time to reflect upon them, or to diſtinguiſh any interval betwixt their growth and the ſenſation that cauſes them.

Fig. 69. Thus, if the eye be at G, it ſees the points P, R, by rays converging towards *p*, *r*; the middle rays in theſe pencils being the lines G*p*, G*r*, which produced backwards are the lines G*i*, G*o*; and ſo, the middle rays of the pencils proceeding from every point of the object PQR, will be refracted to the eye at G in lines, which if produced backwards, will lie orderly between G*i* and G*o*; and therefore the object appears erect, where-ever the point G is taken between the lens and *pqr*. Again, the viſual angle *i*G*o* = *p*G*r* being greater than the angle PGR; the image appears magnified, if it appears as far as the object, which whilſt the eye is pretty near the lens, is generally the caſe. The object to a common eye appears indiſtinct, becauſe the rays of the ſeveral pencils are united in the eye before they reach the retina; and ſo upon the retina the rays from the adjacent points of the object are mixed together; as in the caſe of viſion of remote objects to ſhort-ſighted perſons. As the eye retreats towards *q*, the confuſion increaſes; and before it has got to *q*, the object will be quite loſt, and nothing will be viſible but a glare of light. During this retreat, the viſual angle is conſtantly increaſing, and that the faſter, the nearer the eye has got to *q*; this angle being always equal to *p*G*r*, wherever the place of the eye or the point G is taken. As the confuſion increaſes, the object will ſeem to quit its place, and approach towards the lens, at which it will at laſt ſeem to ſtick. In this caſe, as we are deprived of the ordinary means by which we judge of diſtances, the greater confuſion and the greater viſual angle it ſeems are the cauſes of ſuggeſting the nearer diſtance; as in naked viſion, the confuſion increaſes the more we approach a near object; though in the two caſes, the *foci* of the pencils, after their refractions into the eye, are on different ſides of the retina. But the lens itſelf being an object, between us and which, we perceive nothing; the object looked at through the lens, appears no nearer than the ſurface of that lens.

As the eye retreats beyond *q*, a large, confuſed, and inverted image will begin to appear: During this retreat, the image will continue all the way inverted, and the confuſion will gradually leſſen, till the eye has got as far as the neareſt diſtance of diſtinct viſion, which ſuppoſe to be *q a*. When the eye is at *a*, or farther back than *a*, the image will appear diſtinct, if it appears alſo in its true place, otherwiſe not. The apparent place of the image is at firſt generally near the lens, and the eyes conforming themſelves accordingly, viſion is neceſſarily indiſtinct; but if the object is bright enough, by looking attentively towards the true place of the image, we ſhall ſoon perceive it in that place, and then, and not till then, it will appear diſtinct; the image will alſo appear convex towards the eye, according to its true figure,

if

if the object is a plane facing the lens. The reaſon of all the above phenomena are very manifeſt from an inſpection of the figure; and it is alſo manifeſt, that the field of viſion through a given aperture *g h* is conſtantly decreaſing as the eye retreats backwards from the lens; and when it is a pretty way off, it will be difficult to find the inverted image, for the eye muſt be pretty near the axis before it can meet the rays. At whatever diſtance the eye is from the image it appears erect or inverted; and its apparent magnitude, as in other caſes, will be as its viſual angle multiplied by its apparent diſtance.

When an image appears double.

313. The image *q* of a ſmall object Q, will appear double, as at *m* and *n*, when the axes *d v*, *e v* of the eyes are directed to a point G beyond *q*; which is a caſe that frequently happens, and the reaſon of the phenomenon is very obvious from an inſpection of the figure, and from what has been [d] before explained. The image may alſo appear double when the ſpectator is before it, if the lens is broader than the diſtance between the two pupils of the eyes. This caſe is the reverſe of the preceding, the image *s*, *t* being now on contrary ſides of the optic axes *d v*, *e v*, as when we look ſhort of the object: But this phenomenon of a double image ſeldom happens.

[d] 59. II.

Some farther obſervations relating to viſion through lenſes, will occur under the following propoſitions *.

PROP. XXVII.

314. *A long object placed nearly parallel to the axis of a convex lens, and whoſe fartheſt end is within the principal focus, appears through the lens protracted to a great length: Alſo a ſeries of ſmall objects placed one behind the other in a line near the axis, the remoteſt being within the focus of the lens, appear likewiſe extended to great diſtances.*

The geometrical image of the long object AB or CD, after refrac- Fig. 70.
tion by the lens O whoſe principal focus is F, is [a] *a b* or *c d*: And [a] Conſt.
the geometrical images of the equal objects AC, GG, HH, II, KK, L L, BD, placed parallel to, and at equal diſtances from, one another, are [a] *a c*, *g g*, *h h*, *i i*, *k k*, *l l*, *b d*. In many caſes, the divergencies of the rays in the pencils entering the eye, are not alone ſufficient to give us juſt ideas of the places of the focus's of theſe pencils, eſpecially when theſe focus's are remote from the lens: But in this caſe, being farther aſſiſted by the continuity of the images of the ſeveral parts of the long object, or of the ſeries of the other objects; the apparent places of the ſaid images become thereby protracted to greater

* See alſo prop. XIX, XX. in the preceding ſection, where the parallel caſes of viſion in ſpeculums have been explained.

greater diſtances, ſo as to be nearer the geometric places, than if one part of the long object, or one of the objects placed croſs the axis, had been ſeen ſingly. Thus, the ſucceſſive changes undergone by the eye in order to ſee the ſeveral parts of the image *a b* diſtinctly, or the different diſtinctneſs of theſe parts if the eye remains at reſt, contribute to protract the whole apparant length; that is, the image of the remoteſt point B will in this caſe appear farther, than if the point B only had been ſeen. But the apparent image will not exceed the object in length above a certain proportion, which will be the greater, according as the lens is deeper, and the object is nearer its focus. Alſo, although the apparent magnitudes of the images of the parts of the object continually increaſe, as theſe parts are nearer to the focus; yet, after a certain limit, they increaſe much ſlower than the geometrical images of thoſe parts. If the lens be broad enough to admit both eyes to ſee through it at once, the phenomena will be ſtill more agreeable to the theories of the geometrical images.

Greater indiſtinctneſs of an image cauſes a greater apparent diſtance.

315. *Schol.* 1. If the object AB be placed farther from the lens, ſo that the point B may be nearer the focus F, or even beyond that focus, the apparent image will be ſtill longer; and that not merely becauſe the geometrical images of the ſeveral parts will be farther from the eye, but alſo becauſe they will be indiſtincter and fainter. A greater indiſtinctneſs of an image cauſes it to appear farther, as in like caſes of [b] naked viſion: And when an image is diſtinct, it will ſeldom appear farther than the object is, when equally diſtinct to the naked eye.

[b] See 185, & 188. II.

Phenomena of a box with painted ſcenes explained.

Fig. 71.

316. *Schol.* 2. The annexed figure repreſents the ſection of a box, in which are placed ſeveral ſcenes as AC, GG, HH, II, KK, LL, BD, *&c.* on which are drawn the repreſentation of a camp, a colonade, or rows of trees, *&c.* Theſe ſcenes are open in the middle, excepting the laſt which is whole; that next the eye at AC having the wideſt opening, and the openings of the reſt are gradually diminiſhed, that ſo the picture on each ſide may be ſeen. The front RB of the box is open, but covered with a thin canvas, or oiled paper, to hide the pictures within, and at the ſame time to admit ſufficient light upon them. The box ſhould ſtand upright on account of enlightening the ſcenes; for which reaſon it would be convenient to have the bottom wide enough for a ſupport, without other help. A convex lens may be fixed at the top *r*; but the repreſentation will be more natural, and alſo more ſurpriſing, if the lens be fixed in the ſide, as at O; a plane ſpeculum or looking-glaſs RS, being fixed at half right-angles to the axis of the lens. For then the images of the pictures will appear in an horizontal plane; and prenotions of diſtance from the viſible

length

length or depth of the box being thus removed, the imagination has a greater ſcope in increaſing the apparent diſtances of the images.

In this figure, the focal length of the lens O is *r*F; and the ſcenes are ſo placed that their geometrical images are at equal diſtances from each other; the diſtance of the image of the remoteſt ſcene BD being = O*b* or O*d* *fig.* 70. Theſe diſpoſitions of the ſcenes have been here obſerved, for the ſake of giving an idea of the places where their geometrical images do fall: But it will be beſt to place the laſt ſcene BD nearer to the focus F, or even a little below that focus; and ſo the reſt reſpectively lower down, as experience will direct. For by this means the images of the remoter ſcenes becoming more indiſtinct and faint, they'll appear farther off; and according to the reſpective diſtances the ſeveral ſcenes ſhould appear at, their places in the box muſt be varied, ſo that no general rule for placing them can be here given. Things being properly diſpoſed, if the pictures are good and the ſubject well choſe, the phenomena will be ſurprizing enough. Inſtead of having the ſcenes, the ſides and end of the box might be painted; the box for this purpoſe being made a little taper, otherwiſe the fartheſt end would appear too wide. But the artiſt muſt have regard to the effect of the lens, and conſider the true diſtances of the images of the ſeveral parts, otherwiſe he will be liable to miſtakes, which will hurt his deſign. The ſame regard ſhould be alſo ſhewed both in painting and placing the ſcenes.

The effects of the little ſhew-boxes ſold at the toy-ſhops, having artificial fruits, flowers, *&c.* ſtuck to their ſides, are owing alſo to the cauſes above explained.

PROP. XXVIII.

317. *To explain the phenomena of pictures ſeen through a convex lens.*

If a print or picture, placed upright againſt the wainſcot, be viewed through a convex lens, it will appear a little magnified, and a little improved. A lens uſed for this purpoſe muſt not be too deep; otherwiſe the field of viſion will be too ſmall, and we ſhall be too near the picture. For a ſmall print, a lens of about a foot focal length may be uſed; and for a larger, a lens of about two or three foot focus will anſwer better. But ſuch lenſes not magnifying much, and the adjacent objects to the picture being ſeen at the ſame time or known to be very near; the leſs divergencies of the refracted rays are not ſufficient, in the preſent caſe, to overcome this prejudice.

Let

Let the ſame picture be included in a box, ſo as to be detached from all other objects, and the effect will be very different. A proper box for this purpoſe may be a *camera-obſcura*, which will be deſcribed towards the latter end of this ſection. In this place, it is ſufficient to obſerve that the box may be made on the ſame principle with that *fig.* 71. in which BD repreſents the picture at bottom, RS a reflecting plane; O the lens through which the picture is ſeen erect, and directly before the eye. By this artifice, and the excluding all other objects, the prejudices from prenotions of diſtances are removed; and being here aſſiſted by the painter's art, the picture, if it be a proper one, will be ſurpriſingly improved and magnified.

When the picture is placed at BD *fig.* 71. the diſtance of its geometrical image from the lens is $=$ O*b* or O*d* *fig.* 70. But it will be better to place the picture ſtill nearer to the principal focus F. Now the refracted rays being nearly parallel, they affect the ſight thus far as if they came from a remote object; the image alſo contains fewer viſible parts, than if it was really nearer, and ſo this cauſes ſomewhat of the ſame kind of indiſtinctneſs as we have when we ſee remote objects. We may alſo have the ſame kind of obſcurity, and this will be partly occaſioned by the loſs of light by the refraction of the ſpeculum and refraction of the lens; and the proper degree of obſcurity upon the whole may be obtained, by illuminating the picture in the requiſite proportion. All theſe cauſes conſpire together to make the images appear a great way off, and conſequently to appear proportionably large. The painter alſo, by repreſenting the remoter objects ſmaller, fainter, and leſs diſtinct; enables us to diſtinguiſh, as it were, ſome differences of apparent diſtances. And now the different lights and ſhades, and the reſpective inclinations of different planes, have their due effect, as in naked viſion, in exhibiting the reſpective parts and figures in their due poſitions and attitudes. And ſo upon the whole, inſtead of a picture upon a flat ſurface, we have a lively proſpect of the ſcene repreſented; and we ſee, as it were, real objects upon their own baſis, and detached one from another.

The repreſentation will be ſtill in many caſes better, if the lens be broad enough to admit both eyes to ſee through it; and by retreating a little back from the lens, the viſual angles of the images will be increaſed, and ſo therefore will be their apparent magnitudes.

The apparent image in ſome caſes will be alſo deeper or more hollow, if the print itſelf be made a little concave or hollow †.

318.

† I have not been able to learn when it was firſt obſerved, that pictures are improved by a convex lens; nor have I met with any attempt to explain that phenomenon. The box with the open

318. *Schol.* 1. A convex lens fixed in a frame to be held in the hand, and having before it a plane ſpeculum inclined at the proper angle, or moveable upon a hinge ſo as to be readily ſet to this angle, will make a portable apparatus for the pockets, and may be uſeful for viewing prints we caſually meet with abroad; the inclined ſpeculum being alſo advantageous here, for the reaſons above given. But yet I think, the ſame ſpeculum and lens when thus held in the hand, do not anſwer near ſo well, as when they are fixed or placed at the end of a box; and that for the reaſons given in the preceding.

A portable apparatus for viewing prints.

319. *Schol.* 2. Prints and ſmall pictures may be ſo greatly improved, by a convex lens, that it is well worth the while of thoſe who have collections by them, to have a proper apparatus for the purpoſe. But the prints muſt be judiciouſly choſe: They ſhould not repreſent objects that have great differences of apparent diſtances; nor objects that are too near, or too ſmall. The picture of the inſide of a church, for inſtance, becauſe of the faintneſs and ſmallneſs of the figures in the middle, will appear through the lens a little concave; but not near ſo natural as it does in the concave ſpeculum, the point of ſight here ſeeming much too far off. For the geometrical image of the whole plane being really convex towards the eye, is a cauſe that counter-acts the painter, and does in part fruſtrate his deſign; and the reaſon why it does not wholly overthrow it, is the ſmallneſs of the differences of the divergencies of the rays from different parts of the image of the whole; they being all of them nearly parallel when the image is far off, as it always ought to be. But although theſe differences, ſmall as they are, have their effects; yet are they leſs ſenſible than the ſmallneſſes of the middle figures; and they ceaſe altogether to be ſenſible, when the real objects have nearly the ſame apparent diſtances from the point of ſight. Again, when the objects are near, as within the breadth of a room; the diſtinctneſs with which they appear to the naked eye, cannot be expreſſed in the picture; and the picture itſelf, in this caſe, can be but little improved by the lens. Alſo when ſome of the objects repreſented are very ſmall in proportion to the reſt; their pictures appear through the lens ſo indiſtinct and obſcure, as to have a bad effect.

What prints appear beſt through a convex lens.

In

open ſcenes, *fig.* 71. was firſt brought here from *Holland* in or about the year 1737; and either in that year or the following, Mr. ELLICOTT by the *Royal Exchange* had one made without the bottom F, for the exhibiting of ſmall prints in a book, *&c.* But the *ſhew boxes* now ſo common in our ſhops, and the lens and looking-glaſs in frames joined by hinges, were not introduced here till about the year 1742 or 1743; although I think that *ſhew boxes* ſomewhat analogous to thoſe now in uſe, were carried about here by the poor *Savoyards*, many years earlier.

In general, the objects to be repreſented ſhould not be too remote, or too near the point of ſight, but at ſuch a moderate diſtance as to appear from thence tolerably diſtinct; and there ſhould not be too great a diſproportion betwixt their magnitudes or viſual angles; the prints alſo ſhould not be drawn from too ſmall a ſcale. There are a great variety of landſkip prints or pictures, and alſo ſome hiſtorical ones, that will be greatly improved by the above method of viewing them; and ſeveral of thoſe that receive the leaſt advantage from the convex lens, will be moſtly improved by the concave ſpeculum; and *vice verſâ*. So that by having a concave ſpeculum and a convex lens with the proper apparatus, we may, by one or other of them, greatly improve almoſt all ſorts of drawings or prints, if they are not too ſmall or too big. When they are very ſmall, it is not ſo eaſy to magnify them ſufficiently for repreſenting nature; and if they are too big, they cannot all be ſeen at one view; ſo that a regard ſhould be alſo had to the ſizes of our prints.

PROP. XXIX.

320. *The eye and an object being placed in the axis of any lens, and neglecting the breadth of the pupil: The viſual angle of the object to the naked eye, will be to its viſual angle through the lens or to the viſual angle of its image; as the rectangle under the diſtance of the eye from the principal focus of the lens, and that of its conjugate focus from the object; to the rectangle under the focal length of the lens, and the diſtance between the eye and the object, very nearly.*

Fig. 72, to 75.

Let A be the lens, F its principal focus, and QAF its axis; QP an object facing the lens, the point Q of which is in the axis: And let O be the eye whoſe conjugate focus is *o*; that is, let the point *o* be ſuch, that any ray as P*a* tending directly towards or from *o*, will be refracted to O. From the extream point P of the object to the point *o*, draw the line P*a o*; and from the center of the eye O, to the refracting point *a*, draw the line O*a* and produce it to *x*; from O draw alſo the ſtrait line O*b* P. Now AO*b* or QOP is the viſual angle of the object to the naked eye at O; and AO*a* or QO*x* the viſual angle of its image ſeen from the ſame place; for the ray P*a* is[a] refracted into *a* O; and QA goes directly to O unrefracted. I ſay,

[a] Conſt.

$\angle$ QOP : $\angle$ QO*x* :: OF $\times$ *o*Q : AF $\times$ OQ, nearly.

When the above angles are but ſmall, as in viſion through lenſes the caſe generally is, A*b*, A*a*, may be taken without ſinſible error for their meaſures

meaſures reſpectively. And becauſe of the ſimilarity of the reſpective right-angled triangles, it will be,

A*b* : QP :: OA : OQ; and

QP : A*a* :: *o*Q : *o*A; therefore,

A*b* : A*a* :: OA × *o*Q : OQ × *o*A. But OA : *o*A :: [b] OF : AF; [b] 227. I.

and ſubſtituting the like proportionals, it will be,

A*b* : A*a* :: OF × *o*Q : AF × OQ. *Q. E. D.*

321. *Corol.* 1. When the object is very remote, it will be,

A*b* : A*a* :: OF : AF; for then *o*Q and OQ being infinitely great, the ratio betwixt them is a ratio of equality; and therefore,

(OF × *o*Q : AF × OQ ::) OF : AF :: A*b* : A*a*.

[321.] *Corol.* 2. If the eye O be in the focus F of a convex lens; it will be, A*b* : A*a* :: OA (FA) : OQ (FQ). For *o* being now at an infinite diſtance; *o*Q, *o*A, are in a ratio of equality. Fig. 73. to 75.

PROP. XXX.

322. *The image of an object ſeen through any lens will appear under the ſame viſual angle, and in the ſame poſition in both caſes, if the eye and the object change places, the lens remaining fixed. That is, if the object be tranſlated from* Q *to* O, *and the eye from* O *to* Q; *the lens in both caſes remaining fixed at* A; *the viſual angle of the image to the eye at* Q, *will be the ſame as it was when the eye was in* O; *and according as the image did appear erect or inverted before, it will appear alſo the ſame now.* Fig. 72. to 75.

Retaining the ſame things and references as in the four preceding and correſponding figures, let *q* be the conjugate focus of the point Q; that is, let the point *q* be taken ſuch, that Q*f* : QA :: QA : Q*q*. Alſo, let the object QP tranſlated to O, be O*p*; and ſtrait lines being drawn from *q* to *p* and from Q to *p*, let the points where theſe lines cut the lens be reſpectively, *v*, *y*; and let, as before, the lines A*y*, A*v*, be conſidered as the meaſures of the viſual angles of the object and its image reſpectively. Fig. 76. to 79.

Now, as in the preceding prop. it will be,

A*y* : O*p* :: QA : QO; and

O*p* : A*v* :: *q*O : *q*A; therefore

A*y* : A*v* :: QA × *q*O : QO × *q*A. But QA : *q*A :: [a] Q*f* : A*f*; [a] 227. I.

and therefore, A*y* : A*v* :: *q*O × Q*f* : QO × A*f*.

† Fig. 72 to 75. [b] 320. II. [c] Conſt. [d] 227. I.

Now, † A b : A a :: [b] OF × oQ : QO × AF. But becauſe AF = [c] A f, QO × A f = QO × AF; and therefore what remains to be proved is, that qO × Q f = OF × oQ.

Becauſe, Q f : A f :: [d] AF : F q;

and, A f : fo :: [d] OF : AF. Therefore,

Q f : of :: OF : F q; and,

Q f : (Q f ± of) Q o :: OF : (OF ± F q) O q. Therefore Q f × qO = OF × oQ; and conſequently, A b : A a :: A y : A v; that is, the proportion of the viſual angle of the object to the viſual angle of the image, is the ſame whether the object be at QP and the eye at O, or the object be at O p and the eye at Q. But the viſual angle of the object is the ſame in both caſes, and therefore the viſual angle of its image is alſo in both caſes the ſame. *Q. E. D.*

Again, we are to ſhew that the apparent poſition of the image is not changed, by ſhifting the places of the eye and the object. This

Fig. 72, 76. is evident by comparing the correſpondent figures; or it may be proved thus: If the lens be concave, the image is always erect. And when

Fig. 73. the eye O is between the convex lens A and its principal focus F; the object will appear erect, whatever be the diſtance AQ of the object from the lens. For if AQ be greater than A f, the inverted image

Fig. 77. will be neceſſarily behind the eye, and ſo the object appear erect. If the object be placed at O p, it will then have an erect image, and there-

Fig. 74. fore will appear erect, whatever be the diſtance AQ of the eye from the lens. After the ſame manner, if the object PQ be between the focus f and the lens it will have an erect image: And if the object

Fig. 78. be tranſlated to O, and the eye to Q, the object will ſtill appear erect, becauſe o, the conjugate focus to O, is farther from the lens than Q.

Fig. 75. When the diſtances AQ, AO, of the object and the eye from the lens are ſuch, that the point q, which is the conjugate focus to Q, is between O and the lens; the image appears inverted: And ſo it

Fig. 79. will if the eye be tranſlated to Q, and the object QP to O p. For

[d] 227. I. it being, qF : FA :: [d] A f : fQ; and OF : FA :: [d] A f : fo. The geometrical image is inverted, becauſe AO is greater than AF; and the point o, which is the image of the point O of the object, does neceſſarily fall between f and Q, becauſe OF is greater than qF. Therefore, *&c. Q. E. D.*

323. *Schol.* By the tranſpoſition of the eye and the object, there will be neceſſarily a difference as to the diſtinctneſs in the two caſes, excepting when the lens is in the midway between Q and O; and the apparent magnitude of the object may be alſo different, notwithſtanding the equality of the viſual angles in the two caſes. Theſe differences

rences may not be much, when the lens is concave; but they may be very conſiderable, when the lens is convex. When the eye is at O, Fig. 73. the object QP appears indiſtinct, becauſe it is ſeen by converging rays; and its apparent magnitude will be greater or leſs, as it appears farther from or nearer to the lens. But when the eye is at Q, the object Fig. 77. will be diſtinct, eſpecially if the diſtance AQ is not too great: In this caſe alſo, the apparent magnitude of the image will be generally greater than in the former, as the apparent diſtance is generally greater; and the difference may be very conſiderable. A difference of diſtinctneſs and apparent magnitude, will be alſo obſervable between *fig.* 74, 78; and alſo betwixt *fig.* 75, 79; according to which of the diſtances O*q*, or Q*o*, is the propereſt for diſtinct viſion: And when the image appears in its true place, it will appear bigger from O than from Q, if O*q* be greater than Q*o*; and reverſely if O*q* be leſs than Q*o*.

PROP. XXXI.

324. *Any lens and the eye placed in its axis, being both fixed, while an object from being cloſe to the lens is moved farther from it along that axis: If the lens is concave, or the eye is nearer a convex lens than its principal focus; the viſual angle of the image will conſtantly decreaſe, as the object recedes from the lens: But if the eye is farther from a convex lens than its focus, the ſaid viſual angle will conſtantly increaſe, till the object has got to the conjugate focus of the eye; and after the object has paſſed that focus, the viſual angle of its image will decreaſe perpetually.*

CASE 1. Let A be the lens, whoſe focus is F; O the eye, and *o* its Fig. 72, 73. conjugate focus. Now it is manifeſt that the point *a* where the ſtrait line P*a o* cuts the lens, conſtantly approaches towards A, as the object QP recedes farther from the lens; and therefore the viſual angle AO*a* of the image, continually decreaſes as QP recedes from A. *Q. E. D.*

CASE 2. If, while a line QP is moved perpendicularly along the axis Fig. 74, 75. AO, from A towards *o*, the point P be ſuppoſed to approach nearer towards Q, ſo as to keep conſtantly in the line *a o*; or, if the ſaid line QP be ſuppoſed to decreaſe perpetually till it comes to *o*, ſo that the points Q, P, are always in the lines A*o*, *a o*: The viſual angle AO*a* of the image of QP, will remain invariable; and therefore the viſual angle of the image of an invariable object QP will conſtantly increaſe, as QP recedes from A towards *o*; and the image will alſo appear erect. When the object is at *o*, its image will be cloſe to the eye at O. If the axis A*o*, and the line *a o*, be indefinitely produced; the viſual angle

angle of the image of an object whose extreams Q, P are constantly in the said lines, will be the invariable angle AO*a*: And therefore the visual angle of a given object, constantly decreases after it has past *o*, the farther the object is moved from the lens; and the image will also appear inverted. Therefore, *&c. Q. E. D.*

Fig. 72, to 75. Visual angle of the image always reciprocally as *o*Q.

325. *Corol.* 1. The tangent of the ∠ Q*o*P or A*oa*, is reciprocally as *o*Q, and directly as A*a*: For *o*Q is the co-tangent of that angle to the invariable radius QP; and A*a* is its tangent to the invariable radius A*o*. And therefore the tangent A*a* of the visual angle AO*a* of the image, is always reciprocally as the line *o*Q; that is, as *o*Q is increased or diminished, the tangent of the ∠ AO*a* is diminished or increased in the same proportion. And so when *o*Q is infinitely great, or infinitely small; the said tangent will be accordingly infinitely small, or infinitely great.

326. *Corol.* 2. Because the visual angle of the object is always reciprocally as OQ, and that of its image in the reciprocal proportion of *o*Q; as the object recedes from a concave lens, the visual angle of the image decreases faster, than that of the object would do to the naked eye, if the lens was taken away. And when the eye is between a convex lens and its principal focus, the visual angle of the object to the naked eye, decreases faster than that of its image through the lens. But as AQ increases, the ratio's of *o*Q, OQ approach nearer to an equality; so do also the ratio's of the decrements of the above angles, approach nearer to an equality. In *fig.* 74. the visual angle of the image is increasing, and that at a great rate, while the visual angle of the object is decreasing. And in *fig.* 75. after the object has past *o*, its visual angle decreases slower than that of its image; and that in the proportion as OQ increases slower than *o*Q.

Fig. 72.

Fig. 73.

Fig. 68. To an eye placed in the focus of a convex lens; the visual angle of the image will be the same, at all distances of the object from the lens.

327. *Schol.* If the eye be placed in the focus *f* of a convex lens; the image of a given object PQR will subtend the same visual angle at the eye, wherever in the axis the said object be placed. For conceiving the ray *fv* to be returned back to the lens, it will be refracted in the line *v*PB parallel to the axis; and so wherever in the indefinite line *v*PB the point P be taken, its image will be somewhere in the line *fvp*. After the same manner, a ray proceeding from R will be refracted to *f*. Also the tangents of the visual angles of the object, and its image, will be reciprocally as *f*Q, *f*A; for A*v*, QP are in this case always equal, and therefore *f*Q, *f*A are the co-tangents of the above angles. But the distinctness, apparent place and magnitude of the image, will yet be variable.

PROP.

PROP. XXXII.

328. *The diſtance between a lens and an object being fixed, while the eye retreats back from the lens along the axis: If the lens be concave, or the object be nearer a convex lens than its focus, the viſual angle of the image will conſtantly decreaſe: But when the object is farther from a convex lens than its principal focus, the viſual angle will conſtantly increaſe till the eye has got as far back as the geometrical image; and after the eye has paſt that image, the viſual angle will perpetually decreaſe.*

This prop. is the converſe of the preceding. Let A be the lens, F its principal focus; QP the object, and qp its image. Now when the image is before the lens, as in *fig.* 80, 81. it is manifeſt that the viſual angle qOp conſtantly decreaſes, as the eye O recedes farther from A: juſt as the viſual angles of objects ſeen by naked viſion diminiſh, as the eye retreats from them. When the image is behind the lens, as in *fig.* 82, 83; ſince any ray proceeding from P, does after refraction proceed to the focus p; it is manifeſt that the viſual angle AOB under which the object is ſeen through the lens, is in this caſe alſo always $= \angle qOp$. And conſequently, till the eye has got to the inverted image, the viſual angle conſtantly increaſes; and after the eye has paſt the image, the ſaid angle conſtantly decreaſes. Therefore, *&c. Q. E. D.* Fig. 80, to 83.

329. *Corol.* 1. The tangent of the viſual angle qOp or AOB, is always reciprocally as Oq; or the co-tangent of the $\angle qOp$ is always directly as Oq. And therefore when Oq is infinitely increaſed or diminiſhed, the angle qOp will be the leaſt or greateſt poſſible.

330. *Corol.* 2. As the eye recedes from a concave lens; the viſual angle qOp of the image, decreaſes faſter than the viſual angle QOP of the object, ſeen by naked viſion: And the contrary happens, when the object is between a convex lens and its principal focus. Becauſe in the firſt caſe, the diſtance Oq of the image is leſs, and in the ſecond caſe greater, than the diſtance OQ of the object. But as the diſtance AO increaſes, the ratio's of the decrements of the above angles, will approach nearer to an equality. When the image is behind the eye, its viſual angle increaſes the faſter, the nearer the point O of the eye approaches towards it; and after the eye has paſt the image, the viſual angle at firſt will decreaſe very faſt, and afterwards ſlower by degrees.

Fig. 68. To all diſtances of the eye from a convex lens, when the object is in the focus; the viſual ∠ of the image will be invariable.

331. *Schol.* If an object be placed in the principal focus F of a convex lens; its image will appear under the ſame viſual angle, at all diſtances of the eye from the lens. For in this caſe, the rays P*s*, P*v*, P*t*, *&c.* will be refracted into the rays *sa*, *vf*, *t*G*b*, &c. all parallel to one another, and to the ray *vf*; and ſo wherever in the axis the eye be placed, as at G, *f*, or *a*, &c. the viſual angles AG*t*, A*fv*, A*as*, &c. will be all equal. In this caſe alſo, the image will always appear erect.

But here alſo, the diſtinctneſs, apparent place and magnitude of the object will be variable.

When the object is in one focus, and the eye in the other focus of a convex lens; the viſual ∠ of the image, is double that of the object.

332. *Corol.* When Q is in F, the ∠A*fv* = ∠QAP: For A*v* = QP, and AF = *f*A. And therefore if the eye be in *f*, the viſual angle of the image, will be double the viſual angle of the object, ſeen by the naked eye from the ſame place.

PROP. XXXIII.

333. *The eye and an object remaining fixed in the ſame places, while a lens is moved between them, ſo that its axis is always in the line joining the eye and the ſame point of the object: If the lens be concave, the viſual angle of the image will be leaſt of all, when the lens is in the middle point betwixt the eye and the object; and the viſual angle will continually increaſe, as the lens is moved from the middle towards either extream: If the lens be convex, and its focal length greater than $\frac{1}{4}$ of the diſtance betwixt the eye and the object; the viſual angle will be greateſt of all, when the lens is in the middle; and it will continually diminiſh, as the lens is brought nigher either to the eye or the object: But if the focal length of the convex lens, is leſs than $\frac{1}{4}$ of the diſtance betwixt the eye and the object; the viſual angle of the image will be then leaſt of all, when the lens is in the middle betwixt them.*

Fig. 84, to 91.

Let O be the place of the pupil of the eye, whoſe breadth is here not regarded; Q the object; A the lens in the middle between the eye and the object; that is AO = AQ; the points F, *f*, the principal focus's of the lens when it is placed at A; and G, *g*, theſe focus's when the lens is moved to *a*. When the lens is in A, *o* is the conjugate focus to the eye, and *v* that focus when the lens is at *a*; that is, the incident rays whoſe focus are at *o*, *v*, reſpectively, have their focus at O after refractions.

Now $OF \times fo = {}^{a}OG \times gv = {}^{a}AF^2 = {}^{a}aG^2$; and ſo, [a] 227. I.

$OF : OG :: vg : of$. And when the lens is in $\overset{A}{a}$; it will be,

viſual ∠ of the image : viſual ∠ of the object :: [b] $\begin{matrix} AF \times OQ & OF \times Qo \\ aG \times OQ & OG \times Qv \end{matrix}$. [b] 320. II.

The two firſt of theſe rectangles are equal, becauſe $aG = {}^{c}AF$; [c] Conſt. and they are alſo invariable: Therefore the viſual angle of the image when the lens is in A, or at any other point *a*, will be reciprocally as the rectangle $OF \times Qo$, or $OG \times Qv$. Again $OF = {}^{c}Qf$, and $fg = {}^{c}FG$.

334. CASE 1. *The diſtance between the eye and an object being fixed, while a concave lens is in motion from either of them towards the other; I ſay the viſual angle of the image will decreaſe continually, till the lens has got into the middle, where the ſaid angle will be leaſt of all; and after that it will increaſe continually, and that at the ſame rate that it decreaſed before.*

For $OF : OG \ {}^{*}\sqsubset Qg : Qf$; and therefore $OF : OG \sqsubset Qv$ $(Qg - vg) : Qo\ (Qf - of)$. Wherefore $OF \times Qo \sqsubset OG \times Qv$; and conſequently the viſual angle of the image is leſs when the lens is in A than when it is in any other point *a*; and it is manifeſt that as the point *a* approaches nearer to O, the ratio of $OF \times Qo$ to $OG \times Qv$ continually increaſes. Therefore, *&c. Q. E. D.* Fig. 84.

335. CASE 2. *When a convex lens, whoſe focal length is greater than one fourth of the diſtance between the eye and the object, is placed in the middle between them: I ſay the viſual angle of the image is then greateſt of all; and that this angle continually decreaſes, as the lens is brought nigher either to the eye or the object.* This caſe, as to the conſtructions of the figures, admits of the ſix following varieties.

1, 2. When both the focus's F, G, are before O; or when they are both behind O, and the focus *g* is beyond Q. Fig. 85, 86.

We have $OF : OG \sqsubset Qg : Qf$; and therefore $OF : OG \sqsupset Qv$ $(vg - Qg) : Qo\ (of - Qf)$.

3, 4. When the focus's F, G, are both behind O, and the focus *g* is nearer the eye than Q is; or, the focus G only is behind O. Fig. 87, 88.

It will be, $OF : OG \sqsupset Qv\ (vg + Qg) : Qo\ (of - Qf.)$ Therefore in each of theſe four caſes, $OF \times Qo \sqsupset OG \times Qv$; and conſequently the viſual angle of the image is greateſt when the lens is in A; and

* $\sqsubset$ ſignifies that the quantity before it is greater than that which follows, and $\sqsupset$ ſignifies the contrary: The expreſſion referred to ſignifies that the ratio of OF to OG is greater than the ratio of Qg to Qf; the reaſon whereof is, becauſe $FG = fg$, and $OF \sqsupset Qg$, OF being $= Qf$. The like is to be underſtood in the expreſſions that follow. See alſo note * page 46.

and it is manifeſt that it decreaſes continually, as the point *a* approaches towards O.

Fig. 89. 5. When O is in F, Q will be alſo in *f*. And it will be, when the lens is in $\frac{A}{a}$; *viſual* ∠ *image* : *viſual* ∠ *object* :: $\frac{QF : QA :: {}^{d}2 : 1}{aG \times OQ : OG \times Qv.}$

[d] 332. II.

But $aG \times OQ = 2aG^2$, becauſe $OQ =^{c} 2aG$; and $OG \times Qv = OG \times gv + OG \times gQ$. But $OG \times gv = aG^2$; and therefore $aG \times OQ \sqsupset 2OG \times Qv$. Therefore the viſual angle of the image when the lens in A, is greater than when it is in any other point *a*. *Q. E. D.*

Fig. 90. 6. When O is in G; it will be when the lens is in $\frac{A}{a}$; *viſual* ∠ *image* : *viſual* ∠ *object* :: $\frac{AF \times OQ : OF \times Qo}{OQ : {}^{e}Oa.}$

[e] 327. II.

And $OQ : Oa :: OQ \times Qo : Qo \times Oa$. But $Qo : Oa \sqsupset AF : OF$; For $AF : OF ::$ [c] $Af : fQ ::$ [a] $Ao : (AO)\,AQ :: fo\,(Ao - Af) : Af = Oa$. Wherefore $AF \times OQ : OF \times Qo \sqsubset OQ \times Qo : Qo \times Oa$; and conſequently the viſual angle of the image is greateſt, when the lens is in A. *Q. E. D.*

336. Case 3. *When the diſtance between the eye and the object, is greater than four times the focal length of a convex lens; the viſual angle of the image will be leaſt of all, when the lens is in the middle between the eye and the object.*

Fig. 91. For $OF : OG \sqsubset Qg : Qf$; and therefore
$OF : OG \sqsubset Qv\,(Qg - vg) : Qo\,(Qf - of.)$ Wherefore
$OF \times Qo \sqsubset OG \times Qv$; and conſequently the viſual angle of the image is leaſt when the lens is in A. *Q. E. D.*

In all the above caſes, if the point *a* be taken betwixt A and Q; the caſe will be the ſame as if the eye and the object changed places, whilſt the point *a* retained its former poſition; and which ever ſide of A the point *a* be taken, if the diſtances from A be equal, the viſual angle of the image will be the [f] ſame in both caſes. Therefore, &c. *Q. E. D.* †

[f] 322. II.

Fig. 84, to 90. 337. *Schol.* 1. In the two firſt caſes the object appears always erect, at whatever diſtance from the eye the lens be placed. This is ſelf-evident, if the lens be concave; and when the lens is convex, becauſe $OF \sqsupset FA$, if Q is beyond *f*, *o* will be alſo beyond Q; and therefore the image of Q will be behind O, whence the object neceſſarily appears erect.

Fig. 91. 338. *Schol.* 2. In caſe the third, the image will continue inverted till the lens has got into a certain point B, ſo that $Qb : QB :: QB : QO$, the point *b* being the focus of the lens when it is in B. For then the image

† When $OQ = 4AF$, and $AQ = AO$, the image of Q will be in O, and therefore inviſible.

image of Q will be in O; and ſo it will alſo be, if the point B be taken as far diſtant on the other ſide of A. During the motion of the lens from A to B, the viſual angle of the image was continually increaſing; after this the image will appear erect, and its viſual angle will decreaſe as the lens approaches nearer to O.

To find the point B; take OC = 2AF, and betwixt AO, AC, find the mean proportional AB, which gives the point B required. Becauſe

AB : AO :: [a]AC : AB :: BC (AB—AC) : BO (AO—AB). [a] Hypoth.

Therefore, QB (AB + AO) : AO :: OC (BC + BO) : BO; and

$\frac{QB}{2} : \frac{QO}{2}$ (AO) :: $\frac{OC}{2}$ ([b]*b*B) : BO :: QB : QO :: B*b* : BO. [b] Conſt.

Wherefore, Q*b* (QB—B*b*) : QB :: QB (QO—BO) : QO. *Q. E. D.*

339. *Schol.* 3. In ſome caſes, the viſual angle of the inverted image may be equal to the viſual angle of the object ſeen by naked viſion. For whenever QA : A*q* :: OQ : O*q*; the angles QOP, *q* O*p* are equal. Fig. 83. For, QP : *qp* :: AQ : A*q*; and when QP : QO :: *qp* : *q* O, the triangles QOP, *q* O*p* are equiangular. This can never happen but when the eye and the object, are each of them farther from the lens than twice its focal length; and the diſtance of either of them from the lens being given, to find the place of the other when the ſaid angles are equal.

Let AG and AH, be each equal to twice the focal length AF, of the lens. Say OG : GA :: AH : HQ, and by this proportion we have either the point O or the point Q.

Becauſe OG : GA :: AH : HQ; it will alſo be,

OA (OG + GA) : GA :: AQ (AH + HQ) : HQ.

And OQ (OA + AQ) : AQ :: AQ (GA + HQ) : HQ.

But AQ : Q*q* :: [c]QF : QA. [c] 227. I.

Therefore by multiplying, OQ : Q*q* :: QF : HQ. Again,

O*q* (OQ—Q*q*) : OQ :: AF (QF—QH = FH) QF :: [c]*fq* : A*f*.

Therefore,

O*q* : OQ (:: AF : QF :: *fq* : A*f* :: AF (= A*f*) + *fq* : QF + A*f*) A*q* : AQ.

Q. E. D.

If the ratio of OQ to O*q*, be greater or leſs than the ratio of AQ to A*q*, or of QP to *qp*; the viſual angle of the object, will be accordingly leſs or greater than that of the image.

We have now determined the ratio's of the viſual angles of the object and its image, in all the caſes that can happen: But the apparent magnitudes will not be always as theſe angles; and it will not be amiſs therefore, to add ſome farther obſervations concerning the phenomena themſelves. There will be ſome difference alſo in the proportions of

the viſual angles, on account of the aberrations of the lenſes; but it would have been needleſs to have burdened the above propoſitions, with theſe conſiderations.

When a lens has the leaſt effect.

340. When any lens, either concave or convex, touches the object, it then diminiſhes or magnifies the leaſt poſſible; the object and its image being then nearly coincident, and the neareſt to an equality that it is poſſible for them to be. When the lens is brought as near the eye as may be, without offending it, which leaſt diſtance betwixt the lens and the cornea, may be about one eighth of an inch; the viſual angles of the object and its image are ſo nearly equal, that their difference is ſcarce ſenſible, unleſs the lens has a ſhort focus, as not exceeding a few inches. But yet in this caſe, if the focal length of the lens does not exceed two or three feet; an object ſeen through a concave lens will be ſenſibly diminiſhed, and an object not farther from a convex lens than its focus, will be ſenſibly magnified by it; and theſe differences of apparent magnitudes, are generally more owing to the differences of apparent diſtances of the object and the images, than to the difference of the viſual angles, though theſe alſo contribute their ſhare; that is, becauſe the image ſeen through a concave lens generally appears nearer, and through a convex lens farther, than the object.

Fig. 83.

341. An inverted image, when it ſubtends the ſame viſual angle at the eye that the object does, appears leſs, and generally much leſs, than the object. For the image in this caſe, always appears nearer than the object, its apparent place being generally at or near the lens; and if it be ſeen in its true place, as at qp, its apparent magnitude will be ſtill leſs.

Apparent magnitude through a concave lens, when the lens, the object, or the eye are in motion.

342. To a given diſtance between the eye and an object, a concave lens generally diminiſhes the moſt, when it is brought neareſt to the eye that diſtinct viſion will admit of. Thus, the given diſtance between the eye at a, and the object PQR, being aQ; if the lens A be brought ſo near to a, that aq is the leaſt diſtance of diſtinct viſion, and AQ be greater than aA; the apparent magnitude of the image will be generally the leaſt it can, for that diſtance of the eye and the object. For if the lens be moved nearer towards the object, although the viſual angle will be [a] diminiſhed till the lens gets to the middle between a and Q, yet the image will be enlarged; as if $AQ = Af$, the the image of $PR = HK$ will be $= hk$. But the apparent diſtance of the image from the eye, if the lens is not very flat, increaſes at leaſt nearly, at the ſame rate that the true diſtance aq is increaſed; and

Fig. 67.

[a] 334. II.

and therefore, whilſt the real magnitude of the image is increaſed, its apparent magnitude muſt alſo increaſe.

Again, if while the lens and the object remain fixed, the eye retreats farther back; and if during this retreat, the image appears in the ſame place, as is generally the caſe at leaſt nearly, it will appear all the while of the ſame magnitude; and that for the ſame reaſon, that objects at moderate diſtances from the naked eye, appear ſtill of the ſame magnitude while theſe diſtances are varied. But if the image ſhould ſeem to approach nearer to the lens, this will cauſe it to appear ſomewhat diminiſhed.

If while the eye and the lens remain fixed, the object be moved farther off; the apparent magnitude of the image will be conſtantly diminiſhed. For whilſt its viſual angle is diminiſhed, its apparent diſtance is ſcarce at all increaſed. In this caſe, the geometrical image is diminiſhed, whilſt its apparent and true place are nearly coincident.

Apparent magnitude through a convex lens, is chiefly affected by the viſual angle.

343. In viſion through a convex lens, the viſual angle of the image has frequently a greater ſhare in the idea we form of its apparent magnitude, than when we look through concave lenſes; and that becauſe, when the image is far beyond the object, the divergencies of the rays alone are not ſufficient means for diſtinguiſhing its true place; and the apparent diſtance of the image, when it is before a convex lens, is ſeldom ſo far as its true diſtance.

Apparent magnitude when the object and a convex lens are both fixed, the eye being in motion.

344. The object and a convex lens being fixed at a given diſtance, leſs than the focal length of the lens: As the eye recedes farther back, from being cloſe to the lens, the apparent magnitude of the image will generally increaſe for a little way; for the viſual angle will be diminiſhed in a [a] leſs proportion, than the apparent diſtance is increaſed. If the geometrical image is a good way farther than the object, its apparent magnitude may continue to increaſe, till the eye has got as far as it can ſee the whole object thro' the lens; and the object being ſeen to fill a greater part of the lens, contributes alſo to this phenomenon. But when the object is pretty near the lens, that is, when its image is not far beyond it; after the eye has got to a ſmall diſtance from the lens, the apparent magnitude may vary inſenſibly; becauſe the increaſe of the apparent diſtance, and decreaſe of the viſual angle, become nearer to a ratio of equality; or the increaſe of the one and decreaſe of the other, may for a while continue exactly proportional, as the image may ſeem to move a little nearer to the object. Or the apparent magnitude of the image may decreaſe, by its ſeeming to approach ſo towards the object, that the increaſe of the apparent diſtance ſhall be in a leſs proportion, than the decreaſe of the viſual angle. And hence, it

[a] 330. II.

it is eaſy to account for all the different phenomena, that may happen by placing the object at different diſtances from the lens, towards its focus; or by uſing different lenſes.

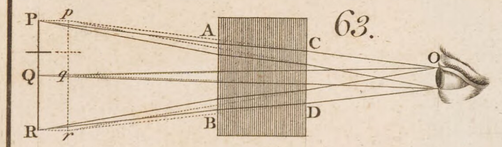
If the object be placed in or very near the focus, the apparent magnitude will increaſe all the way as the eye recedes from the lens. For
[c] 331. II. the viſual angle of the image [c] remains the ſame, whilſt by retreating from the lens, its apparent diſtance is increaſed. And if the object is placed beyond the focus; the apparent magnitude will be neceſſarily increaſed, as the viſual angle will be increaſed, till the eye has got as far back as the image. After the eye has paſt the image, its apparent magnitude will remain the ſame, if it be ſeen conſtantly in its true place; but if it ſeems to hover about the lens, as the caſe frequently is, its apparent magnitude will decreaſe, as the viſual angle will diminiſh faſter, than the apparent diſtance is increaſed.

The eye and the lens being fixed, while the object is in motion.

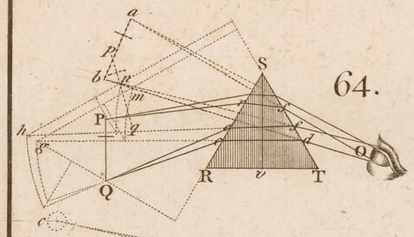
345. The diſtance between the eye and a convex lens remaining invariable, whilſt the diſtance betwixt an object and the lens is continually increaſing: If the eye and the object are pretty cloſe to the lens, the apparent magnitude will continue, for a while, pretty much the ſame; for in this caſe, the increaſe of the apparent diſtance and decreaſe of the viſual angle, will be nearly proportional: But if the object be moved ſo far, that its apparent diſtance through the lens, increaſes ſlower than its true diſtance; the apparent magnitude will decreaſe. If the ſtation of the eye be nearer the focus of the lens, the phenomena will be different; and the apparent magnitude of the object, as it retreats from the lens, will conſtantly increaſe, at leaſt till it has got to a certain diſtance off. Becauſe here, the viſual angle decreaſes ſlower than the apparent diſtance increaſes; and when the eye is in the focus, that
[d] 327. II. angle [d] continues invariably for all diſtances of the object from the lens.
If the eye be without the focus, as the object is moved from the lens,
Fig. 68. the apparent magnitude will increaſe ſtill faſter: For in this caſe, the viſual angle itſelf will be increaſed; as the image of a point P moving parallel to the axis A q, will be always ſomewhere in the line fvp; and therefore when the eye is at a point a, further from the lens than f; the farther is the point p, the greater will be the viſual angle qap. And whether the eye be within or without the focus, if it be not far within it, the apparent magnitude will increaſe after the object has paſt the focus; for the viſual angle continually increaſes, as the image behind advances forwards towards the eye; and till the image has got before the eye, the apparent diſtance of the object is never leſs than that of the lens.

In this and the preceding, there will be ſome difference in the phenomena, on account of the aberrations of the lens: Alſo there will be

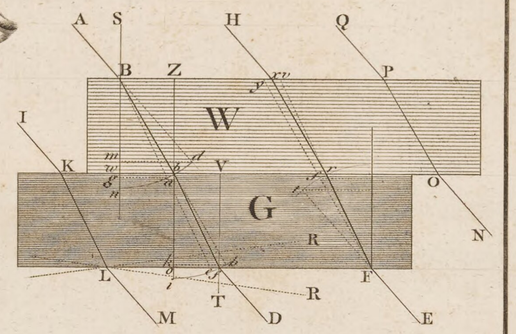
63.



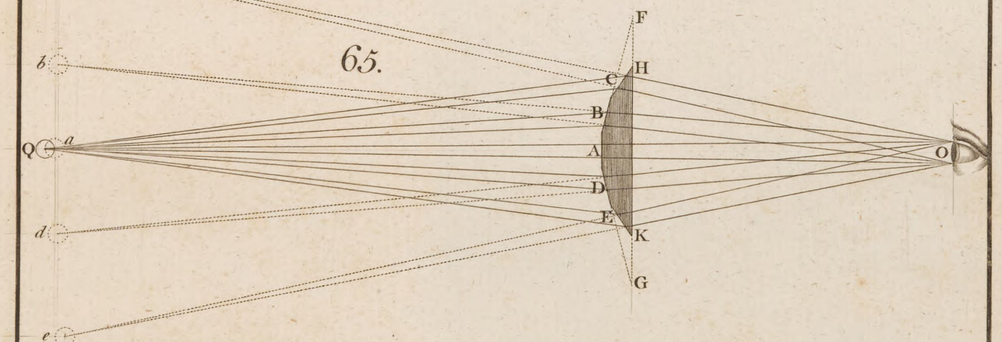
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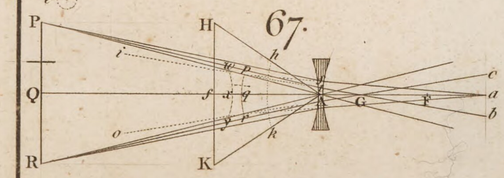
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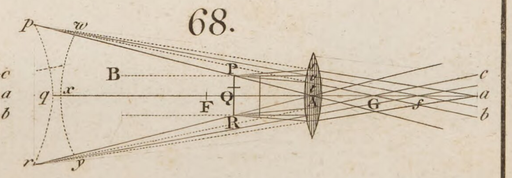
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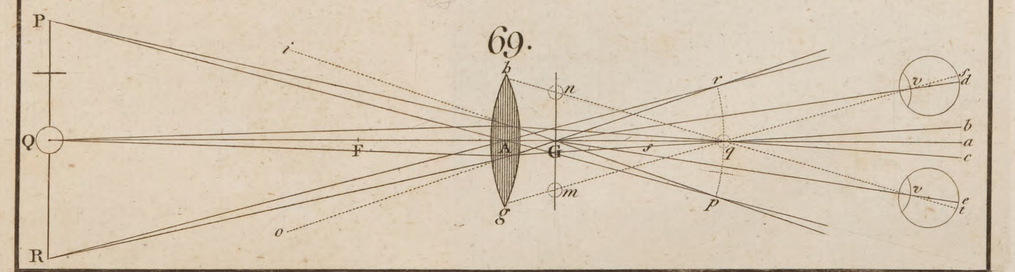
67.



68.



69.



be frequently a difference in the apparent place of the image, according as the eye is vigorous or fatigued, *&c.* and that will cauſe a difference in the apparent magnitude.

Phenomena when the eye and object are fixed, while the lens is in motion.

346. It has been [e] ſhewed, that when the diſtance between the eye and an object, is leſs than four times the focal length of a convex lens; the viſual angle will be greateſt of all, when the lens is in the midway between them: The apparent magnitude in this caſe, will be alſo greateſt of all, when the lens is not very near either to the eye or to the object, but not always when it is in the middle betwixt them. For as the lens is held at different diſtances from the eye, the apparent diſtance of the object will be different; and the apparent magnitude will be greateſt of all, when the rectangle under the viſual angle and the apparent diſtance, is greateſt. The apparent diſtance is commonly greateſt of all, when the lens is ſo placed, that the object is near its principal focus: And when the diſtance between the eye and the object, is much above twice the focal length of the lens, as three times or more that length; the viſual angle varies but ſlowly, as the lens is moved a little either way from the middle. And hence, when the diſtance between the eye and the object, is ſenſibly greater or leſs than about twice the focal length of the lens; the apparent magnitude is commonly greateſt, when the lens is accordingly nearer the object or nearer the eye, than the middle. But in looking at an object, we naturally place the lens ſo, as upon the whole the object appears beſt; and this it does not always when it is moſt magnified. For if the eye is ſuited for viſion at common diſtances, as in all the above obſervations were ſuppoſed; the object muſt be placed a little within the principal focus, that it may appear to the beſt advantage.

[e] 335. II.

An object may be more magnified by a ſhallower lens than with a deeper, and why.

347. From the preceding obſervations about viſion through convex lenſes, it is manifeſt that an object may appear bigger through a lens of two or three feet focus, than through a lens of half that focal length; according as the proportions of the diſtances betwixt the eye, the object, and the lenſes are differently circumſtanced. But yet there is another cauſe that may produce theſe phenomena. An image ſeen with both eyes through a broad convex lens, will ſometimes appear two or three feet or conſiderably more, beyond the object, if the lens has not a very ſhort focus: And generally, if the focal length of the lens does not exceed a certain limit as about arm's length; the longer it is, the farther off beyond the object the image may be perceived. Thus, through a lens of a foot focus, the apparent diſtance of the image beyond the object, may be proportionably greater than if we look through a lens of ſix inches focus; the diſtances betwixt the eye,

the object and the lenſes, being proportional in both caſes. The reaſon whereof ſeems to be, that the flatteſt lens being fartheſt from the object, the eyes adapt themſelves the eaſier for viſion at a greater diſtance; and the rays being refracted more uniformly by the flatteſt lens, may probably be another reaſon of our being the better able to diſtinguiſh the places, from whence they diverge to the eyes. If the proportion of the apparent diſtance through the flatteſt lens to that through the deepeſt, be greater than that of the viſual angle through the deepeſt to that through the flatteſt lens; the object will appear biggeſt through the flatteſt lens. Theſe phenomena happen only in particular caſes, and not frequently.

Magnifying and diminiſhing powers of lenſes, reciprocally as their focal lengths, nearly.

348. Hence it appears, that the proportion of the apparent magnitudes of an object ſeen through two different lenſes, cannot be exactly aſſigned by having given the focal lengths of theſe lenſes, and the diſtances of the object from them. However, it may be reckoned in the general, that the magnifying powers of convex lenſes, and the diminiſhing powers of concave lenſes, are reciprocally as the focal lengths, nearly. Let an object be placed ſucceſſively in the focus's, or at ſimilar diſtances from the focus's of two convex lenſes; the eye being alſo placed ſucceſſively in the other focus of each lens: The viſual angles of the images in theſe two caſes, will be reciprocally as the focal lengths of the lenſes; and therefore if the apparent diſtances of the images from the lenſes, were equal in the two caſes; the apparent magnitudes would be reciprocally as the focal lengths of the lenſes: But as the image ſeen through the deeper lens, will hardly ever appear ſo far as it will through the other; ſo the ratio of the apparent magnitudes through the two lenſes, will be nearer to an equality than that of the viſual angles.

The apparent magnitude of a remote object, ſeen through two concave lenſes of ſhort focus's, will be very nearly in the reciprocal proportion of their focal lengths: The diameters of the images will be exactly in that proportion; and if the eye be at proper diſtances, they will appear very nearly in their true places, that is, in the focus of each reſpective lens. But when the object is near, the deeper lens will not diminiſh ſo much more than the other, as in the above proportion.

PROP.

PROP. XXXIV.

349. *To determine the viſible area, or breadth of the field of viſion, through a given * aperture of a given lens; the place of the eye, and of the indefinite plane or object, being alſo given.*

Let A be the lens, whoſe center is A, and focal length is AF; AB the ſemi-aperture of the lens, O the place of the eye, and PQ the indefinite object. Having drawn the indefinite image pq, draw the line OB, and produce it till it cuts the image in p; through the point p draw the axis Ap, and produce it, as the caſe requires, till it cuts the indefinite object in P. Now it is manifeſt that pq, contained between the axis OQ and the line OBp, is the breadth of the viſible image through the ſemi-aperture AB, and that PQ is the part of the object whoſe image is pq. Therefore PQ is the viſible area through the ſemi-aperture AB. *Q. E. I.* Fig. 80, to 83.

The above conſtruction, regards only the extream rays that enter the center of the pupil; but if the breadth of the pupil be conſidered, the viſible area will be ſomewhat larger. Let, as before, O be the center of the pupil, and Ob that ſemi-diameter of it which lies on the contrary ſide of the axis that the point P is of. By drawing the line bB till it cuts the image, and a line from the center of the lens through this point of interſection till it cuts the object, we have the viſible area Qx, as before.

350. *Schol.* 1. Hence the viſible area is greateſt when the eye is cloſe to the lens: And the lens and object being fixed; the part in view through a given aperture, will decreaſe perpetually while the eye recedes from the lens, unleſs the image is behind the eye; in that caſe it will decreaſe only till the eye has got to the image, and after that it will increaſe perpetually. But notwithſtanding, as the eye recedes from O farther back; the decreaſe of the viſible area in *fig.* 80, 81, and the increaſe of it in *fig.* 83, has but a ſmall limit. For, ſuppoſing the pupil contracted into a point, the image pq in both caſes will be equal to AB when O is at the greateſt diſtance; that is, when OA, OB are parallel. Whence by making qp = AB, and drawing the axis ApP; we have the ſmalleſt field in the one caſe, and the largeſt in the other, that can poſſibly happen. In *fig.* 82. when the eye is in q, qp cannot exceed Op the ſemi-diameter of the pupil: But in this caſe indeed, nothing can be ſeen on account of the indiſtinctneſs.

Field of viſion the greater, the nearer the eye is to the lens.

351.

* By the aperture of a lens or ſpeculum is meant ſo much of it as is open and not covered.

Fig. 81. Field of vision = aperture of a convex lens, when the eye is in its focus.

351. *Schol.* 2. When the eye is in the focus *f* of a convex lens, the visible area of an object placed any where in the axis on the other side, will be equal to the aperture AB. For in this case, if the ray OB or *f*B be supposed to return backwards to B, the refracted ray BP will be parallel to the axis AF; and therefore where-ever PQ is placed in the indefinite axis A*q*, the ray PB will be refracted to O, and the visual angle *p*O*q* will be invariable.

Fig 80, to 83.

352. *Schol.* 3. If the places of the lens, the eye and the object be given; and also the breadth of the object, to find the least aperture of the lens through which it may be seen. Find the image *qp* of QP, and draw the lines *q*O, *p*O; and that part of the lens which is included between them, as AB, will be the aperture required. In like manner, if QP and AB are both given; and it be required to find the point O, where the rays QA, PB intersect after refraction by the lens. Produce *q*A, *p*B till they meet, and that will give the point required.

An object seen through a lens moved up, or down, or sideways, seems also to move, and which way.

353. *Schol.* 4. If a lens held before the eye be moved quickly up, or down, or sidewise; an object seen through it will also seem to move or dance, at the same time. And if the lens is concave, or the eye is behind the inverted image of a convex lens, as in *fig.* 83; the object will seem to move the same way that the lens is moved: In the other cases, as in *fig.* 81, 82; the apparent motion of the object seen through a convex lens, is the contrary way.

Fig. 80, to 83.

From the point P of the object, draw the line P*a*O to the center of the pupil O. Now let the lens be moved upwards till its center A is in *a*; and the point P which before was seen in the line OB, will be now seen in the line O*a*; P*a*O being now the axis of the pencil of rays proceeding from P to O: And consequently P appears in a different place from what it did before; and for the same reason it will seem to move, and also every other point of the object, all the while the lens is moved. The phenomena will be same, if while the lens is fixed, the spectator moves his head.

How to find whether a flattish glass is a lens.

354. *Cor.* And hence we have a ready way of finding whether a flattish glass, has its sides truly flat and parallel; or whether the glass is a concave or a convex lens. For if the glass has its sides flat and parallel, the apparent place of the object will remain invariable while the glass is moved before the eye; because the rays are refracted alike by every part of it. And if while the glass is moved, the object seems to move, the glass has a focus; and by observing whether the motions of the object and the glass are the same way or contrarywise, we know whether the glass is a concave or convex lens.

355.

355. *Schol.* 5. If the diſtance AO of the eye from a concave lens be ſuch, that AB = B*a*; the field of viſion through the given aperture AB, will be twice as wide, as it would be to the naked eye looking through an aperture of the ſame ſize, and at the ſame diſtance from it. And the point P will continue viſible to the eye at O, through the ſmall aperture AB, while the center of the lens is moved throughout the whole ſpace A*a*. But the point P, ſeen through the convex lenſes, *fig.* 81, 82, continues viſible through the large aperture AB, only while the center of the lens moves through the ſmall ſpace A*a*: And in the figures laſt referred to; the field of viſion through the given aperture AB, is only about half the breadth of what it would be through the ſame aperture, if the lens was taken away. And hence it is, that an object continues viſible through a concave lens, while either the lens or the eye is moved ſideways conſiderably, or whilſt the axis of the lens is conſiderably inclined: And this motion may be continued much longer before the object is loſt, than it could if the lens be removed whilſt the ſame aperture remained. And the contrary happens in looking through a convex lens. Fig. 80.

PROP. XXXV.

To find the focal lengths of lenſes by experiments.

This propoſition may be ſolved by different methods, as followeth.

356. The focal length of a pretty broad glaſs lens, may be obtained near enough for ſome purpoſes, thus: Meaſure exactly the thickneſs of the lens in the middle and at the edge, and alſo its breadth; then it will be, By the thickneſs and breadth of the lens.

As the difference of the above thickneſſes, is to half the breadth; [a] *ſo is half the breadth, to the focal length required.* [a] 202. I.

By a proper gage made for the purpoſe, the thickneſs of a lens may be obtained to the 500th part of an inch; and its breadth might be alſo meaſured to a tolerable exactneſs. But leſt the figure of the lens ſhould be faulty near the edge, the thickneſs ſhould be taken at ſome diſtance therefrom, and alſo in two places diametrically oppoſite; half the ſum of theſe two thickneſſes is to be deemed the thickneſs at the points where they were taken, and the diſtance betwixt theſe points is to be reckoned the breadth of the lens. By this method, an eſtimate may be made how the lens is centered, and whereabouts is its vertex; and it may be uſeful on ſome occaſions, as

it may always be put in practice: But if great exactneſs be required, this method is not to be depended upon; becauſe a ſmall error in the *data*, may produce a great one in the focal length.

By having the radii of the ſides.

357. If we have the radii of the ſides of the lens given; its focal length may be found by the rules prop. XXIV. or prop. XXV. of Book I. according as the lens has one ſide plane or both ſpherical; remembring for *m*, *n*, if the lens is of glaſs, to uſe the numbers 31, 20, or * 17, 11, as being more exact than the numbers 3, 2. But this method in practice is very precarious, as it is difficult to find the radii of the ſides of the lens to a ſufficient exactneſs: And even the focal lengths of two lenſes formed by the ſame tools, will be different; ſo that the workmen themſelves before trial, do not know the exact focal lengths of their lenſes, though they know to what ſpheres their tools were formed. In practice therefore, we need only uſe the following rule, according as the centers of the ſides are on different ſides, or on the ſame ſide of the lens;

As the ſum or difference of the radii of the ſides, is to one of them;
[b] 192. I. *ſo is double the other, to the focal diſtance from the lens.*

If the curvities of each ſide is the ſame, the radius of that curvity may be deemed the focal length. And if one ſide of the lens is plane,
[c] 179. I. the focal length may be reckoned equal [c] to twice the radius of the ſpherical ſide.

The focal length of a convex lens may be alſo found by the following methods.

Focus of a convex lens found by the image in a dark room of a given object.

358. Let the room be darkened ſo that no light comes into it, excepting through a hole in one of the window-ſhutters: The lens being placed againſt and cloſe to this hole, move a white paper behind it, till the picture of a particular object, directly facing the lens, appears moſt diſtinct, and there fix the paper; then meaſuring the diſtances betwixt the paper and the object each from the lens, there will be a ſufficient data for finding the focal length required.

Fig. 83. Let QP be the object without doors, A the lens in the window-ſhutter, *qp* the place of the paper where the image of QP is diſtincteſt, and QA*q* the axis of the lens. Having meaſured the diſtances
* 227. I. QA, *q*A, it will be, Q*q* : QA :: [d] QA : QF; and QA — QF = AF the focal length required †.

359.

* If theſe numbers be admitted, a double convex or concave lens, whoſe thickneſs is inconſiderable and whoſe ſides have equal curvities, will have its focal length to the radius of its curvity as 11 to 12; as will appear by applying the rule 185. I. t being neglected.

† The diſtance betwixt a fixed object without doors and the window-ſhutter, being once accurately meaſured; the focal lengths of other lenſes may be afterwards found with little trouble, unleſs A*q* be greater than the breadth of the room.

359. *Schol.* 1. In ſtrictneſs the focal length of any lens[e], is the diſtance from its principal focus to the reſpective focal center. And in the example above, the diſtances betwixt the focal centers of the lens, ſhould have been[f] ſubtracted from Qq; and for QA, we ſhould have taken the diſtance from Q to the neareſt focal center. But in theſe ſort of experiments, thoſe niceties would be ſuperfluous.

[e] 215. I.

[f] 226. I.

360. *Schol.* 2. In the above experiment, there will be a certain part of the axis, extending on each ſide the geometrical focus q, within which where-ever the paper is placed, the picture as to ſenſe will be equally diſtinct. Having found nearly the place of q; move the paper from thence both ways along the axis, till an indiſtinctneſs begins to be perceptible in a particular ſmall part of the picture; and having noted theſe two places, the middle point between them may be taken for the focus q. In making this experiment with lenſes of about 12 foot, 8 foot, and 4 ½ foot focus, the lengths of the axis within which the picture of a remote object appeared equally diſtinct, were reſpectively about ‖ 6 or 8 inches, 3 or 4 inches, and 1 or 1 ½ inch. I could not determine the focal length of the flatteſt lens, or that having the longeſt focal length, to leſs than about 2 or 3 inches; the next I could define its focus within about an inch; and the focal length of the deepeſt lens, or that having the ſhorteſt focal length, I could aſcertain to leſs than half an inch. The apertures of each of theſe lenſes was about 1 ½ inch; and this being ſo ſmall in proportion to the focal lengths of the two flatteſt lenſes, is a reaſon why, by this experiment, their focal lengths could not be aſcertained ſo near in proportion, as that of the deepeſt lens.

361. *Schol.* 3. If the focal length of the lens does not exceed 2 or 3 feet, it may be found without darkening the room, by holding the lens at ſuch a diſtance off, that the image of the window-ſaſh may be diſtinct upon the oppoſite wainſcot; then computing as above, the window-ſaſh being now the object.

And for common purpoſes; when the focal length of a lens is but ſhort, this length may be reckoned the diſtance from the lens, to the place where the image of a remote object appears diſtincteſt. For, if the object be diſtant from the lens a 100 or 1000 times the focal length; the image will be beyond the principal focus, only the[g] 100th or 1000th part of that focal length.

[g] 227. I.

362. The focal length of a lens may be found, and that without computation, by making the flame of a candle the object, thus: Move the

Focal length found by the image of a candle, without computation.

‖ Theſe meaſures will be variable, in uſing the ſame lens and object, according to different brightneſs of the object, and different darkneſs of the room. Alſo there will be a difference, if the ſame lens be uſed with different apertures, the other circumſtances being alike.

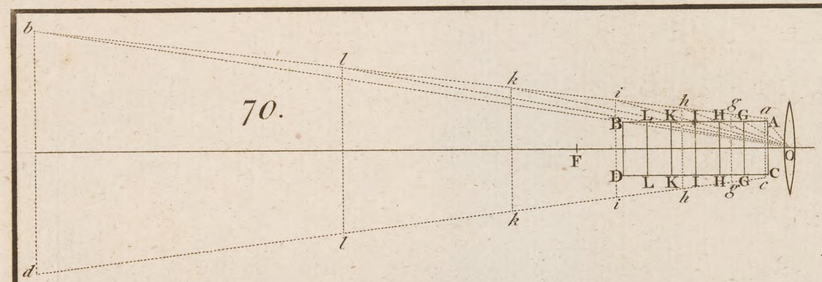
the lens or the candle, and the paper for receiving its image, ſo, that, when the image is diſtincteſt, the lens be exactly in the middle betwixt the other two; then half the diſtance betwixt the object or its image
Fig. 82. and the lens, is the focal length required. Thus, if $Qq = 2AQ$, AQ muſt be $= 2AF$. Or, if of the lens, the candle and paper for receiving its image, any one be moved while the other two are fixed; having meaſured the reſpective diſtances, the focal length of the lens
[h] 358. II. may be found, as has been [h] before ſhewed. Or, if a ſmall hole, of about a $\frac{1}{4}$ or $\frac{1}{8}$ of an inch broad, be made in the window-ſhutter of a darkened room, and a lens and a paper be held behind this hole at proper diſtances; the place where the image of the hole is diſtincteſt, may be determined very critically; and ſo the focal length of the lens may be found, as above.

By the ſun's image. 363. Place the lens ſo that its axis may point, as near as may be, towards the ſun; then holding a paper behind it, the burning point, or where the ſun's image is ſmalleſt, and where its limb appears diſtincteſt, is the focus of the lens.

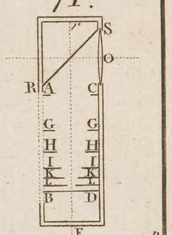
This method of finding the focus's of lenſes that are broad in proportion to their focal lengths, ſuch as common ſpectacles and reading-glaſſes, is ſufficiently accurate. But for lenſes having long focus's, unleſs they are ſo long as to exhibit the ſolar ſpots, the above method will not anſwer at any rate: For the ſun's image being only, as it were, a glare of light, without any diſtinguiſhable parts; we can perceive no difference as to diſtinctneſs, even at the limb, while the paper is moved either way along the axis to conſiderable diſtances. And the different magnitudes of the white circle upon the paper, as it is moved along the axis, will not be ſufficient to determine the place of the focus: For if the lens be no broader than the ſun's image at the focus, the ſtream of light paſſing through it will be a cylinder; and if the lens is not ſo broad as above, the magnitude of the white circle will increaſe all the way, as the paper is moved farther from the lens. Theſe inconveniencies may be removed by the following method.

Fig. 92. 364. Let A be the given lens, whoſe axis is Ab, and principal focus is v or the center of the circle snv; and let snv be the ſun's image, or let the $\angle sAn$ be equal to that ſubtended by the ſun's diameter. Let the lens be covered with thin paſt-board or opaque paper, in which are two round holes a, b, near the edge of the lens: Theſe holes ſhould be at the ſame diſtance from the edge, and in a line paſſing through the middle or the vertex of the lens.

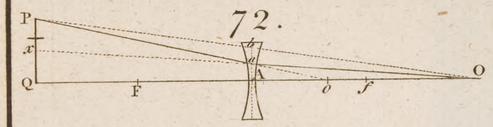
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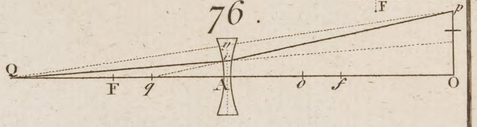
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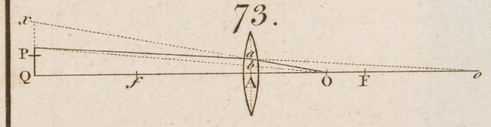
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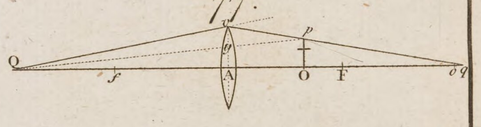
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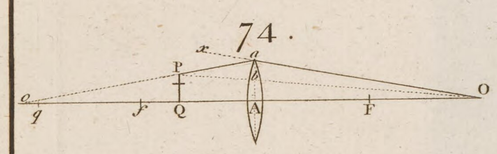
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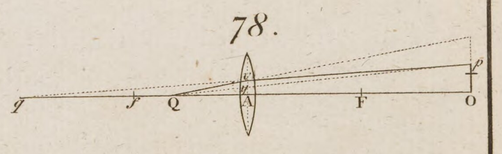
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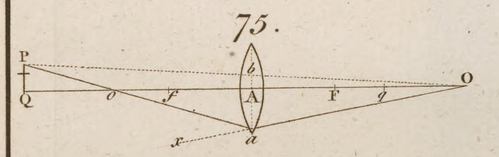
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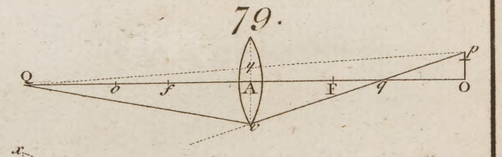
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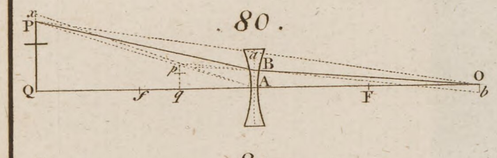
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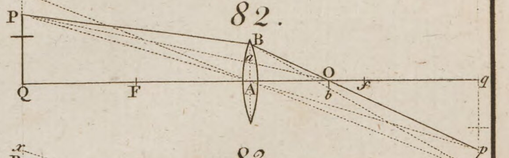
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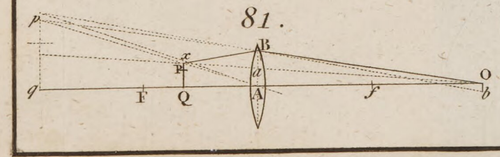
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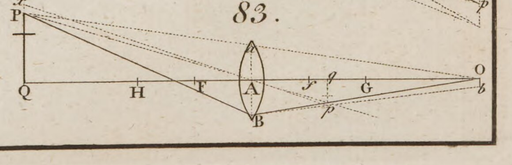
82.



81.



83.



The lens being thus covered, place it ſo that its axis may point directly towards the ſun; then through the holes *a*, *b*, there will iſſue the ſtreams of light *s a n*, *s b n*; the rays belonging to the upper and lower limb of the ſun, having their *foci* reſpectively at the points † *n*, *s*. Now if a paper be held behind the lens, theſe ſtreams of light will form two white ſpots or circles upon it, which gradually approach nearer together, as the paper is moved toward *v*; at a certain point *d* their limbs will touch; after this they will incroach one upon another, and the part common to each will be brighter than the reſt; at *v* theſe circles will coincide; if the paper be moved farther, they will begin to ſeparate, and at *h* their limbs will juſt touch. But as it is difficult to determine preciſely, when the two circles are perfectly united; obſerve where their limbs are at a ſmall and equal diſtance aſunder, as at *o* and *p*; then biſecting *o p* we ſhall have the focus *v* required. If that point be called *x*, where the lines *d a*, *h a* produced either way, as the caſe may require, would interſect; it will be $d v : v h :: {}^{1} x d : x h$; and ſo $x d + x h : x d :: d h : d v$. The holes *a*, *b*, ſhould be cloſe to the edge of that aperture which the lens is to have when applied to its uſe; and they ſhould be alſo of a proper ſize; not too ſmall, for then the white circles will be dark and ill defined; and to make them large would be diſadvantageous, as their centers muſt be nearer together. The experiment will ſucceed better, if the room be darkened. Inſtead of having the paper with the holes in it; the lens might be left open, and covered only with two round opaque circles: But the holes do better.

[1] 3.VI. *Eu.*

By this method the focal length of a lens may be found, I think, more accurately than by either of the preceding ‖. By making the experiment with the lenſes before mentioned of about 12, 8, and $4\frac{1}{2}$ feet focal lengths; I could determine theſe lengths reſpectively within about $\frac{3}{10}$, $\frac{2}{10}$, $\frac{1}{15}$ or $\frac{1}{20}$ of an inch. The limits betwixt the places where I could perceive the images to ſeparate, were reſpectively about 3 inches, 2 inches and $\frac{3}{10}$ of an inch; and had the flatter lenſes been wider, theſe limits would have been more contracted.

The focal length may be alſo found in the ſame manner, by expoſing the lens to the moon, and obſerving where its image is leaſt; or where the bright ſpots coaleſce, if the lens is covered as above. But we cannot be ſo exact here as with the ſun, eſpecially if the focal length

† The incident rays *k a*, *k b*; and *l a*, *l b*; proceed reſpectively from the upper and lower limbs of the ſun; *l a*, *l b*, being parallel to A *s*, and *k a*, *k b* to A *n*.

‖ Other methods for finding the focal lengths of the object-glaſſes, of teleſcopes, and of microſcopic lenſes, will be ſhewed in the next book.

length exceeds two or three feet; and if the lens be covered the holes muſt be large.

An apparatus for holding the lens.

365. In making the above experiments; the lens and the paper for receiving the images, ſhould have proper ſupports; ſo that they may remain fixed at any given ſtation, while the reſpective diſtances are meaſured.

Fig. 93.

The annexed figure repreſents an apparatus, I made uſe of, for holding lenſes of various ſizes. A is the pillar of a wooden candleſtick, on which ſlides the wooden bar *c d*, which may be ſtayed at any height by means either of a ſpring or a wooden wedge at *c*, bearing againſt the pillar. The lens is to be placed between the upright and wooden pieces *a*, *b*; the piece *b* being moveable on *c d*, till it touches the edge of the lens. The fore and inſide parts of *a*, *b*, and alſo the upper ſurface of *c d*, have narrow and thin ledges for the lens to bear againſt; and for confining it on the other ſide, are ſlips of wood ſliding on the inſide ſurfaces of *a*, *b*. Theſe ſlips have a motion of about half an inch, on account of receiving a thick lens, or a lens in its cell; and they are confined at proper places, by a ſcrew in each going through ſlits made acroſs in *a*, *b*, and having a nut bearing againſt the out-ſide of the ſaid pieces *a*, *b*. This little apparatus I had made ſo as to hold any lens not wider than ſix inches: And that the lens might be ſo placed as directly to face any object, or that its axis might have any elevation; the bar *c d* turns ſtifly in a round hole made in the ſocket ſliding on the pillar A; and as a guide to point it aright, there is a ſhort index fixed to the top of the immoveable piece *a*, having on it two ſights to guide the eye, or to let the ſun ſhine through. An † *ox-eye* fixed to a window looking eaſterly or weſterly, ſo as it may face the ſun when he is low, will be very convenient for finding the focal lengths of ſuch lenſes as may be readily ſcrewed to it. Or a ſhort tube faſtened to the window, and having without a ‖ plane ſpeculum which may be ſo inclined as to reflect the ſun's rays directly upon the lens, will do ſtill better.

The paper for receiving the light ſhould be parallel to the lens; and how to ſupport it in this manner, there may be different methods, which it would be needleſs to deſcribe ‡.

To

† See 397. of this.

‖ See the deſcription of the ſolar microſcope in the next book.

‡ Having miſlaid the above deſcribed apparatus, I have ſince got made a more convenient one for finding the focal lengths of lenſes not exceeding two feet. Within about eight inches of one end of a ſtrait wooden bar 32 inches long, was fixed acroſs a ſhort bar, having ſliding cheeks, *&c.* for confining lenſes of different ſizes, like *c d*, *a*, *b*, before deſcribed. The long end of the bar had a groove in it, in

To find the focal length of a concave lens.

366. Let the lens be covered with paper having in it two ſmall circular holes, as at O, *v*; and on the paper for receiving the light, deſcribe two ſmall circles as *a*, *b*, whoſe centers let be at twice the diſtance aſunder of the centers of O, *v*; and it will be beſt if their diameters are alſo double thoſe of O, *v*. Then moving the paper to or fro, till the middle of the ſun's light coming through the holes O, *v*, falls exactly on the middle of the circles *a*, *b*; that diſtance of the paper from the lens will be the focal length required. For it is manifeſt that $ab : Ov :: aF : OF$: Or whatever is the diſtance of *a*, *b*, it will be, $ab - Ov : aO :: Ov : OF$, the focal length of the lens. Inſtead of the paper with the holes, ſmall patches may be ſtuck on the lens. Fig. 94.

Focus found by candle-light.

367. The focus may be alſo found by candle-light, thus: Let Q be the place of the flame; *c*, *d*, the bright ſpots upon the paper, and *q* the point where the lines *vd*, O*c* produced backwards would interſect. To find the point *q*, it will be $cd - Ov : Ov :: Oc : Oq$. Then $Qq : QO ::$ [a] $QO : Qa$, and O*a* is the focal length required.

[a] 227. I.

By joining the lens with a convex one of a deeper focus.

368. The focal length of a concave lens may be readily found, by joining it to a convex one having a ſhorter focus.

Let *q* be the image of any object formed by the convex lens A, fixed at A; and let Q be the image of the ſame object, when the concave lens O is joined to A. Having meaſured the diſtances Qq, qO; to find the focal length OF of the concave lens, it will be, as in the preceding, $Qq : QO :: QO : QF$; then $QF - QO = OF$. *Q. E. I.* Fig. 95.

§. 3. *Of viſion when the object or the eye or both, are in a medium denſer than air.*

369. The appearances of objects placed in a denſe medium, whoſe refracting ſurface is plane, to a ſpectator in the common medium of air,

in which was a ſlider divided into inches and parts; over both theſe was an upright piece which moved the whole length, and which might be readily fixed in any place. The ſide of this piece facing the lens was covered with white paper, for receiving the ſun's image; and for meaſuring the exact diſtance between the paper and the lens, at one end of the long ſlider was fixed occaſionally a piece at right-angles, which by moving the ſlider would touch the thickeſt part of the lens, and then the diviſions cut by the paper would ſhew the diſtance between them. This apparatus was fixed occaſionally to the top of a ſtand for carrying a ſmall reflecting teleſcope, by which it might be elevated to any angle.

air, may be eaſily underſtood from prop. V. and its coroll. of Book I. And I think the phenomena agree in all caſes with the theory, as it has been there explained.

PROP. XXXVI.

370. *To explain the phenomena of viſion of objects placed in a ſpherical denſe medium, to a ſpectator in the medium of air.*

Phenom. Objects placed in ſuch a medium to a ſpectator without, appear magnified, and that in different degrees according to their diſtances from that part of the ſurface which is next the eye; thoſe that are in contact to that point of the ſurface not being magnified at all, whilſt at all other places they will be magnified, and that more and more the farther they are from the ſaid point of the ſurface; ſo that when an object is in contact with the remoteſt point of the ſurface from the eye, it is magnified moſt of all.

Fig. 96. Let ADO be the ſection of a refracting ſphere whoſe center is O; as for inſtance, the ſection of a decanter of water; the ſpectator being on the ſide A, and ſomewhere in the axis A*f*. Let F be the focus of rays, coming from without parallel to the axis *f*OF, after their refraction near A; or F the focus of thoſe rays ſuppoſing the medium continued beyond F; and *f* the focus of rays within the ſphere parallel to the axis DOA, after their emerſion near A. And let Q 1, Q 2, O 3, Q 4, Q 5, D*v*, be the diameter of an object, or the diameters of as many equal and ſimilar objects, ſituated at theſe reſpective places. Now to find the points *q* of the reſpective images *q* 1, *q* 2, O 3, *q* 4, *q* 5, *q* 6; it will be, QF : OF :: [b] AQ : A*q*. Having thus found the reſpective points *q*; through the extreams 1, 2, *&c.* of the objects Q 1, Q 2, *&c.* draw the ſtrait lines O 1, O 2, *&c.* and produce them till they meet *q* 1, *q* 2, *&c.* drawn parallel to the reſpective objects; and *q* 1, *q* 2, *&c.* will be the geometrical images of the ſaid objects, or of their diameters. Or in this caſe, becauſe the objects are all equal and contained between the parallel lines AD, *dv*; their images, if we neglect the aberrations of the refracted rays, will be contained between the axis *f*AF, and the line *fd* produced, as to 6. For *f* is the [c] focus of a ray *vd* coming to *d* parallel to the axis; and ſo the rays coming to *d* from the extreams 1, 2, *&c.* of the ſeveral objects will be all retracted into *df*; whence the extreams 1, 2, *&c.* of all the images are in the line *fd* produced.

[b] 144. I.

[c] Conſt.

Hence when Q is in A, or the object is in A*d*; *q* is alſo in A, or the image is equal and coincident with the object. But as the object is

removed from A towards D *v*, its image increaſes, and that more and more as it advances towards D; the image being greateſt of all when the object is in D *v*. All which are exactly agreeable to the phenomena. *Q. E. D* ‖.

371. *Corol.* 1. The refracting ſurface A *d* remaining the ſame; if the medium be indefinitely continued on the oppoſite part, the images will ſtill increaſe in magnitude as the object advances farther from A, until it has got to F. If the object be placed beyond F, viſion will be indiſtinct; becauſe then the refracted rays will enter the eye, placed near to *A*, converging.

372. *Corol.* 2. The denſer is the medium, the more it magnifies objects placed within it: For then the leſs is the focal diſtance A *f*; and conſequently the diameter of the object = A *d* remaining the ſame, the greater is the angle A *f d*.

373. *Schol.* 1. Whilſt the object is between A and the center O, the image is nearer to A than the object; at O they both coincide, and the image is farther when the object is beyond O. And this is exactly agreeable to the phenomena; the ſemi-diameter O A next the eye, appearing much ſhorter than the ſemi-diameter O D, which ſeems extended as far as the line *q* 6. By covering the bottom of a decanter of water with ſmall objects, as peas, *&c.* the phenomena here explained will be very remarkable, and quite agreeable to the above theory, if the eye is nearly in the ſame plane with the objects. But if the eye is much above that plane, ſo that the remoter objects are ſeen through the belly of the decanter, whilſt the nearer ones are ſeen through a ſmaller ſegment; the remoter objects will appear accordingly ſo much the more remote: For OD, D *e* are in a conſtant ratio; that is, the greater is OD, the greater alſo in proportion will be D *e*.

374. *Schol.* 2. The images of a given ſmall object placed at the centers, or at like diſtances in proportion to the radii from the refracting ſurfaces, of any ſpheres made of the ſame medium, are equal.

All ſpheres of the ſame medium, magnify objects having very ſmall breadths, equally.

Let O be the common center of the ſpherical refracting ſurfaces AD, *a d*, and F, *f* their principal foci; and let O *y*, the breadth of the given object placed any where between the parallel lines AB, DC, be ſo ſmall, as not to differ ſenſibly from the tangents of the arches AD, *a d:* I ſay the images formed by the refracting ſurfaces AD, *a d* will be equal, if the diſtance of the object in each ſphere from the reſpective ſurfaces AD, *a d*, be as the radii OA, O *a*. Fig. 97.

‖ The lines O *a b*, QB, *q* B *b* are for illuſtrating the theory according to Prop. XXI. Book I. This being a different caſe, from either of the figures referred to in that propoſition, but nevertheleſs falling under the ſame rules.

Through the reſpective points D, *d*, draw the lines FDH, *fdh*; and where-ever the objects are placed in the reſpective ſpheres, their images will be contained between the lines FG, FH, and *fg*, *fh*. Let the object placed within the ſphere ABO be called X, and its image *x*; and the object within the leſſer ſphere *ab*O be called Y, and its image *y*. Now becauſe,

[d] Hypoth. 144. I. FX : OF (:: [d]*f*Y : O*f*) :: [b] AX : A*x* (:: [b] *a*Y : *a y*), it will be, A*x* : *ay* :: AX : *a*Y :: [d]FA : *fa* (:: A*x*+FA : *ay*+*fa*) :: F*x* : *fy*. But,

[e] 4. VI. *Eu.* FA : F*x* :: [e]AD : *x*; and *fa* : *fy* :: [e] *ad* : *y*; and becauſe,

FA : F*x* :: *fa* : *fy*; it will be AD : *x* :: *ad* : *y*. But AD = [d] *ad*, and therefore *x* = *y*. *Q. E. D.*

Thus, if the ſmall object be at the center, as O*y*, its image in each ſphere will be = O*x*; and if it be at the peripheries *bc*, BC, its images will be *gh* = GH.

375. *Schol.* 3. When the object is ſo large, that the verſed ſine *av* has a ſenſible magnitude; the image formed by the refraction of the leſſer ſphere, will be leſs than that formed by the greater ſphere. For ſuppoſing the object O*y* to be at the center, for inſtance; it will be, *fv* : *f*O :: *vd* : O*x*; and therefore the greater is the ratio of *fv* to *f*O, the greater will be the ratio of O*y* to O*x*. And hence, equal objects having ſenſible magnitudes, appear ſomewhat more magnified the larger is the ſphere. Add to this, that the greater obſcurity of the thicker medium, may ſomewhat enlarge the apparent diſtance of the image; and this will proportionably increaſe the apparent magnitude.

376. *Corol.* And hence, from theſe two cauſes conſpiring together; the ſmall bores of capillary tubes, or ſuch as are uſed for mercurial thermometers, appear bigger in proportion the thicker are the tubes.

377. *Schol.* 4. A broad ruler with parallel ſides placed upright in the middle of a conical veſſel, as a glaſs of water, will at firſt ſight appear broadeſt where the glaſs is narroweſt. This deception is chiefly owing, to the breadth of the ruler being referred to that of the glaſs; the diſtance that is obſerved between them, being partly attributed to the figure of each. But holding the glaſs even with the eye, and looking attentively, the apparent taperneſs of the ruler will ſoon vaniſh.

Fig. 97. 378. *Schol.* 5. The greateſt proportion that the breadth of the image of an object placed within a ſphere of water, can bear to the breadth of the object, is that of 2 to 1. For the image is greateſt, when the object is at the remoteſt ſurface BC. Let K be the principal focus of rays coming from without upon the ſurface AD; and

it will be, AB : AG :: [f]KB : KO :: [5]2 : 3 :: [5]AB : AF. Wherefore FA : FG :: 1 : 2 :: AD : GH. But at firſt ſight one is apt to imagine that the diameter of a round object, as a pea, is magnified much more than in the above proportion. We ſee that the whole bulk is greatly ſwelled; the apparent magnitude in this caſe, being not much leſs, than as the cube of the diameter, as on account of the diſtance between our two eyes, we ſee at once more than half the pea; and by ſo much as is ſeen, the idea of the inviſible part is readily ſuggeſted. And hence, the bulk and diameter are ſuppoſed at random, to be magnified in near the ſame proportion. We commit the ſame miſtakes in naked viſion, the reaſon indeed in both caſes being the very ſame. But if we attentively compare the breadth of the magnified pea, with the extent of two contiguous peas ſeen by naked viſion, we ſhall ſoon rectify our former miſtake.

[f] 144. I.
[5] Conſt.

If the refracting ſphere is ſolid glaſs, then GH : AD :: 3 : 1.

If the object be at the center of a ſphere of water, its diameter will be to that of its image as 3 to 4; and if the ſphere is ſolid glaſs, the proportion between them will be as 2 to 3.

379. To a ſpectator in a denſe medium, as for inſtance in water, all objects within that medium will be confuſed; and the confuſion may be ſo great, that one object cannot be diſcerned from another, as in the caſe of a very flat eye that cannot diſcern any thing without ſpectacles. The caſe will be much the ſame, if the objects are without that medium, if its refracting ſurface is flat, as that of ſmooth water.

Viſion in a denſe medium confuſed.

The reaſon of which is, that the refraction between the denſe medium, and the cornea and the aqueous humour, is leſs than between air and the cornea; and ſo the rays not converging enough at their entrance into the chryſtaline humour, are not united upon the retina, but have their focus's beyond it. If the refractive power of the denſe medium, is equal or greater than that of the cornea and aqueous humour; the rays will accordingly proceed to the chryſtaline humour either unrefracted or more diverging.

380. The beſt way for *divers* is to ſuit themſelves with glaſſes by experiment; for according as they are *long*, *common*, or *ſhort*-ſighted; or as the refractive power of their † *cornea* may be greater or leſs in proportion to that of their chryſtaline humour, they will require glaſſes of greater or leſs convexities. But, if the focal length of the cornea in our natural element, air, is known; a lens which will reſtore diſtinct viſion in a given medium, may be found by the following *prop.*

PROP.

† By the refractive power of the cornea, is properly meant that of the aqueous humour behind it.

PROP. XXXVII.

381. *The focal length of a lens in one medium, being given; to find its focal length in any other given medium.*

Let the medium whereof the lens is made, be called G, one of the mediums wherein it is placed A, and the other medium W. Let the greater and leſſer of the ſines of incidence and refraction between G and A, be reſpectively m and n; and between G and W, M and N; and the focal length of the lens in A be $=f$, and in W $=$ F. And it will be, $f : F :: \frac{n}{m-n} : \frac{N}{M-N}$ nearly.

Fig. 98, 99. 1. Let the lens be a plano-convex, with its plane ſide to the incident rays; Oa the radius of its convexity, af its focal length in the medium A, *fig.* 98; and aF its focal length in the medium W; to find which in the reſpective mediums, it will be accurately:

[a] 175. I. Oa : af :: [a] $m-n$: n; and Oa : aF :: [a] M—N : N; whence

$af = \frac{Oa \times n}{m-n}$; and $a\,F = \frac{Oa \times N}{M-N}$; and therefore,

$af : a\,F :: \frac{n}{m-n} : \frac{N}{M-N}$. *Q. E. D.*

If the lens be reverſed, its focal length in each medium will be [b] 177, & 211. I. ſtill the ſame, [b] reckoning that length the diſtance between the focus, and the neareſt focal center of the lens.

And in all lenſes, whoſe ſides are both ſpherical, if we neglect their thickneſſes as inconſiderable, it will be,

[c] 185. I. $f : F :: [c] \frac{n}{m-n} : \frac{N}{M-N}$. Therefore, *&c. Q. E. D.*

382. *Corol.* Hence if the lens is glaſs; the medium A is air, and the medium W is water; it will be $f : F :: 1 : 4$.

For $m-n : n ::$ [d] $1 : 2$; and M—N : N :: [e] $1 : 8$; whence,

[d] 35. I. [e] 302. II. $\frac{n}{m-n} : \frac{N}{M-N}$ $(:: \frac{2}{1} : \frac{8}{1} :: 2 : 8) :: 1 : 4$.

383. *Schol.* 1. Any lens being given; we may find another lens which being joined to the former, both together ſhall have a given focal length in any given medium.

Fig. 99. Let aF be the focal length required of the two combined lenſes, in the given medium. 1. Find [f] the focal length of the given lens in the given medium W; if the lens has no † focus, that is, if it tranſmits the rays unrefracted; then it is manifeſt that aF is the focal length of the lens

[f] 381. II.

† This cannot happen but when the medium of the lens has the ſame refractive power as the medium W; as is ſuppoſed to be caſe of the aqueous humour of the eye and common water.

lens required in this medium, whence its focal length in air is [f] readily found. [f] 381. II.

If the given lens has a real focus, as g; then ſay, $gF : ga :: {}^{g}ga : gG$; or if its focus is G, that is, if it tranſmits parallel rays diverging as from G; it will be, $GF : Ga :: {}^{g}Ga : GH$; [g] and aG or aH will be accordingly, the focal length in the medium W of the lens required. Whence in all caſes, its focal length in air may be found. [g] 227. I.

384. *Schol.* 2. Hence having the focal length of the *cornea* in air, we may alſo have its focal length in any other given medium; and ſo, from the preceding, ſuit ourſelves with a lens that will reſtore diſtinct viſion in that medium. A proper lens for divers.

Thus, if the focal length of the *cornea* in air is $= a$F, and its refractive power be the ſame with that of water; a lens that will procure diſtinct viſion in water, will have its focal length therein $= a$F; and if the lens is glaſs, its focal length in air will be $= {}^{h} \frac{1}{4}$AF. If the focus of the cornea a, when under water be g, or G; the focal length in water of a lens that will reſtore diſtinct viſion, will be accordingly aG, or aH. [h] 382. II.

385. *Schol.* 3. If the cornea be not concerned in the change made in the eye, for procuring diſtinct viſion at different diſtances; the lens above found will enable the eye to ſee diſtinctly in water, at all ſuch diſtances as it can in air: But if the cornea is concerned in that change of configuration of the eye, as moſt probably it is; the latitude of diſtinct viſion through the above lens in the denſe medium, will not be ſo extenſive as it is to the naked eye in air.

386. *Schol.* 4. Hence again, if a perſon under water had ſufficient light, and was exactly fitted with a lens for ſeeing an object at a given diſtance diſtinctly; we might from thence find pretty nearly the focal length of the cornea in air: And if the ſaid perſon could alſo determine, how much farther and nearer than the ſaid given diſtance, he could alſo ſee diſtinctly; we might alſo make an eſtimate of the ſhare, which the cornea has in the change of configuration of the eye. But the want of light, and of ability of ſtaying at eaſe any time under water, are difficulties that would render very precarious, the concluſions drawn from the obſervations of a perſon under theſe circumſtances.

§. 4. *Of camera-obſcura's.*

The theory in general of the *camera-obſcura* has been already explained in Book I. *art.* 243 to 246. I am here to add ſome farther particulars, not there taken notice of, with a deſcription of different contrivances for forming the *dark chamber*.

387. The pictures exhibited in the *camera-obſcura* are very entertaining; eſpecially if they repreſent objects in motion, as carriages either by land or water, people walking, the wafting of the boughs and leaves of trees, *&c.* And if all circumſtances are favourable; the pictures ſeem to appear more lively and diſtinct, than the objects do to a naked eye looking at them, from the place where the lens for forming theſe pictures or images was fixed. For the picture being diminiſhed, the rays forming it are condenſed; and ſo the colours, ſhades, *&c.* are proportionally brighter and more vivid. The picture being alſo a near object, is ſeen free from that faint hue wherewith objects without doors are commonly tinged; thoſe rays that occaſion theſe appearances, being as it were abſorbed in the picture, or becoming too feeble, after they are there diſperſed, to be perceived.

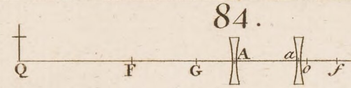
Objects at a moderate diſtance make the beſt picture.

388. All other circumſtances being alike, the pictures of objects that are near, as within 5, 10, or 20 yards, appear more vivid than thoſe of objects that are more remote. And univerſally, the pictures will be more diſtinct and pleaſant, when the objects are at ſuch moderate diſtances in proportion to the focal length of the lens, as to exhibit ſmall parts, as the features of a perſon's face, the bricks and tyles of houſes, *&c.* If the light without is favourable, and the ſpectator has been ſome time in the dark; it is ſurprizing how diſtinct and bold the pictures of ſmall objects will appear, that are diminiſhed at leaſt 20 or 30 times; and a perſon may be known, when his features are no bigger than in that proportion. The lights, ſhades and tints being here exquiſitely delicate and perfectly juſt, at the ſame time, that the picture is diminiſhed; the *relievo's* of objects are ſurprizingly bold, and when the natural ones are but moderate, thoſe in the picture ſeem to come up to the real ſize. A diſtant proſpect appears perfect enough, though it does not make ſo entertaining a picture.

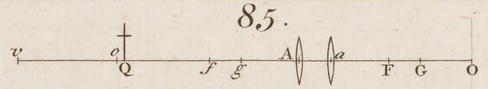
The *camera* ſhould be as dark as poſſible.

389. The room wherein the picture is formed ſhould be as dark as poſſible; that is, all light ſhould be excluded from it but what comes thro' the lens; for in proportion as the field about is darker, the objects will appear brighter, as the ſtars do in a dark night. And for this purpoſe, the lens, beſides having its aperture properly limited, ſhould

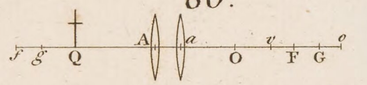
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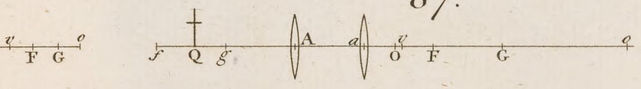
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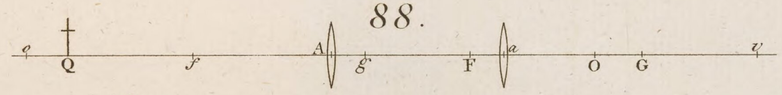
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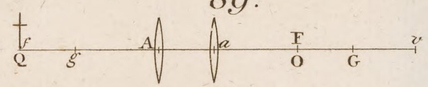
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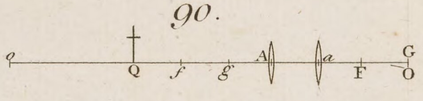
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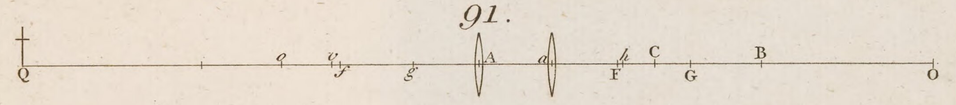
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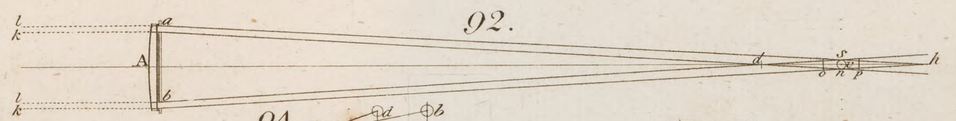
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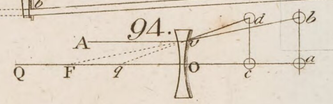
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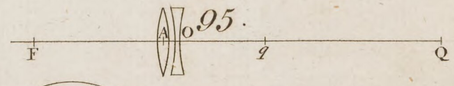
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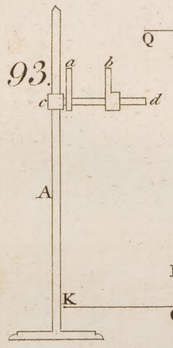
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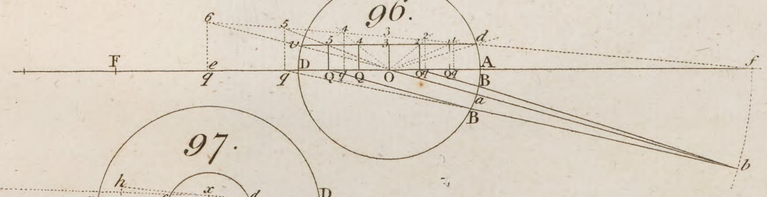
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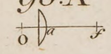


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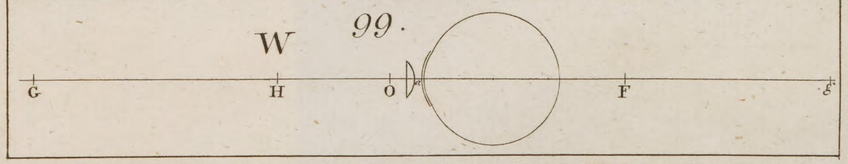
97.

98.A



W

99.



ſhould have a ſhort tube before it, ſuch as will exclude all light but what is contained in the field intended to be taken into the picture. This tube ſhould be black on the inſide; and it would do ſo much the better, if the whole inſide of the dark room was alſo black. The ſpectator himſelf, at leaſt whilſt he looks at the picture, ſhould be alſo in the dark; for by this means, the pupils of his eyes will enlarge, and as they do enlarge, the apparent brightneſs of the picture will increaſe, and being alſo free from extraneous light, the impreſſions upon the retina's will be the more vivid and ſenſible.

And the objects well illuminated.

390. The objects ſhould be alſo well enlightened; otherwiſe, notwithſtanding all the above requiſites have been taken care of, if the weather is dark, or the objects are in the ſhade, the picture will be obſcure, dull, and no ways agreeable. This phenomenon therefore of the camera-obſcura, ſhould never be attempted but in a clear day; and it is beſt of all when the ſun ſhines upon the objects from the oppoſite coaſt; that is, if the proſpect is weſtern, the phenomena will be beſt in the morning or forenoon; if eaſtern, in the afternoon; if northern, about noon. A ſouthern aſpect is the worſt of any for the camera-obſcura, becauſe the ſun is never oppoſite to it; and in this caſe, the experiment will ſucceed beſt either betimes in the morning, or late in the evening; that is, when the ſun ſhines moſt directly upon the objects. In all caſes, a great deal of light is ſtopped by lenſes, as is manifeſt from the vividneſs of the images of objects ſeen in them by reflection: And in the preſent caſe, the light impinging upon the paper, or whatever the obſtacle be at which the picture is formed, is there diſperſed, ſo as to be much rarified before it comes to the eye. And hence it is, that the *chamber* requires to be very dark, and the objects well illuminated.

Proper apertures of lenſes for the camera-obſcura.

391. The lens, uſed for this purpoſe, ſhould have a proper aperture. If the aperture be too ſmall, the picture will be dark and obſcure, and upon that account indiſtinct and unpleaſant; and if it be too large, the picture will be indiſtinct on account of the aberrations of the extream rays, and alſo becauſe the room will be too much enlightened by adventitious light entering into it; and this laſt cauſe has no ſmall ſhare in marring the pictures, as it proportionably renders them more obſcure: But when the aperture of the lens is moderate, the vitious rays on account of the aberrations are, in the preſent caſe, at their entrance into the eye very dilute and rare, and therefore ſcarce ſenſible. But the apertures here will admit of ſome latitude; and they may be a little narrower or wider, according as the objects are more or leſs illuminated; and according as they are nearer

or farther. It is beſt therefore to leave the aperture of the lens, juſt wide enough for the darkeſt weather it is to be uſed in, and to have a narrow ring to clap on occaſionally.

By making obſervations as carefully as I could, in a pretty clear day, the objects being at about 150 yards diſtance, and the ſun ſhining obliquely upon them: A lens of 18 inches focal length, I thought did beſt with an aperture of about 1 inch or $1\frac{1}{4}$ inch in diameter, according as the weather varied; and the weather being overcaſt, an aperture of $1\frac{1}{2}$ inch ſeemed to do better than a leſs: And had the objects been nearer with the ſun bright upon them, I doubt not but that an aperture of ſomewhat leſs than an inch would have been beſt. This lens when open was $2\frac{1}{4}$ inches wide, and it ſeemed then to form a picture pretty diſtinct; but on clapping to it a ring having a proper aperture, the difference was very conſiderable. The ſun ſhining upon the objects; a lens whoſe focal length was about 30 inches, required an aperture at the leaſt of $1\frac{1}{4}$ inch; and it would bear at moſt, one of about $1\frac{3}{4}$ inch. The picture was thrown into a box placed in a light room; but had the room itſelf been dark, thoſe apertures would have bore, and even required to have been, conſiderably leſs. In this laſt caſe, an aperture of $1\frac{1}{2}$ inch ſeemed to ſuit very well with a lens of eight foot focus.

From theſe obſervations, an eſtimate might be made of the proper aperture of a given lens, the object being under given circumſtances. But as the focal lengths of the lenſes are increaſed, there will be no great nicety required in ſettling theſe apertures; only I think they ſhould not be taken much wider, if any, than in the ſubduplicate proportion of thoſe lengths: And let us do what we can, the apparent brightneſs of the picture will decreaſe nearly, as the ſquare of the focal length of the lens is increaſed. For this apparent brightneſs will be nearly, as the denſity of the light in the picture, divided by the denſity of the adventitious light in the room. And whatever is the focal length of the lens, the denſity of the adventitious light in the room, will be nearly as the ſquare of the linear aperture of the lens; and to preſerve the ſame denſity of the light in the picture, the linear aperture muſt be as the focal length of the ſame lens. Wherefore, if the ſame aperture be retained, that is, if the adventitious light in the room remains the ſame; the apparent brightneſs of the picture, which in this caſe is as the denſity of the rays that form it, will be reciprocally as the ſquare of the focal length of the lens: The apparent brightneſs of the picture will be alſo in the ſaid proportion, if the denſity of the light upon it be preſerved the ſame; that is, if the

linear

linear aperture of the lens be taken as its focal length; for in this caſe, the adventitious light will be directly as the ſquare of the focal length. And in ſettling the linear apertures of different lenſes, we ſhall not much mend the matter, by taking any proportion between the ſimple and ſubduplicate of their focal lengths; the apparent brightneſs being here, I think, the chief thing to be regarded, the aberrations not being ſenſible; and this brightneſs, as near as I could eſtimate, is nearly in the proportion above aſſigned.

Limit of the focal lengths of lenſes for the camera-obſcura.

392. And hence, without ſome method of * preventing the adventitious light, from diminiſhing in ſo great a degree the apparent brightneſs of the picture; we ſhall ſoon come to a limit, beyond which it would be uſeleſs to extend the focal lengths of our lenſes. If the picture be received upon paper, a lens of about 8 foot focus, I take upon the whole to be preferable to one that is longer; this will repreſent objects in a landſkip, larger than we commonly ſee them in pictures; and the picture will be much brighter, diſtincter and pleaſanter, than if the lens had been flatter. A lens of about 8 foot focus, in the preſent caſe, has manifeſtly a very conſiderable advantage above a lens of 12 foot, and more ſtill proportionably above a lens of a longer focus. As we decreaſe in our focal length, the images will appear brighter, ſharper, and rather more diſtinct and pleaſant, till we come to a certain limit; after this, the picture will be leſs pleaſant and ſeemingly leſs diſtinct, becauſe we ſhall loſe ſome of the ſmaller parts of the objects. This limit as to the leaſt focal length, I do not take to be leſs than 2 or 3 feet; eſpecially if the picture be received upon paper within a room, for in that caſe, there will not be a convenient ſpace for the ſpectator to view it in, if the lens be ſhorter than as above. Upon the whole, I take it, that a lens from about 4 to 6 or 8 foot focal length, is the beſt ſize of any for the camera-obſcura. But in ſome caſes, a lens of a yet longer focus will do better; and in ſome others we may deſcend to ſo ſmall a length as 12 or 8 inches focus, and the picture will be pleaſant enough.

Extent of the field that may be taken in the picture.

393. In ſome caſes, the breadth of the picture may be about two-thirds of its diſtance from the lens, and even more if the paper for receiving it be made a little concave; that is, the picture may take in a field or angle of near forty degrees. But in moſt caſes when the field is ſo wide, the middle half of the picture will be diſtincter than towards the extreams; and therefore generally it is beſt not to take in a larger angle than about 20 or 30 degrees.

* How this may be partly remedied will be ſeen a little farther on.

Glaſs poliſhed only on one ſide for receiving the images.

394. If inſtead of the white paper or cloth for receiving the picture, be placed a glaſs having both its ſides ground flat and parallel, and poliſhed only on one ſide; a ſpectator behind the glaſs will ſee the images vaſtly more vivid, diſtinct and bright, than by the reflection from the paper. The rays of light in their paſſage through the rough ſurface of the glaſs, are there broke and diſperſed, after the ſame manner as they were by the paper, and ſo as to paint there as it were the images of the objects from whence they came. But here the rays tranſmitted are not ſo much diſſipated, as they are by the reflection of an opaque ſurface; and ſo the rays being denſer, the ſpectator behind receives a greater quantity in each pencil, than when he ſees the picture upon the paper *. In this caſe, the ſpectator is alſo leſs offended with extraneous light, and it would be eaſy to exclude from him all ſuch light, excepting what falls on the glaſs itſelf, and is tranſmitted through it; and hence, from theſe two concurring cauſes, we may have a good picture by a much ſhallower or a much deeper lens, than when the rays are reflected from the paper.

The phenomena farther explained.

395. It might be expected that if the flat glaſs be taken away, and a tranſparent one be placed in its room, or if nothing be placed there, we might yet ſee ſuch a picture as we did before. We ſhall indeed ſee a narrow image ſomewhere before us; but its true place will not be readily found; and when it is found, it will not be ſo well defined, diſtinct or bright, or ſo like a real object, as what we ſaw on the rough ſide of the glaſs or on the paper. In this latter caſe, the rays diverge to the eye from the rough ſurface as if that was an object, or a picture like what we ſee was painted upon it; and this ſurface being uniform and colourleſs, the rays coming to the eye ſtill retain the original colours, *&c.* of the objects.

Fig. 100. Thus, the rays PB, PA, PC; QB, QA, QC; RA, RB, RC; proceeding from the extreams and middle of a remote object, are refracted by the convex lens BAC ſo as to unite in the reſpective focuſes *p*, *q*, *r*. Now if the eye be placed behind the image *p q r*, as at O, it is manifeſt that only a very ſmall part of it can be ſeen at once; and upon this account it will ſcarce have the reſemblance of a picture. But if the glaſs GK poliſhed on one ſide be placed in the *foci p*, *q*, *r*; the rays in the ſeveral pencils inſtead of proceeding directly onwards, as repreſented at *p* and *r*, will be inflected various ways, as repreſented at *q*: So that the image *q* will be viſible on either ſide

* In conſequence, and as a fuller proof of this, the ſpectator cannot ſee ſo much of the picture at one view, as he could when it was reflected from the paper; and here he muſt move his head, ſo as to receive the rays in a nearer direction to that which they had when they reached the glaſs.

ſide the axis, within a certain limit EEE. And the rays here having undergone no other modification excepting that of being inflected, the phyſical point *q* retains ſtill the figure and colour of the original point in the object. And as the rays belonging to every point in the image, are inflected in like manner, all diverging different ways from their reſpective focus's in the glaſs; ſo to an eye ſomewhere at E, they exhibit a viſible picture, of a conſiderable breadth, as if it was really painted on the glaſs. In like manner if GK be a white paper, the rays falling on every point of it will be inflected different ways; as if they diverged from the paper, as repreſented by the rays *q* D. But, as before obſerved, there will not ſo many rays come now to the eye from each point of the image, as when they were tranſmitted through the glaſs; and for which reaſon the picture will be leſs bright and vivid.

Camera-obſcura how made.

396. The window-ſhutters of the room intended for a *camera-obſcura*, muſt be made to ſhut very cloſe; and if there be any creviſes, they ſhould be ſtopped by nailing ſlips of black cloth or leather over them. This being done a round hole is to be made in one of the ſhutters, SH, againſt a ſaſh or caſement which muſt be left open during the experiment; and cloſe to the ſaid hole, a convex lens BAC is to be fixed. The paper ſkreen or glaſs half poliſhed GK, muſt have a proper ſupport to hold it in the focus *p q r*, which focus is readily found, by moving the ſkreen to or fro till the picture is moſt diſtinct. Fig. 100.

Scioptric-ball.

397. For holding the lens, there is a little convenient apparatus to be had ready in the ſhops, called a *Scioptric-ball*, and by ſome an † *Ox-eye*. This conſiſts of a wooden ball BB, confined in a hollow zone of wood CCDD; the flat ſide of which CC, is laid cloſe to the window-ſhutter, againſt a hole made big enough to receive ſo much of the ball as projects forwards; and the whole may be confined by three buttons faſtened to the ſaſh, as *a*, *a*, *fig.* 100, made and placed ſo as to turn over the rim *a a*. The wooden zone conſiſts of two parts Fig. 101.

† This aptly enough may be conſidered as a kind of artificial eye; the wooden zone may repreſent a fruſtrum of the eye's orbit, and the voluble ball within, that of the natural eye. A lens whoſe focal length is about = E A being ſcrewed at the end E of the hole in the ball, may repreſent the cryſtaline humour; and fixing in a ſhort tube at A, a glaſs with parallel ſides and poliſhed only on one ſide, the pictures of objects will be ſeen upon it, as they are upon the retina; and to make the thing more natural, the ſides of this glaſs may have a ſphericity equal to that of the ball. If we ſucceſſively turn the lens towards objects at different diſtances, we ſhall ſee a neceſſity of a different configuration in the eye for viſion at different diſtances; and one way how diſtinct viſion may be reſtored, as by altering the length of the axis; as will appear by drawing in or out the tube at A till the picture is diſtinct upon the glaſs. If a lens having a longer or ſhorter focus be ſcrewed at E, we ſhall have the caſe of a long or ſhort-ſight perſon; and by holding a proper convex or concave lens before that at E, we ſhall ſee the effects of ſuch glaſſes in reſtoring diſtinct viſion.

parts CC, DD, ſcrewed together after the ball is placed between them, and ſo as to admit the ball to turn eaſy round its center. The ball has a large cilindrical hole, at each end of which is cut a female ſcrew, for receiving a lens as A fixed in its cell. By the motion of the ball, the axis of the lens may be turned different ways; and the ſphericity of the zone, prevents any light from being tranſmitted betwixt it and the ball. There are commonly two lenſes, of different focal lengths, fitted to the ball; and by uſing them both at once, there will be in effect a third, having a ſhorter focus than either. When one lens only is uſed, it ſhould be fixed to the end A, that ſo the ſhort tube may exclude ſome of the extraneous light out of the room.

Picture may be ſeen out of another room.

398. If the picture upon the glaſs half poliſhed GK, be at a moderately ſmall diſtance from the partition, dividing the dark room from an adjacent one; a perſon in the adjacent light room, may ſee this picture very well through a hole oppoſite to it in that partition. This hole, which may be ſhut cloſe at other times by two folding-doors black on their inſide, and covered with a picture-frame, or &c. ſhould be big enough to receive part of the perſon's head, ſo that when he looks he may be quite in the dark; the little folding-doors contributing much to this end, and ſomething black may be alſo placed over them. The picture may be thrown to either ſide of the dark room, by a plane ſpeculum properly placed, as may ſuit beſt with reſpect to an adjacent light room. A contrivance of this kind may be entertaining, and the more ſo, as the phenomena will be leſs expected than in the dark room.

How to ſet the picture upright.

399. An inverted picture does not look near ſo pleaſant as an erect one, eſpecially if it repreſents live animals; and a perſon cannot be known near ſo readily in the inverted picture, as after the ſame picture is made upright; though in the former caſe, the images be much brighter and ſharper than in the latter.

If a perſon facing the picture, whether it be upon paper or on the glaſs half poliſhed, looks into a looking-glaſs held before him with its face towards the picture and inclining downwards; the images ſeen in the glaſs will be erect, and in their due poſition as to right and left if the picture be on an opaque ſurface, otherwiſe they will be inverted. In this caſe the looking-glaſs ſhould be pretty large; but as ſuch are cumberſome and ſeldom true, it will be more convenient to place a ſmall plane ſpeculum near the lens, ſo as to inflect the rays before they form the picture. In the annexed figures, PA, QA, RA, are the axes of pencils of light coming from an object without doors, which after paſſing through the convex lens A, are reflected by the plane

Fig. 102 to 105.

plane ſpeculum *s t*, into the lines *s p*, *v q*, *t r*; and the *foci* of theſe pencils being at *p*, *q*, *r*, an image of the object is formed upon the plane G K. In this caſe, the lens and the plane ſpeculum may be both placed in a ſhort ſquare tube, open only at the furtheſt end, and on the ſide facing the ſpeculum. By this means, the room will be much darker, and the picture proportionably livelier, than when the light is tranſmitted directly into it through the lens. The ſpeculum may be ſo fitted in its box, as it may be eaſily inclined to any angle; and being but ſmall it may be made of ‖ metal, which reflects abundantly more light than glaſs; or if it be glaſs, its ſides ſhould be exactly flat and parallel. The tube may ſlip into a ſquare box made in a board fitted to the place made for the ſcioptric-ball; or the tube might be round and fitted to the hole in that ball.

400. In all caſes, the reflecting plane ſhould be ſo inclined as to reflect the principal pencil of rays, or that whoſe axis is coincident with the axis of the lens, nearly at right angles to the plane GK; for by this means, the focus of all the pencils will be nearly in that plane, which is perpendicular to the axis *v q*, and ſo the whole picture will be diſtinct. In *fig.* 102, 103, the reflecting plane makes half a right-angle with the axis QA; and ſo the rays coming along that axis are thrown directly upwards or downwards. In the one caſe the picture is formed upon the cieling, or on a paper parallel to it; and in the other caſe upon a paper on or parallel to the floor; and in both theſe caſes the tube muſt reach a little way into the room. The firſt method is inconvenient, as it is aukward and troubleſome to look directly upwards; and the ſecond method will be alſo inconvenient, unleſs the lens and ſpeculum can be placed high enough, and far enough into the room; to admit the picture to be about the height of a table above the floor, and to give room to the ſpectator to ſtand or ſit betwixt it and the window. If this be done, the picture will lie very convenient for copying; and in this caſe, the figures will be alſo in their due poſition as to right and left. In *fig.* 102, by placing another plane ſpeculum *x* above *s t*, and making an angle with it a little † obtuſe; the picture may be ſhown on a white plane *g k*, inclining a little from the window. The two ſpeculums and the lens might be fitted in the ſame box; and the picture thus formed would be very pleaſant, if both the

‖ That a hard whitiſh metal well poliſhed, reflects abundantly more light than quick-ſilvered glaſs, ſeems to me very manifeſt; and of this Mr. *Short*, who has had much experience in theſe matters, is very certain. The contrary opinion, which for a long time prevailed, muſt have aroſe from the compariſons having been made with improper metals.

† The angle ſhould be a little obtuſe, that the picture may be a little higher than the ſpeculum.

the ſpeculums are good, and the objects without are well illuminated by the ſun; otherwiſe it will be dark, becauſe of the loſs of light by the double reflection.

Fig. 104. 401. In the annexed figure, the paper-ſkreen GK is ſuſpended obliquely from the cieling. This method gives a very agreeable picture, as the images are not only erect, but alſo in their due poſition as to right and left.

A ſkreen for optical uſes. A large ſkreen would be uſeful here, and alſo for ſome other optical purpoſes; and it would be alſo convenient to have it portable. The ſkreen may be made of any ſize by ſheets of white paper paſted ſmoothly on cloth, to which ſhould be fixed a ledge and roller, as is commonly done to large maps. The two ſupporters I had made were about $5\frac{1}{2}$ feet high, and in the top of each was a round pin drawn tightly, by a wedge thro' a mortiſe in the end of it, againſt a ſhoulder on the other ſide; at the other and inner end of each pin was a groove, in which ſlid a rail of about ſix foot long. Between theſe two rails was fixed occaſionally the rail of the ſkreen, being there confined by two wedges going through the ends of it; and in the bottoms of the ſaid ſide-rails, the roller turned ſtiffly for bringing the ſkreen ſmooth, being drawn tight in the holes by two wedges as above. Through the groove in the inner end of each of the center-pins, and through one of various holes made in the ſide-rails, went a ſmall peg by which the ſkreen could be ſuſpended at various heights; and by turning the ſaid center-pins, it was eaſy to give the ſkreen any deſired inclination. The whole was eaſily put together, or taken aſunder, ſo as to take but little room when out of uſe.

Fig. 105. 402. In this figure, GK is a glaſs poliſhed only on one ſide, which ſhould be placed rather higher than a perſon's head, that he may ſee the picture by rays coming through the glaſs, pretty directly. The ſpace between KG and the lens ſhould be incloſed on all ſides, ſo that no light can come into the room but through the flat glaſs GK. By this means, we ſhall have the moſt lively picture that can be exhibited in the camera-obſcura, tho' it will be inverted as to right and left; and perſons at a moderate diſtance without, may be known as well as if we looked at them through the window. To put this method in practice in a common room; the lens, plane ſpeculum, and the flat glaſs, ſhould be fixed in a kind of priſmatical box, black on the inſide, which may be readily hooked againſt the upper part of the window, ſo that the tube containing the lens fits to a hole made in the ſhutter to receive it; the diſtance between the lens and GK being before

before adjuſted, according to the diſtances of the objects that are to be taken in, and the upper window-ſaſh let down. If the objects are lower than the window, the plane ſpeculum *svt* need be but ſhort. Thoſe who have a ſpare room or cloſet facing a good near proſpect, by letting ſuch an apparatus ſtand in it, might at any time agreeably entertain themſelves, by only ſhutting the door upon them; there being ſomething more entertaining in ſuch a picture as this, than in ſeeing the real life †. Either another window or the bottom of this, might ſerve to enlighten the room at other times.

Picture by a concave ſpeculum.

403. The entertainment which the *camera-obſcura* affords to all claſſes of people, is what led me to be ſo particular in its deſcription ‡. And before I diſmiſs this ſubject, I ſhall juſt obſerve, that if a concave ſpeculum be placed againſt a ſmall hole in the window-ſhutter of a dark room; a picture, like that we have been deſcribing, will be formed on a ſheet of paper placed near the principal focus of the ſpeculum.

Portable *camera's*.

404. One conſiderable uſe that the *camera-obſcura* has been applied to, is in aſſiſting young ‖ artiſts to draw in perſpective; and for this purpoſe different ſorts of portable boxes have been contrived, the moſt common of which is that I am now going to deſcribe. But firſt let it be obſerved once for all, that all boxes made for this purpoſe ſhould be cloſe, and black within; and that the ſpectator alſo when he looks, ſhould be ſkreened from the light as much as poſſible.

The annexed figure, repreſents a vertical ſection of the box through the middle: ST is a looking-glaſs received in grooves on each ſide of the box; and making half a right-angle with the axis Q A *b* of the convex lens A, fixed in the ſhort and ſliding tube *d g f e*. GK is a glaſs poliſhed Fig. 106.

† A compleat thing of this kind, is a little room built by Dr. *Hoadly* at *Chelſea*. The roof is pyramidical like that of a ſummer-houſe; and out of the ſides of it are projections like thoſe of dormer lights or windows; in each front is placed a lens with a plane metallic ſpeculum fixed at the proper angle for throwing the picture upon a large glaſs half poliſhed, fixed againſt the cavity made by the ſaid projection cloſe to the cieling: There is another apparatus of the ſame kind at top, the lens and little ſpeculum being made to turn all round; ſo that there are five pictures in the room at once, and which give ſufficient light for reading a ſmall print.

‡ *J. Baptiſta Porta* is, I believe, the firſt author that takes notice of the inverted pictures of external objects formed upon the wall of a dark room, by rays coming through a ſmall hole in the oppoſite ſide. Theſe phenomena he deſcribes at large in his *Magia Naturalis, lib.* xvii. *cap.* 6; and alſo ſhews how theſe pictures will be improved by a convex lens, and made erect by a plane ſpeculum, *&c.* This work conſiſting of twenty books, Mr. *Molyneux* quotes as being printed in 1589; but *Hartley's Catalogus Univerſalis* places the firſt edition at *Francford* in 1591. *P. Gaſpar. Schottus* ſays that four of theſe books were publiſhed at *Antwerp* in 1560, but theſe books are on ſubjects no ways relative to the paſſages here referred to.

‖ It is certain that the camera-obſcura is very uſeful for this purpoſe, and alſo that in many caſes it exhibits what may be deemed a fine picture; but how far it may be uſeful to a painter is another conſideration, and what is beſide my preſent purpoſe to diſcuſs.

poliſhed only on one ſide for receiving the image *p q r*, formed by rays which before their reflection by the ſpeculum ST, tended to form an image *a b c*. LM is the lid turned upwards, and which now ſerves to ſcreen the picture and the ſpectator from the light, the better to effect which, there are alſo folding leaves on each ſide; ſo that the whole ſpace LMNB above the box is incloſed, excepting in the front NB and the top MN. The two glaſſes when out of uſe, are put into a drawer ſliding in the cavity EPCD, with ſomething ſoft under and between them to prevent their breaking in carriage. Alſo the pyramidical trunk EFHI is uſually made to hook on and off, ſo as when out of uſe to be placed in the box. But this ſeems to be needleſs.

Another box.

405. As it is difficult to have a large looking-glaſs that is true, a ſmall one may be made to anſwer the purpoſe; as in *fig.* 102, G*ab*K may repreſent the ſection of a box, at the bottom of which is fixed at the proper angle the plane ſpeculum *s v t*; the lens A is moveable along with a ſhort tube or drawer to the proper diſtance, and GK receives the picture as in the preceding.

How far the tube need ſlide.

406. In all caſes, the tube carrying the lens ſhould have a ſmooth and ſteady motion; to effect which, the principal bearings ſhould ſpring againſt it, and it would be beſt to have the tube of braſs. The tube need not have a motion of above a $\frac{1}{4}$ part of the focal length of the lens, at moſt; and for common purpoſes it need not ſlide near ſo much. Matters ſhould be ſo ordered, that when the tube is puſhed as far in as it can, the paper or glaſs for receiving the images, ſhould be then in the principal focus of the lens; and objects that are beyond the other focus of the lens a 1000, 100, 50, 10, *&c.* times the focal length of the lens, will have their pictures in the box diſtinct, when the tube is drawn out, the $\frac{1}{1000}$, $\frac{1}{100}$, $\frac{1}{50}$, $\frac{1}{10}$, *&c.* part of that focal length. If the pictures of objects that are nearer than about 6 or 8 times the focal length of our lens are wanted, it will be beſt to uſe another lens of a ſhorter focus. By this means the ſhort tube will ſtill ſerve; for it would be inconvenient here to have the tube long, as it muſt be proportionably broad.

Fig. 106.

407. The rough ſide of the glaſs GK is uſually placed uppermoſt, for the ſake of copying with a pencil the picture upon it; and ſometimes inſtead of the glaſs, is uſed oiled paper ſtrained in a frame, for the ſame purpoſe.

If the board BP be made to ſlide out, and the glaſs GK be placed in its ſtead; the ſpeculum ST being taken away, the lid LM ſhut, and the glaſs GK ſcreened from the light coming to it from without;

out; we ſhall have an inverted picture much ſharper and diſtincter than the former, and alſo much properer for the imitation of an artiſt, as the images here are duly ſituated notwithſtanding their being upſide down †. A draftſman, I ſhould think, would make no other uſe of the *camera*, than to aſſiſt his imagination in drawing upon another paper he has before him. But when the picture is drawn upon the rough ſide of the glaſs or on the oiled paper, as it muſt be again copied, the error cauſed by the looking-glaſs may be then rectified. But for the purpoſe of drawing, the following conſtruction of a box ſeems to be much more convenient.

408. The annexed figure is the box *fig.* 106, ſet upright upon the bottom, neither of the glaſſes GK or ST being here uſed. Inſtead of ST is here uſed the ſmall ſpeculum *s t*, which reflects the rays coming through the hole or opening *m n*, directly upon the lens A; and which are there refracted, ſo as to form a picture upon paper placed at the bottom BC of the box. To a ſpectator with his back towards the objects, this picture will be erect, and the images in their due poſition as to right and left. For viewing the picture, a ſlit may be made at I, which may be covered occaſionally by two ſlips of boards ſliding to each ſide: And for drawing, the front BL, which before was the upper part, muſt be opened; the lid ML ſupported by the folding leaves on the ſides, ſerving here alſo to ſcreen the artiſt from the light, and which with the addition of a piece of looſe black cloth thrown over his ſhoulders, will keep it out effectually. The opening *m n* is eaſily encloſed, when the tube is uſed as in *fig.* 106. Fig. 107. A proper box for drawing.

409. If the ſpeculum *s t* is a good one, it is indifferent whether the rays from the objects come to it, or the lens firſt; but if it be faulty, it will do leſs harm when placed after the lens, as repreſented in *fig.* 108; and therefore this laſt conſtruction, for the purpoſe of a *camera obſcura*, is always to be preferred.

410. The annexed figure is a larger box, made upon the ſame principles, and anſwering the ſame purpoſes, with the laſt deſcribed. The bottom part of it is the [a] *ſhew-box* for holding pictures before the concave ſpeculum; and the top of it is the box, *fig.* 106, ſet with the face LB downwards, the lid LMN being now taken away. Or it will anſwer the ſame purpoſe, if the ſmall box be ſet upright, as in *fig.* 107, the bottom BC being taken away. I had the little box deſignedly made Fig. 109.

[a] 286. II.

† For this purpoſe of copying, the paper on which the picture is received, may be divided into ſquares, either by croſs lines or by a frame with croſs threads or wiers laid on occaſionally: This frame would ſerve for dividing the glaſs half poliſhed, if it be uſed inſtead of the paper; and the paper on which the copy is to be made being divided into ſquares in like manner, and theſe numbered; in either caſe it will be eaſy to take a correct copy of the principal lines, and the whole may be afterwards finiſhed at leiſure.

a cube, that it might fit on either way. By either of theſe methods, we ſhall have a large picture on a ſheet of paper placed at the bottom of the box, and which will appear very diſtinct and erect through an opening at LM, or thorough the front MN: The ſaid opening being ſhut, and the folding doors before [a] deſcribed laid open; a piece of looſe black cloth thrown over the tops of the doors and the ſpectator's ſhoulders, he may eaſily gather about him, ſo as to exclude all light from the box; and in this poſition fall to work with his pencil. If there be creviſes left betwixt the box and the edges of the doors when they are open, there ſhould be thin ſlips of black leather nailed over them. A box made only for this purpoſe, might ſpread wider towards the bottom; but as the doors in that caſe cannot open ſo conveniently, I think this figure is preferable.

[a] 286. II.

To make the boxes more portable.

411. There may be various methods of putting boxes together, ſo as to be reducible into a leſs bulk for the conveniency of carriage. There is one which has obtained in our ſhops for many years, called a *Book-camera*, from its being made to fold exactly in the ſhape of a folio book; and to make a kind of *rebus*, it is bound with leather and lettered on the back, in the ſame manner †. To erect it into a box, one of the lids or cover of the book ſerves for the back, and the front makes part of the top, to which is joined with hinges a board having its form ſemicircular, and reaching beyond the bottom; to this top is fixed black leather or cloth to hang over the draftman's ſhoulders when he draws, and which ſerves as a front to the box: Two boards faſtened by hinges to the ends, turn up and form the ſides and alſo ſupport the top; and by way of a tube for holding the lens, there is faſtened to the top a leathern collar, which may be raiſed or depreſſed, by an arm coming from the frame of the lens to an upright board or bar ſliding in one of the ſides; the plane ſpeculum is fixed in a frame ſo as to be ſet to any angle, and turning round the lens to different aſpects. This box is founded upon the ſame principles with that *fig.* 107, the paper for receiving the picture being placed at the bottom; and the whole contrivance is not amiſs, if care be taken that the parts are ſo well fitted and proportioned, that the box is ſteady when in uſe. But for home purpoſes, one would prefer a box remaining faſt together.

Book-camera.

Another portable camera.

412. In contriving things for carriage, the weight as well as the bulk ſhould be reduced as ſmall as poſſible, and at the ſame time there ſhould be as little trouble as may be in fitting the machine for uſe; and

† Inſtead of this leathern cover, a naked box would be better, ſo that it might be ſent by a carrier without other package.

and when in uſe it ſhould be alſo of a convenient height, if it may be, without a foreign ſupport. I do not know a better method for obtaining all theſe ends in a portable *camera-obſcura*, than the following *. In the annexed figure BD is the ſection of a ſquare board, to Fig. 110. each corner of which is faſtened a ſtiff braſs joint, wherein is fixed a round ſtaff, of about the length and ſize of thoſe uſed to ſupport ſurveying inſtruments, and which like them conſiſt of two parts ſcrewed together. The board BD is of a ſize juſt big enough to admit a proper tube for carrying the lens A and reflecting plane *s t*, and which is made to ſlide ſmooth in another ſhort and ſpringing tube fixed in the ſaid hole; or inſtead of theſe tubes may be uſed a leathern collar, like that to the book camera: Or rather, the lens ſhould be placed firſt, as in *fig.* 108. The oblong board GK, which may conſiſt of 2 or 3 pieces joined together either by hinges or pins, may have holes in the corners through which the legs at the joints may ſcrew againſt ſhoulders left in the braſs part for that purpoſe; or the legs may be eaſily faſtened to the board by other methods, but the above I think is the beſt, as it will be a great ſtiffening to the whole. The legs as low as the board GK, on which the paper for receiving the picture is placed, is covered with a black leathern conical bag; which on the back and two ſides may be hooked under the board GK to keep it tight; but on the fore part, the bag muſt be wider and hang lower, ſo as to admit a perſon's head and arm, and cover his ſhoulders ſufficiently to exclude extraneous light from the picture.

If the machine has four legs, one of them ſhould be made ſhorter than the reſt, and have a ſliding foot which may be brought to ſtand faſt upon any uneven floor. Or three legs might be made to anſwer the purpoſe here; and in either caſe, this ſtand might ſerve to carry a ſerveying or levelling inſtrument, *&c.*

413. To a perſon who is accuſtomed to draw by his eye, a camera of about 6 or 8 inches long and 2 or 3 inches wide, which he can eaſily put in his pocket, may perhaps anſwer all his purpoſes †. This may be made to be uſed like *fig.* 106, and with or without the plane ſpeculum S T. But a painter will uſe the *camera obſcura* with diſcretion; and there are many caſes wherein a convex ſpeculum, or a concave lens, may perhaps anſwer his purpoſe much better, Pocket *camera.*

414.

* I had an intimation of this contrivance ſome years ſince by Mr. *Dugal Campbel*, one of his Majeſty's engineers. But one that he got made being much too ſlight, did not anſwer.

† Theſe ſmall *camera's* neatly and conveniently made, are now to be had at our ſhops. They have been made with a lens of about $1\frac{1}{2}$ or 2 inches focus, and fitted to the heads of canes; but theſe are much too ſmall to anſwer any purpoſe.

Shew-boxes.

414. Any camera-obſcura, wherein the picture is received upon paper at the bottom, may be eaſily converted into a *ſhew-box*, or one for improving prints. And it is pity that the common ſhew-boxes, are not made to anſwer both theſe purpoſes. The ſame lens may ſerve for both; but in the one caſe it muſt have a limited aperture, and in the other, the lens ſhould be broad enough to admit both eyes to ſee, through it, the print or picture at once. All that is required for obtaining theſe two ends, is to have either an aperture, or two lenſes with their proper apertures, to be ſhifted on or off occaſionally.

Fig. 107, 109.

The print to be viewed is to be placed at the bottom of the box with its top inwards, where it is enlightened thro' the front, now left open, either by day or candle-light; but it ſhews much more natural and pleaſant by day-light, as we are not accuſtomed to ſee proſpects by any other: The opening *m n* in the tube *fig.* 107, or the top box *fig.* 109, are to be turned the contrary way to what they are in theſe figures, that the box may ſtand between the ſpectator and the light. For the ſake of ſuiting different eyes, the tube ſhould have a little motion, that the lens may be brought either nigher to or further from the bottom than its focal length. But thoſe who are too ſhort or too long ſighted to ſee diſtant objects without ſpectacles, they muſt uſe the ſame here. The box ſhould be big enough to take in a large print or a book of prints; and it would be of ſome advantage, if two thin blacked boards be faſtened by hinges to the ſides near the top, to be ſet to the margins of the print; ſo as to limit the ſight, as much as may be, to a view of the print only.

In a ſhew-box, it is beſt to place the plane mirror outermoſt, as in *fig.* 107; becauſe any faults in that mirror will be leaſt prejudicial, when it receives the rays after they are refracted by the lens. The

[b] 350. II.

field of viſion will be the [b] more extenſive, the nearer the eye is to the lens; but the box *fig.* 107 will exhibit a print ſufficiently large; and therefore upon the whole, that conſtruction as to the placing of the lens and the looking-glaſs, is the beſt of the two for this purpoſe of viewing prints.

Inconvenience of reverſing the pictures, how to be remedied.

415. I have above, referred only to thoſe boxes which had been before deſcribed for other purpoſes, for this alſo of improving prints. But theſe have this grievous inconvenience, that the plane looking-glaſs reverſes the print or picture as to right and left: The remedy is indeed obvious enough; another looking-glaſs will ſet all to rights, by reverſing the reverſion made by the firſt. The looking-glaſſes may have different poſitions; but in all caſes of this nature, it is beſt to incline a plane ſpeculum ſo as to make about half a right angle with the

the principal pencils. This diſpoſition of a ſpeculum will reflect more light than when it ſtands more oblique, and there will accrue the leſs damage from the imperfections of its figure. Let one of the plane mirrors be placed like TS *fig.* 109, and let the lens A be placed nigher to it, as in the line EL; then a plane looking-glaſs placed upright before the lens A, and inclined ſo as to make about half a right angle with its axis, will exhibit the picture in its due poſition †. By having the looking-glaſs before the lens to ſhift in and out occaſionally, it will be very eaſy to adapt a *camera* like that *fig.* 109, in the manner and for the purpoſe laſt deſcribed. A box of this ſort is now got into our ſhops, and was firſt made by Mr. *Nairne*, optical inſtrument-maker in *Cornhill*, in the year 1753. But this contrivance has its inconveniencies. The field of viſion is rather too much contracted: From the almoſt unavoidable irregularities of the figures of looking-glaſſes, an indiſtinctneſs ariſes; and from the loſs of light at their ſurfaces, the picture, unleſs it be ſtrongly illuminated becomes too dark and obſcure; ſo that upon the whole it does not appear ſo diſtinct and pleaſant as with only one looking-glaſs.

In the *Gentleman's Magazine* for *April* 1753, we have the deſcription of a box for improving prints by Mr. S. PARRAT of *Killerton*, near *Kirby-Lonſdale* in *Weſtmoreland*. And I am apt to think, that the ſhew-box which one Mr. *Johnſon* from *Berwick* had laſt ſpring at *Charing-croſs*, was much of the ſame ſort. Having already dwelt long upon this ſubject, I might perhaps have contented myſelf with only referring to Mr. *Parrat*'s own account of his box, if I did not think the following conſtruction to be more advantageous.

A new ſhew-box.

416. The box I got made is 16 inches long, 18 inches wide, and 24 inches deep. Within this box, and parallel to the top and bottom, is fixed a partition leaving a cavity above about 10½ inches deep; into this cavity ſlips another box of the ſame length with the outer one, ſo that both together may draw out to the length of about 27 inches, which is the proper diſtance betwixt my lens and the print to be viewed. The lens is fixed in the end of this inner box, a little nearer the bottom than the top, having behind it an opening about 4 inches wide and 1½ inch deep. The lens is included within a ſhort tube, blacked on the inſide, and of ſuch ſize as to hide all the box, excepting the print itſelf at the farther end; and for confining the view to prints of different widths, the ſides of this tube or box have an angular motion. To prevent the inner box from dropping when drawn out,

† Or, the lens might be placed firſt, as in *fig.* 107, and the two ſpeculums, inclined in the manner above deſcribed, behind the lens. This diſpoſition of the lens and ſpeculums, will give a diſtincter picture than the other, but the field of viſion will be more contracted.

out, its lower end reaches down to the table, ſo as always to ſupport it level. The farther end of the outward box, againſt which the prints are placed, is fixed to a board ſliding ſmoothly upon the bottom above-mentioned, ſo as it may be drawn out far enough to take in a large book of prints; and that it may be ſupported, this end alſo reaches down to the table, being ſtrengthened by two brackets fixed to it below and to the ſliding board. At the ends of the two ſides, are fixed two ſlips or cheeks about $\frac{3}{4}$ of an inch wide, for confining the prints or a book betwixt them and the ſliding end; theſe ſlips ſlide in grooves made in a ſmall bar fixed acroſs the top, and another lying over the ſliding board, ſo as to hide all except the print itſelf; an advantage not to be ſlighted ‖. For illuminating the prints, about half the top turns up ſtiffly upon two pegs, ſo as to ſtand at any angle, and when upright it drops $1\frac{1}{4}$ inch within the box; the under-ſide of this lid is covered with white paper for reflecting light upon the print, which together with that falling directly upon it through the aperture now made, will illumine it ſufficiently. The whole together is, I think, very convenient for the purpoſe, and has a better effect than any I have yet ſeen. The box was made ſo deep as above deſcribed, that when placed upon a common table, a perſon ſitting upon a chair might look into it at his eaſe, without much ſtooping. The bottom part, which is every way incloſed and having a door on one ſide, is big enough to hold ſeveral hundreds of prints; and for the ſake of diſtinguiſhing them into ſeveral ſortments, there are ſeveral ſliding ſhelves which will draw out at the door, which is about $2\frac{1}{2}$ inches narrower than the ſide, to make room for the brackets before mentioned for ſtrengthening the ſliding end. This box is blacked on the inſide, and is eaſily converted into a *camera-obſcura*.

A box of a ſmall ſize.

417. If the bottom part be omitted, the box above deſcribed will be reduced to about half the ſize; and a proper caſe or cheſt for holding prints, may now be kept in the inner box. This box, but with ſome diſadvantage, may be yet reduced into a much ſmaller ſize: Let the inner box before deſcribed, be made no deeper than you would have a drawer for containing prints, and let the outer box be made juſt big enough to contain this drawer. Let the lid of the caſe, excepting a ſlip of about 2 inches wide at the front which ſhould remain fixed to the ſides, turn upon joints, and † be ſupported upright,

‖ Theſe ſliding cheeks have fixed to them, and alſo to the box, ſlips of black leather, which will allow them to move in the whole 5 or 6 inches.

† The method I uſed was by two joint rules having their ends turned upon pins fixed to this lid and the ſides, and ſo as to lie within the thickneſs of theſe ſides. This method may

upright, which may be done various ways. Againſt a proper aperture made in this lid, the lens is to be fixed; or it may be fixed in a board ſliding to different heights, that it may be the better adapted to prints of different ſizes.

The drawer alſo has a lid, which turns upright upon pegs made within about 1½ or 2 inches of the farther or lower end, which end when the lid is upright bears againſt a ledge or falſe end fixed acroſs the drawer; and in this poſition the lid is faſtened by a button in the ſaid ledge. Againſt this lid the prints or books with prints are to be placed, the bottom of a book bearing upon the above-mentioned ledge; and the ſides and top being all free from obſtacles, a book may be here placed of any ſize. The prints ſhould be ſupported at top; for this purpoſe, let there be two narrow boards ſliding upright in grooves on the back of the lid, and to the tops of theſe ſliders, let there be fixed a narrow tranſom the whole width of the lid: By a ſpring, ſcrew, or a wedge bearing againſt one of theſe ſliders, the tranſom may be ſupported at various heights for holding prints of various depths. Along this tranſom, let there be a thin ſlip either ſpringing, or turning ſtiffly, againſt it for holding the prints; this ſlip ſhould turn upon joints, and ſo as to be eaſily opened, which may be effected by different methods. Within the upper half of the tranſom, let there be two ſlips ſliding to each ſide, the fore-ſide of their outer ends being in the ſame plane with the fore-ſide of the lid or tranſom; at theſe ends let there be wooden ſlips drawn cloſe to them by ſprings, for faſtening the edge of a print. By this means, a very long or very deep print may be held ſtraight at the top; and, were it neceſſary, it might be alſo ſupported at bottom, by two ſliders drawing out to the ſides, as above; and having grooves in them, or by two ſlips turning upon joints. For confining books of different thickneſſes; there is a narrow board or falſe end, to be placed occaſionally into different correſpondent grooves, made a little aſunder in the ſides of the drawer a little before the lid; and ſo as the top of the ſaid end, when there placed, may ſtand about an inch or better above the ſides of the drawer. The light is now to come upon the print ſideways; and having placed the lens and print at their due diſtance aſunder, move back the table againſt the pier, or ſhut ſo much of the window till you have the proper degree of light; too much light being here very prejudicial, and there is but little danger of having too little.

This little apparatus, when out of uſe, may be placed as a drawer in a cheſt or under a table; and it will anſwer extremely well, if due care

may be uſeful in many caſes, where one thing is to be held at a given angle to another.

care be taken as to the proper degree of light upon the prints: It has one conſiderable advantage above the box deſcribed in the preceding, unleſs that be made very big, the taking in of very large prints. But the excluding of all objects from the ſight, excepting the print itſelf, and preventing the glare incident to near and bright objects, are advantages not eaſily counter-ballanced.

Having a lens fixed on a pedeſtal ſtanding on a table, it would be eaſy to ſupport prints againſt a wainſcot having a proper light, ſo as to exhibit the phenomenon we have been conſidering, without the aid of any box.

The concluſion.

418. I ſhall now conclude this ſubject with the following obſervations:

1. In all caſes, a lens for improving prints, ſhould be broad enough to admit both eyes to ſee through it at once, or about 4 or 4 ¼ inches wide. This enables us the better to judge of the greater diſtance of the image of the picture, on which the effect greatly depends, as in like caſes of naked viſion.

2. For common ſized prints, which are about 14 or 16 inches wide, a lens of about 24 or 26 inches focal length, will beſt anſwer the purpoſe. If the lens be much more or much leſs convex, the prints will accordingly either appear coloured and diſtorted towards the edges, or the effect of the lens will not be ſufficiently ſenſible. If the prints are much ſmaller or much larger, the lens ſhould accordingly have a much ſhorter or longer focal length. So that to a given lens, the prints ſhould be nearly of a ſize, and alſo done nearly on the ſame ſcale.

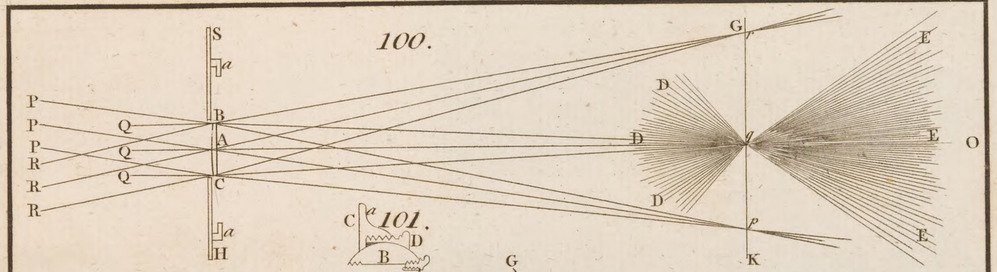
3. The prints ſhould be moderately and properly enlightened, and ſo as not to have any glare, as near objects are apt to have. Upon this the phenomena greatly depends; the degree of light received from remote objects, ſuch as are repreſented in the print, is uſually very moderate. And it may be laid down as a general rule in optics, that a ſpectator ſhould be ſkreened, as much as may be, from all light but what he receives from the object he looks at.

In the caſe before us, the lens increaſes the viſual angle but very little, and yet the print appears greatly magnified, inſomuch that the objects repreſented may appear of their natural ſize. The principal cauſes of this phenomenon have been already explained*: And it may be further obſerved, that the caſe here ſeems ſomewhat analogous to that of viewing objects at different diſtances by the naked ſight: Theſe appearing ſtill of the ſame magnitude, whilſt the firſt diſtance or viſual angle is varied a great number of times; and to ſome one of thoſe ſtations, the circumſtances of the print may be exactly correſpondent, with perhaps the addition of ſome favourable advantages.

* 317. II.

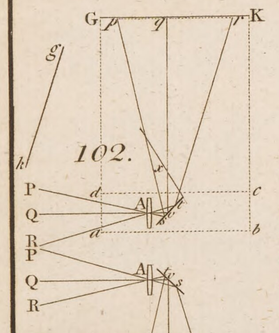
FINIS.

100.



101.



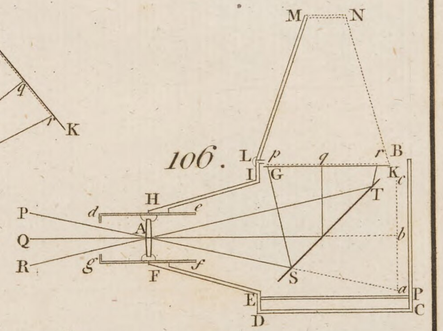


102.

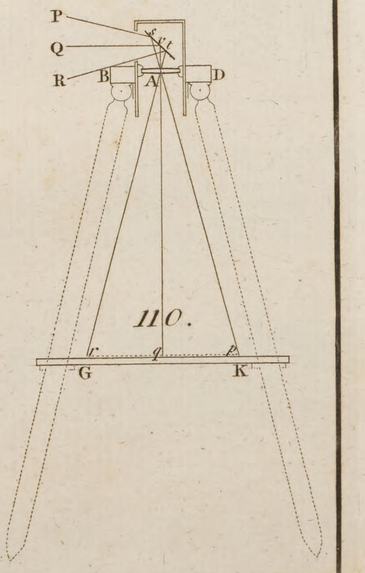
103.

104.

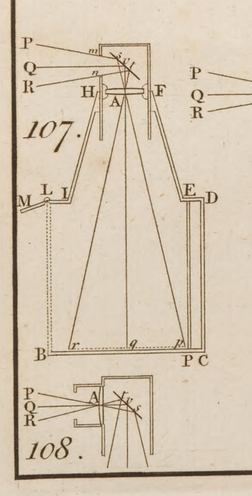
105.



106.

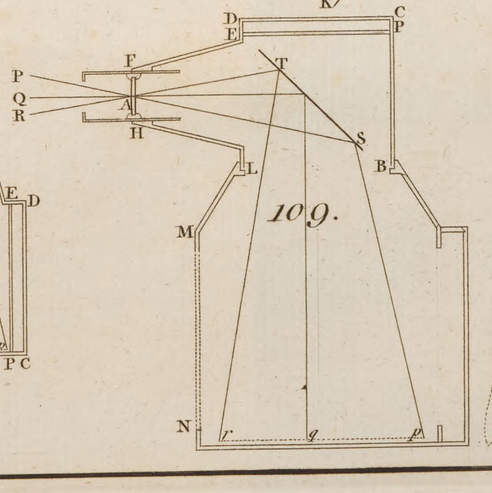


110.



107.

108.



109.

BSB Bayerische StaatsBibliothek
MDZ Münchener DigitalisierungsZentrum Digitale Bibliothek

Harris, Joseph

A treatise of optics containing elements of the science ; in two books

London 1775
4 Phys.sp. 116 u
urn:nbn:de:bvb:12-bsb10058420-5